ECE 301 - Lecture #10 (1)

Exam 1 - Monday Feb. 11

2 Problems Similar to HW1/Q1

2 Problems Similar to HW2/Q2

2 Problems Similar to notes (until today)

Each is 25 pts.

and the Max is 125 pts.

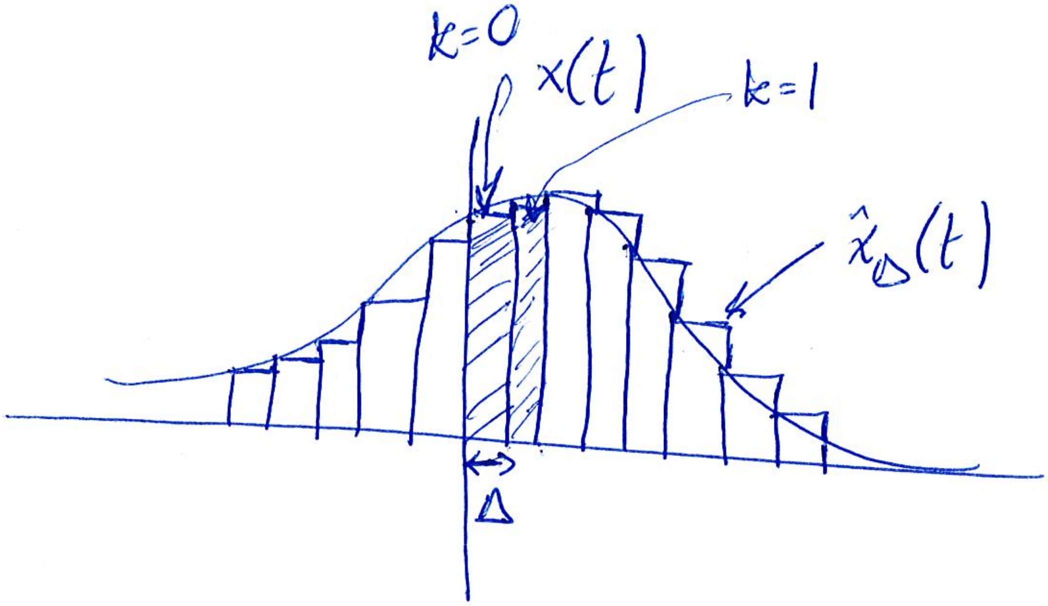
Office Hours

Tue. 1-2 pm

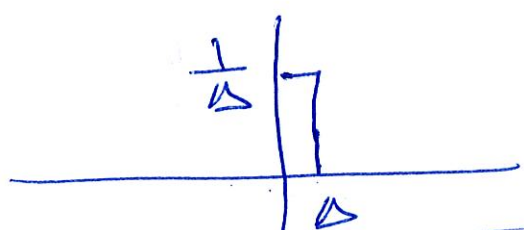
Wed. 11-12

One Crib Sheet Front and Back

Continuous-Time LTI Systems



$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & , \quad 0 \leq t < \Delta \\ 0 & , \quad \text{otherwise} \end{cases}$$



$$\boxed{\hat{x}_\Delta(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\, \cancel{\delta_\Delta(t)}\, \delta_\Delta(t-k\Delta)\, \Delta}$$

$$x(t) = \lim_{\Delta \to 0} \hat{x}_\Delta(t)$$

$$x(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

Sanity check

$$\int_{\tau=-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(t)$$

$$\int_{\tau=-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{\infty} x(t)\, \delta(t-\tau)\, d\tau$$

$$= x(t) \int_{\tau=-\infty}^{\infty} \delta(t-\tau)\, d\tau$$

$$= x(t)$$

$$\lim_{\Delta \to 0} \hat{x}_\Delta(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\, \delta_\Delta(t-k\Delta)\, \Delta$$

