# ECE 301 - Lecture #11



## Example 2.8

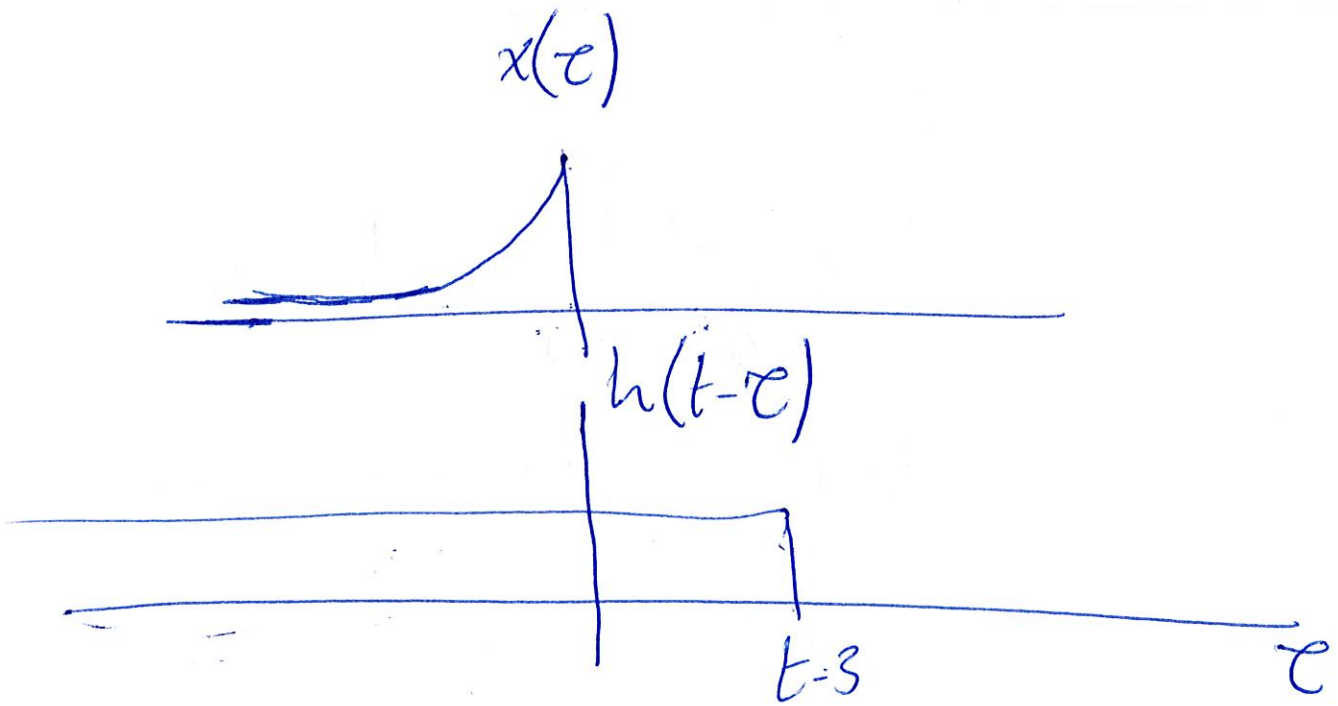
CT - LTI System

$x(t) = e^{2t}u(-t)$ Input

$h(t) = u(t-3)$ Impulse Response

$y(t)$?

