ECE 301 - Lecture #12

Properties of LTI Systems

Stability

For any bounded input, the output is bounded $\Longleftrightarrow$ Stable System

Consider a bounded input

$$|x[n]| \leq B$$

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \right|$$

(2)

$$|y[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \right|$$

$$\leq \sum_{k=-\infty}^{\infty} |h[k]|\, |x[n-k]|$$

$$\leq B \sum_{k=-\infty}^{\infty} |h[k]|$$

Absolute Summability

$\Rightarrow$ If $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$, then

the LTI system is stable

If the LTI system is stable, then

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

For CT-LTI System

System is stable $\Longleftrightarrow$ Impulse response is absolutely integrable

$$\int_{\tau=-\infty}^{\infty} \left| h(\tau) \right| < \infty$$

Invertibility

LTI

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

If it's invertible

$$y(t) \longrightarrow \boxed{h^{-1}(t)} \longrightarrow x(t)$$

And this holds for all $x(t)$

If $x(t) = \delta(t)$

$$\delta(t) \longrightarrow \underset{LTI}{\boxed{h(t)}} \longrightarrow h(t)$$

If it's invertible

$$h(t) \longrightarrow \boxed{h^{-1}(t)} \longrightarrow \delta(t)$$

$\Downarrow$

$$\boxed{\text{There exists a signal } h^{-1}(t) \text{ such that } h(t) * h^{-1}(t) = \delta(t)}$$

Condition for invertibility of LTI Systems

$$x_1[n] \longrightarrow y_1[n] = n^2 x_1[n]$$

$$x_2[n] \longrightarrow y_2[n] = n^2 x_2[n]$$

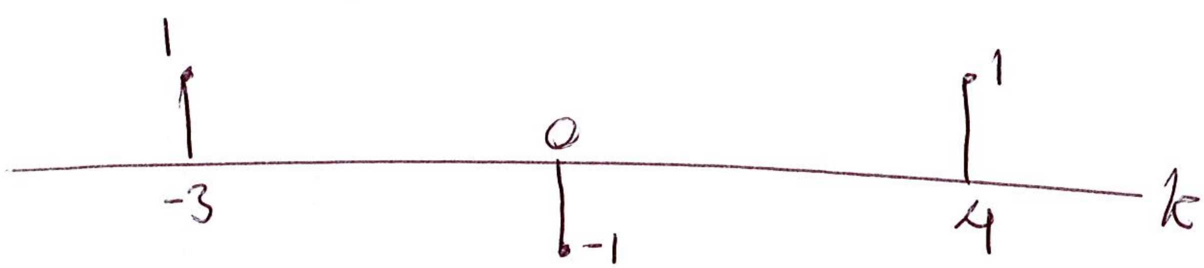
$$x_3[n] = x_1[n] + x_2[n] \longrightarrow y_3[n] = n^2 x_3[n]$$

$$\stackrel{?}{=} y_1[n] + y_2[n]$$

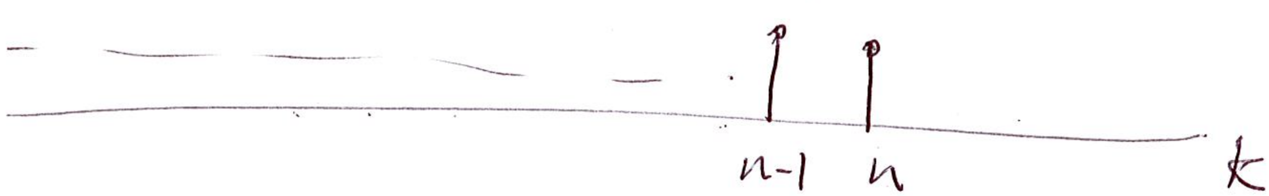
HW 2 P. 4 a

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$x[k]$



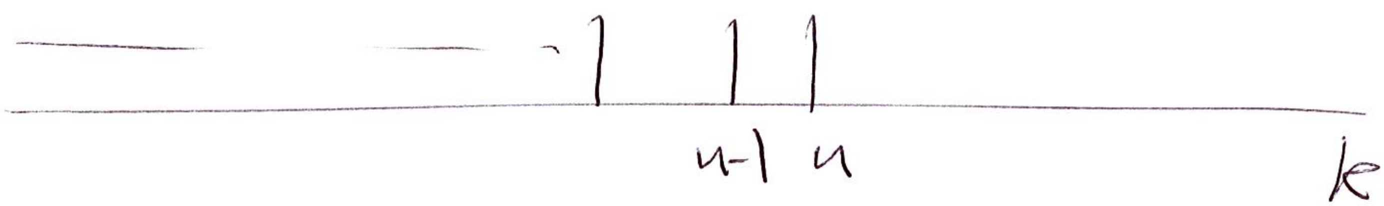
$h[n-k]$



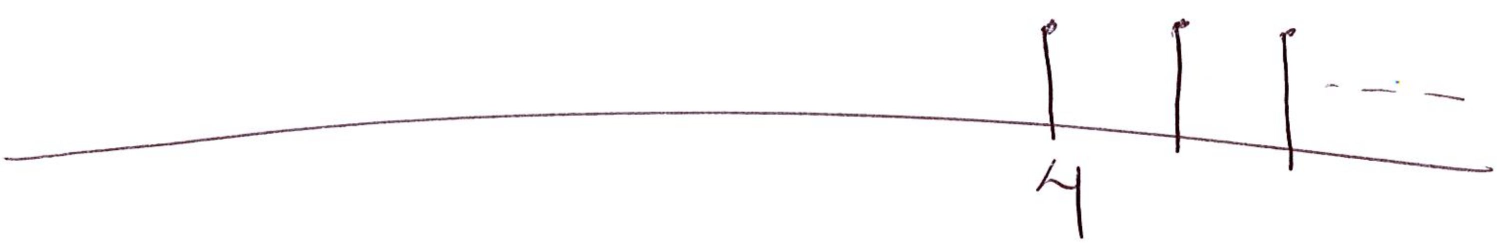
(6)

$$x[k] = \delta[k-4]$$

$$h[n-k] = u[n-k]$$



$$y[n] = \sum_{k=-\infty}^{\infty} \delta[k]\, u[n-k] = u[n-4]$$



$$x[n] = \delta[n-m]$$

any shift

$$y[n] = x[n] * h[n]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \delta[k-m]\, h[n-k]$$

$$= h[n-m]$$

4.c

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$