$\boxed{\text{LTI System}}$

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\, h(t-k\Delta)\, \Delta$$

$h(t)$ as the response to $\delta(t)$

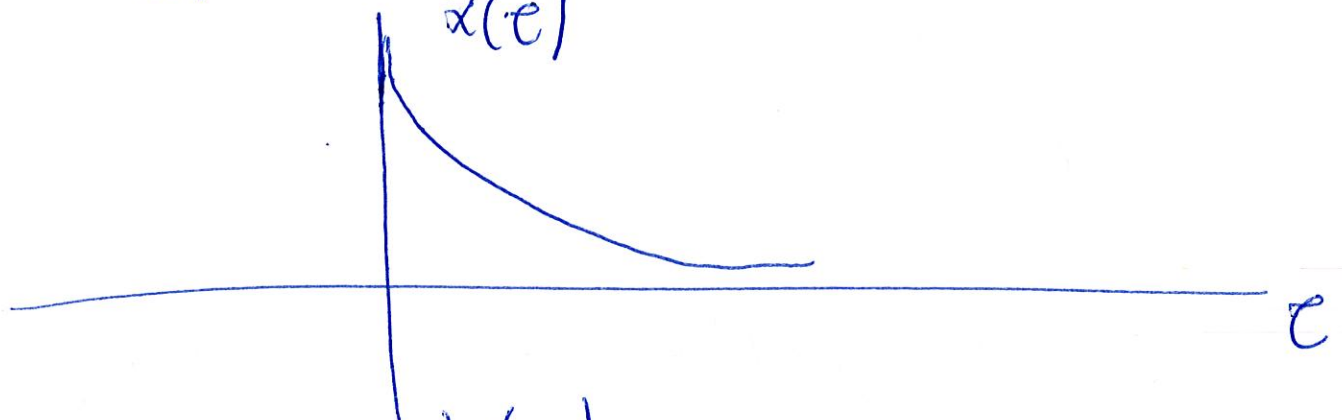
$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

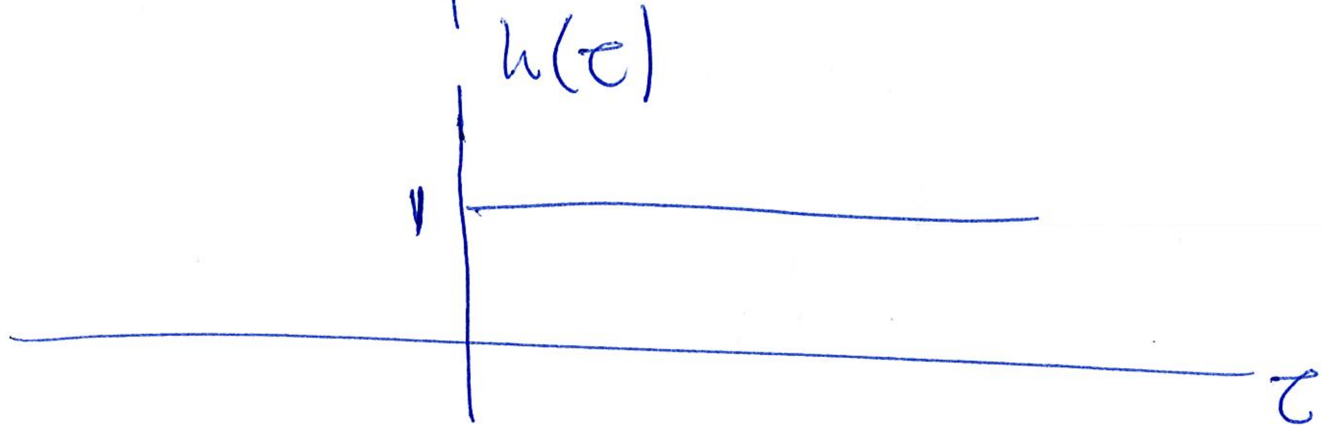
$$\triangleq x(t) * h(t) \quad \boxed{\text{Convolution Integral}}$$