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$



For $t-3 < 0$

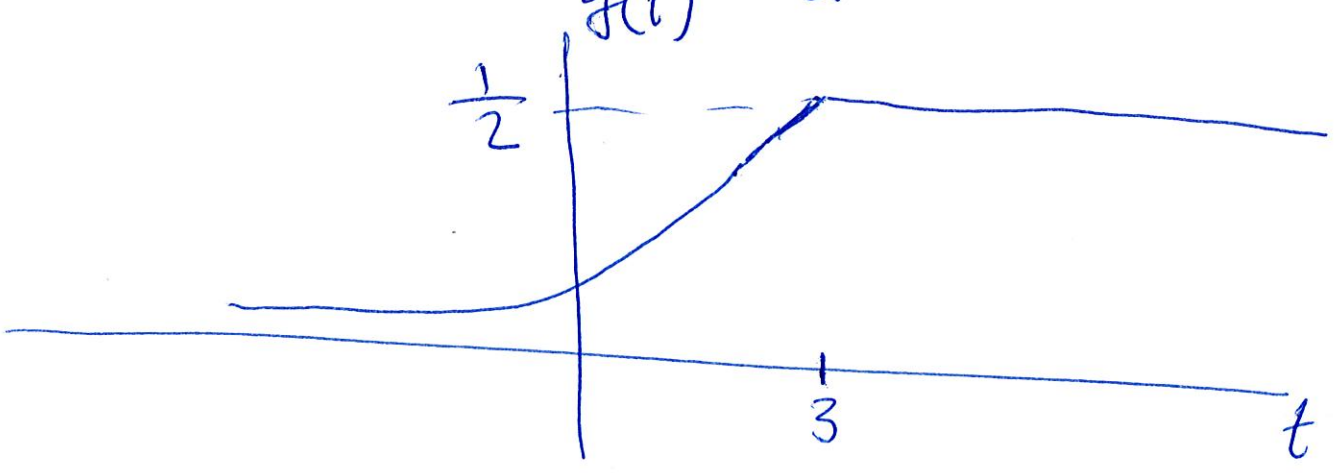
$$y(t) = \int_{\tau=-\infty}^{t-3} e^{2\tau}\, d\tau$$

$$= \frac{1}{2} e^{2\tau}\Big|_{-\infty}^{t-3} = \frac{1}{2}\left[e^{2(t-3)} - 0\right]$$

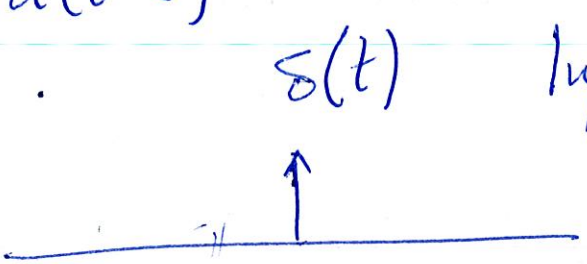
For $t-3 \geq 0$

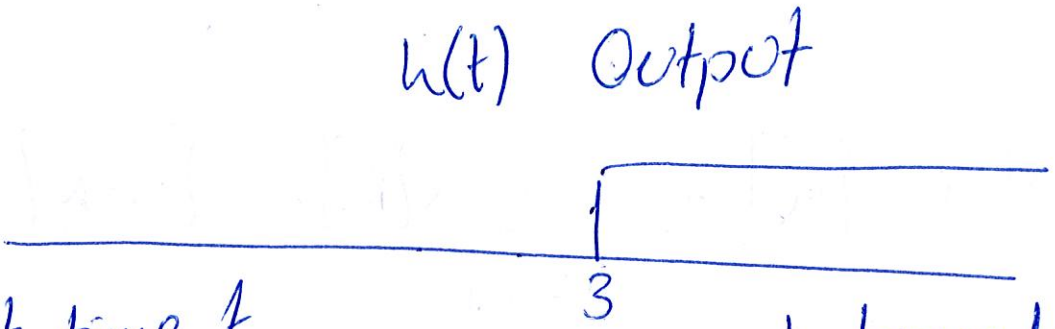
$$y(t) = \int_{\tau=-\infty}^{0} e^{2\tau}\, d\tau$$

