ECE 301 - Lecture #13

Stability and Invertibility of LTI Systems

Stability

DT $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$

CT $\int_{\tau=-\infty}^{\infty} |h(\tau)| < \infty$

Invertibility

There exists a signal $h^{-1}(t)$ such that $h(t) * h^{-1}(t) = \delta(t)$

Impulse Response of the Inverse System

Example 2.13

$$h[n] = \delta[n - n_0]$$

fixed

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} \delta[n-n_0] = 1 < \infty$$

$\Rightarrow$ Stable

$$h[n] = \delta[n-n_0]$$ Invertible

$$h^{-1}[n] = \delta[n+n_0]$$ Impulse Response of the Inverse System

$$h[n] * h^{-1}[n] = \delta[n-n_0] * \delta[n+n_0]$$

$$= \delta[n-n_0+n_0] = \delta[n]$$

Another Example

$$h[n] = u[n]$$ Accumulator

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} 1 = \infty$$ Unstable

Invertibility

$$h^{-1}[n] = \delta[n] - \delta[n-1]$$ Impulse Response of the Differentiator

$$h[n] * h^{-1}[n] = u[n] * (\delta[n] - \delta[n-1])$$

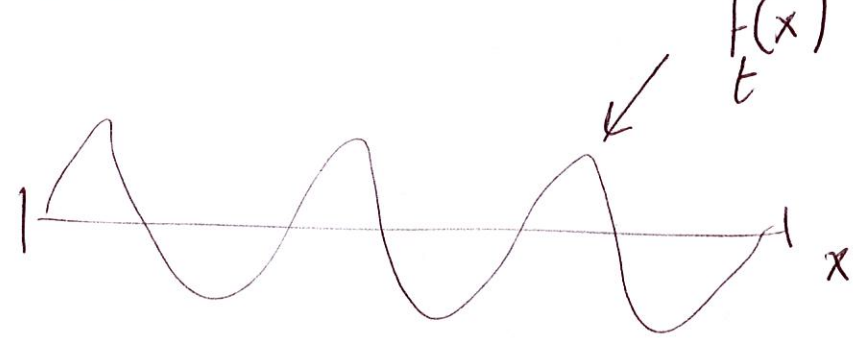
$$= (u[n] * \delta[n]) - (u[n] * \delta[n-1]) = u[n] - u[n-1]$$

$$= \delta[n]$$

Foeurier Series Representation of Periodic Signals

* Notes #10

Euler



Euler's observation

If $f_t(x)$ is a linear combination of sinusoids, then $f_{\tilde{t}}(x)$ is also a linear combination of sinusoids for any $\tilde{t} > t$

Why are linear combinations of sinusoids easier to analyze

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purely imaginary e^{st} $\longleftarrow$ Sinusoid in real / Imaginary

$$e^{st} \longrightarrow \underset{}{\overset{\text{LTI}}{\boxed{h(t)}}} \longrightarrow ?$$

$$y(t) = x(t) * h(t)$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= h(t) * x(t)$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{st}\, e^{-s\tau}\, d\tau$$

$$= e^{st} \underbrace{\int_{\tau=-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau}_{H(s)}$$

$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow e^{st} H(s)$$

$H(s)$

Eigen function of the system

Eigen value of the system

Frequency Response