Example 2.6

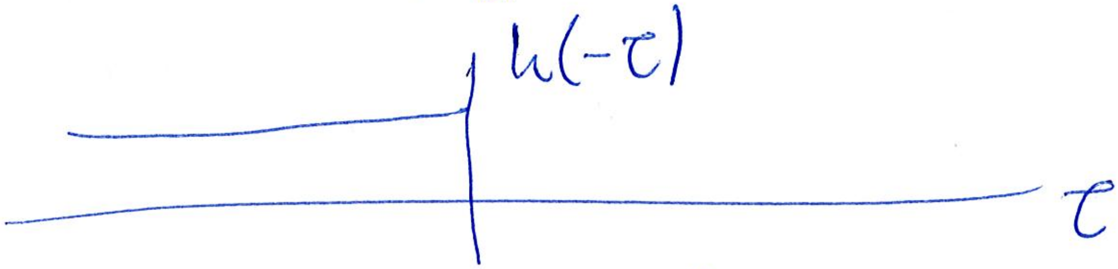
$$x(t) = e^{-at}\,u(t)\,, \quad a > 0$$

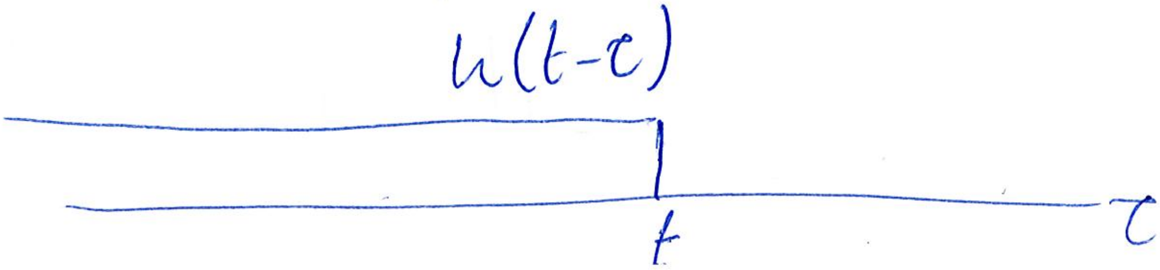
$$h(t) = u(t)$$

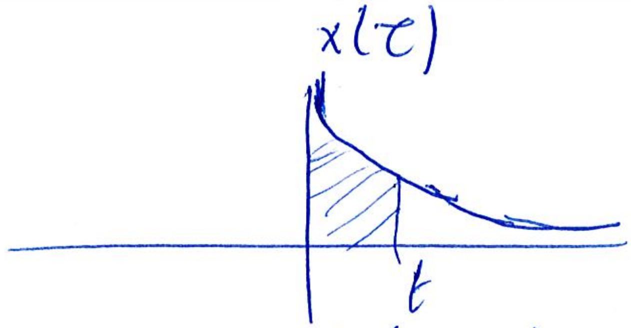


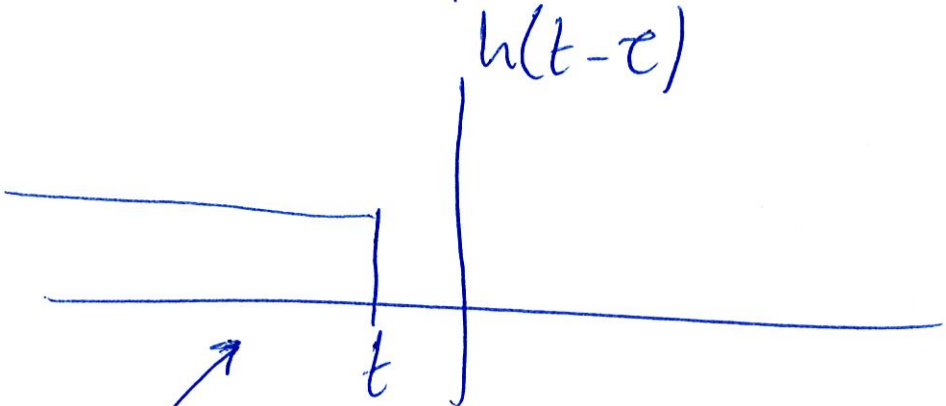


$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$