$$= \frac{1}{2} e^{2\tau}\Big|_{-\infty}^{0} = \frac{1}{2}$$



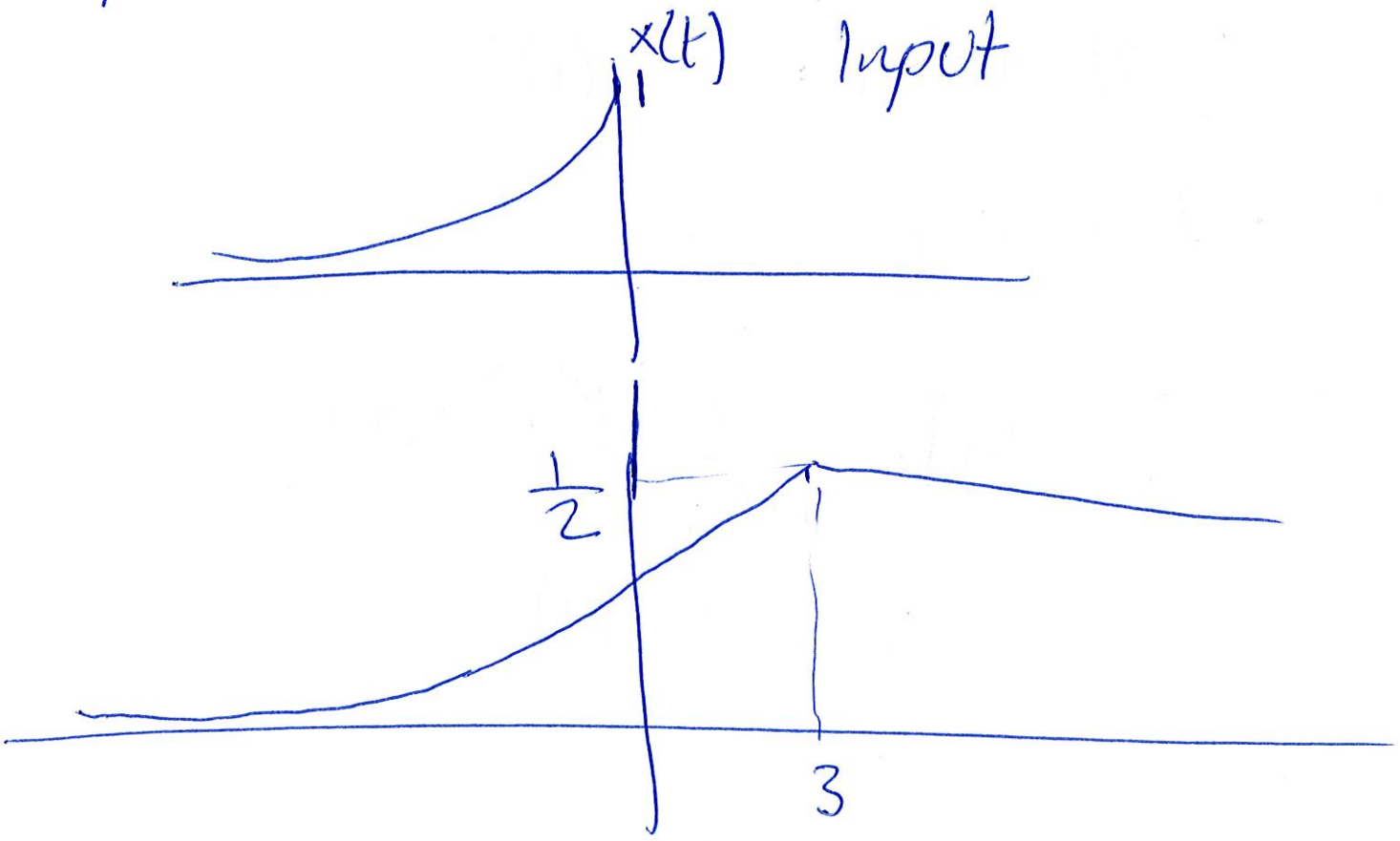
$h(t) = u(t-3)$




at time $t$ Integrating

Output is obtained by accumulating input from $-\infty$ to $t-3$



# Properties of Convolution

## Commutative? Yes

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \quad (1)$$

$$h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \quad (2)$$

Is (2) the same as (1)?