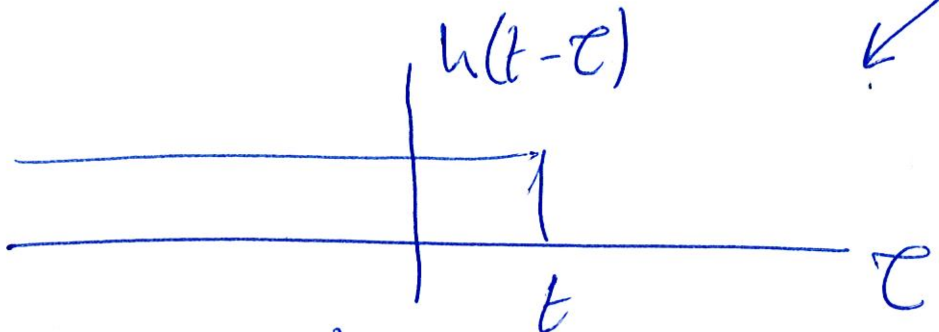






$t < 0 \Rightarrow y(t) = 0$

$t > 0$

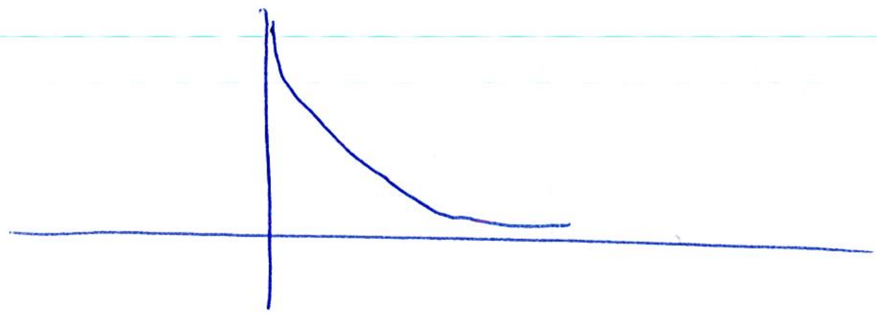


$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

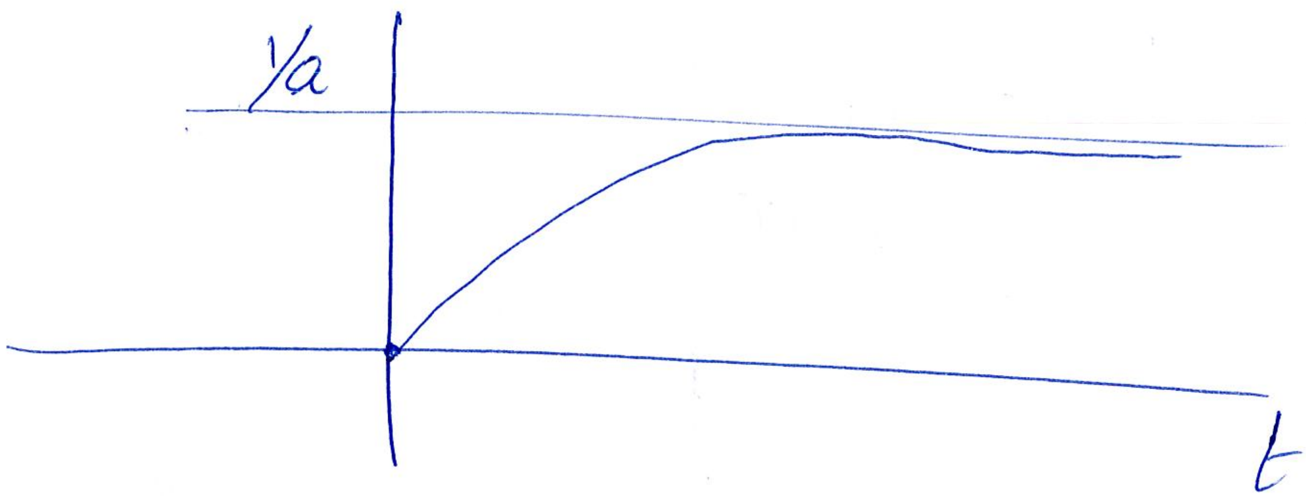
$$= \int_{\tau=0}^{t} e^{-a\tau}\, d\tau = -\frac{1}{a} e^{-a\tau}\Big|_0^t$$

$$= -\frac{1}{a}\left[e^{-at} - 1\right] = \frac{1}{a}\left[1 - e^{-at}\right]$$

$x(t) = e^{-at}\, u(t)$



$y(t) = \frac{1}{a}\left[1 - e^{-at}\right] u(t)$

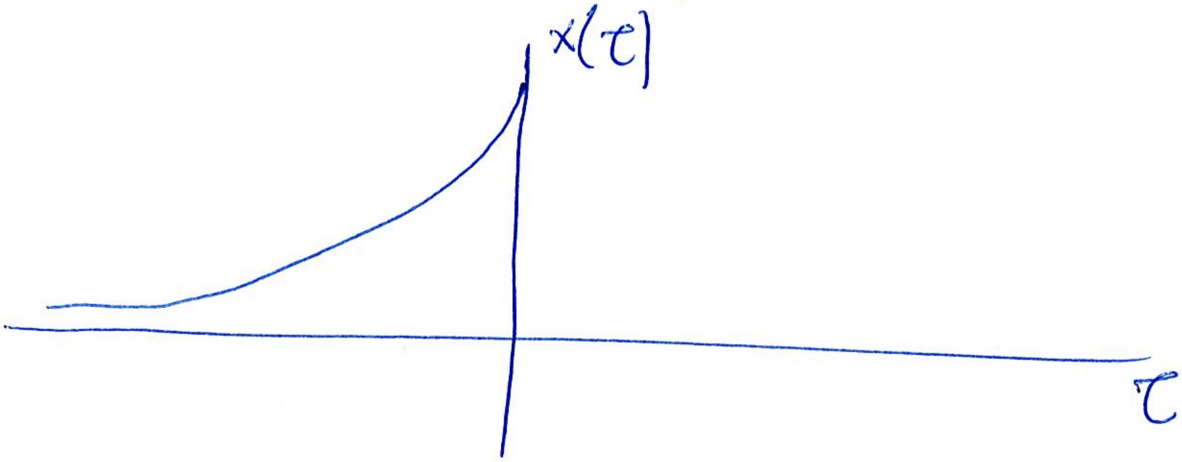


$\boxed{\text{Accumulator}}$

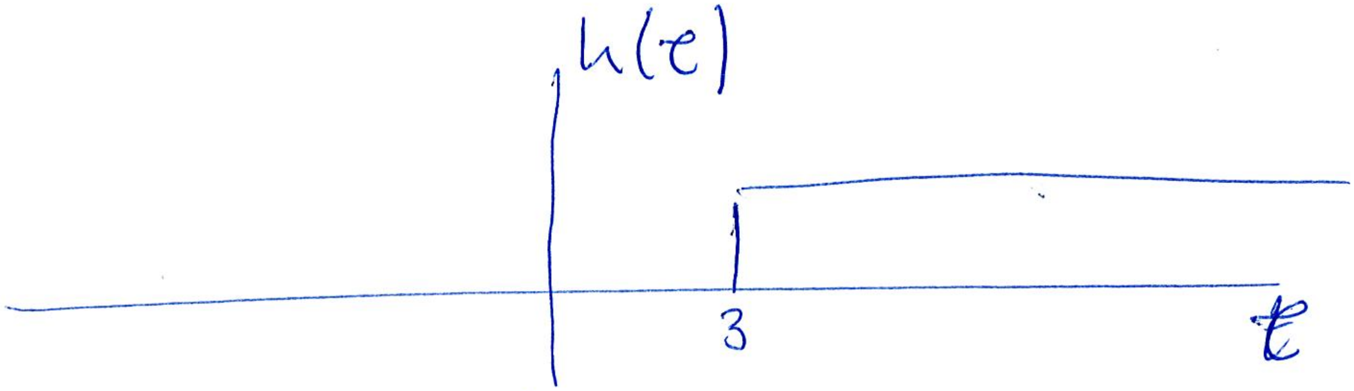
Because $h(t) = u(t)$

Example 2.8

$$x(t) = e^{2t}\, u(-t)$$



$$h(t) = u(t-3)$$



$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)$$