Let $r = n-k$

$$h[n] * x[n] = \sum_{r=-\infty}^{\infty} h[n-r]\, x[r]$$

Same as (1)

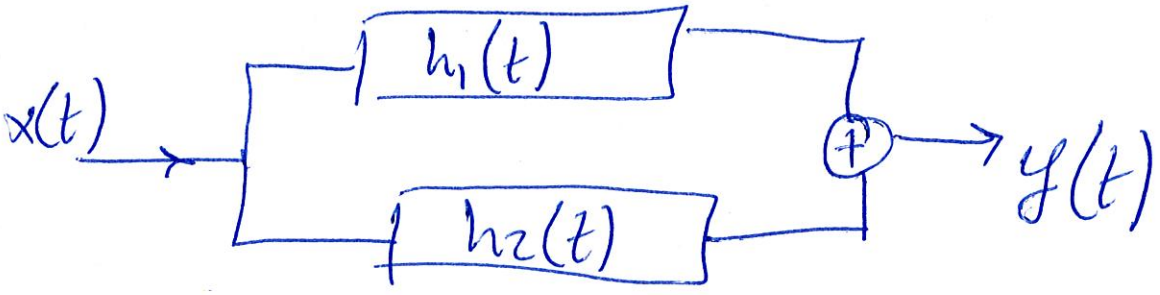
# Distributive? Yes

$$x(t) * \left[ h_1(t) + h_2(t) \right]$$

$$\stackrel{?}{=} x(t) * h_1(t) + x(t) * h_2(t)$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau) \left[ h_1(t-\tau) + h_2(t-\tau) \right] d\tau$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau) h_1(t-\tau)\, d\tau + \int_{\tau=-\infty}^{\infty} x(\tau) h_2(t-\tau)\, d\tau$$



Associative? **Yes**

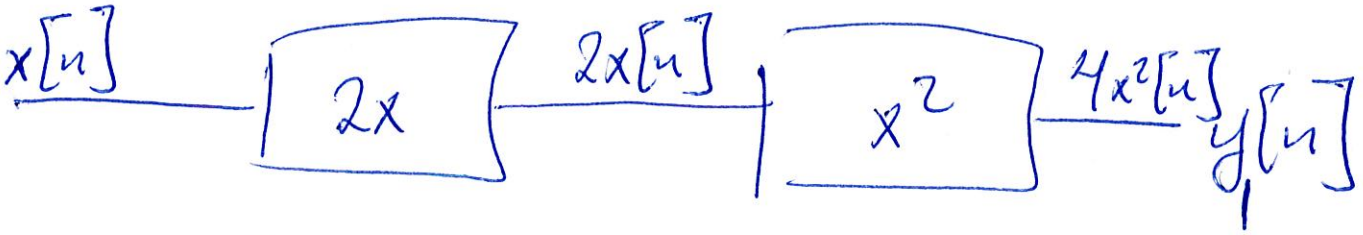
$$x[n] * \left( h_1[n] * h_2[n] \right)$$

$$\stackrel{?}{=} \left( x[n] * h_1[n] \right) * h_2[n]$$

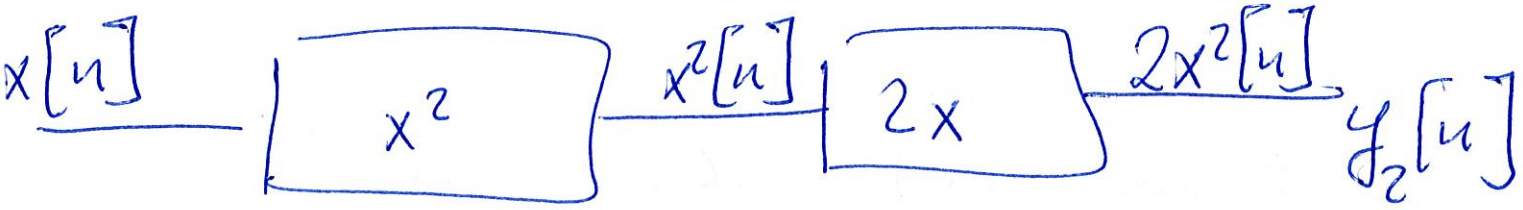
---

Because Convolution is commutative and associative, then the impulse response of a cascade of LTI systems, is the convolution of the individual impulse response (order does not matter)

# Does not apply to non-linear systems (7)




$$y_1[n] = 4x^2[n]$$




$$y_2[n] = 2x^2[n] \neq y_1[n]$$

# Properties of LTI Systems

## Memoryless

Output at time $n$ depends on input only through its current value $n$

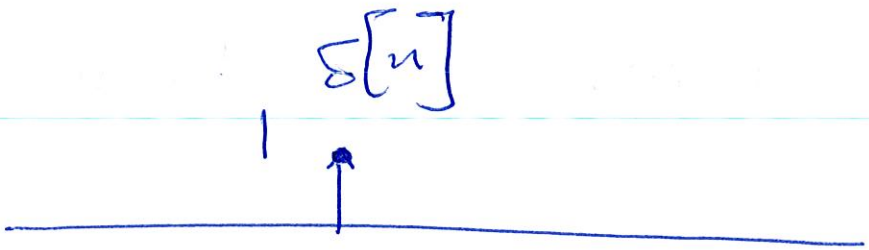
What does that imply on $h[n]$?

$$h[n] \neq 0 \text{ only at } n=0$$

For any memoryless LTI System

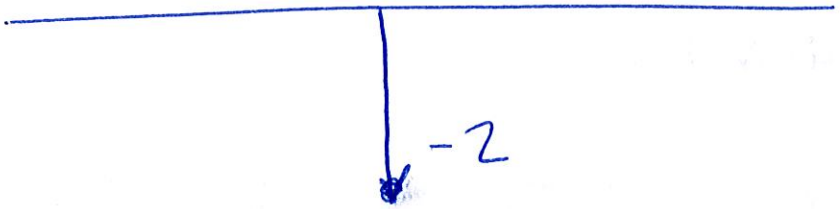
$$h[n] = k\,\delta[n]$$

$k$: Scaling Coefficient



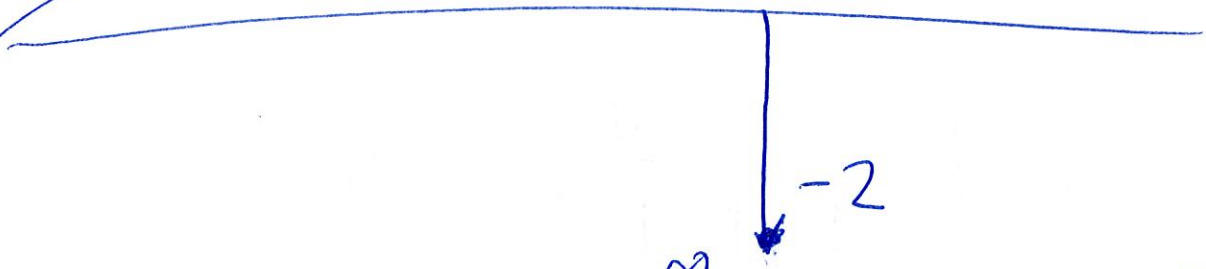

$k = -2$

$h[n] = k\,\delta[n]$




$x[n] = \delta[n-3]$


$y[n] = \delta[n-3] * -2\,\delta[n]$




$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\; h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \delta[k-3] - 2\,\delta[n-k]$$

## Causal?

Output at time $n$ depends on the input only through its current and past values

What does that imply on $h[n]$?

$h[n] \neq 0$ only for $n \geq 0$

Ex.

$h[n] = u[n]$ Accumulator

$h[n]$ Not causal