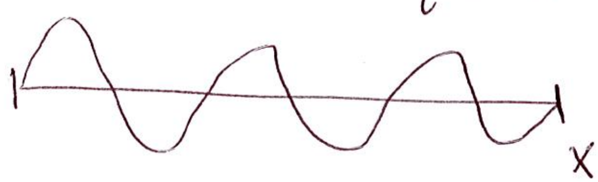
ECE 301 - Lecture #14

Fourier Series Representation of Periodic Signals

Euler's Observation

$f_t(x)$



If $f_t(x)$ can be represented as a linear combination of sinusoids then $f_{\tilde{t}}(x)$

↑

Harmonically related

can also be represented as a linear combination of the same set of harmonically related sinusoids, for any $\tilde{t} > t$. Moreover, the coefficients of $f_{\tilde{t}}(x)$ can be computed from those of $f_t(x)$ is a straight forward way

~~Crit~~ Lagrange's Criticism

Any signal with a sharp discontinuity cannot be represented as a linear combin. of harmonically related sinusoids

Fourier's Discovery

* Useful for describing distribution of heat throughout the body

* Any periodic signal can be represented (or approximated) by a linear combination of harmonically related sinusoids, as long as it satisfies a certain set of conditions.

<u>CT</u>

$$x(t) = e^{st} \longrightarrow \overset{\text{LTI}}{\boxed{h(t)}} \longrightarrow$$

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{\tau=-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} H(s),$$

where $H(s) = \displaystyle\int_{\tau=-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$

<u>DT</u>

$$x[n] = z^n$$

$$y[n] = x[n] * h[n] = h[n] * x[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] = \sum_{k=-\infty}^{\infty} h[k]\, z^{n-k}$$

$$= z^n \sum_{k=-\infty}^{\infty} h[k]\, z^{-k}$$

Example

$$y(t) = x(t-3)$$

$$x(t) = e^{j2t}$$

$$\rightarrow y(t) = e^{j2(t-3)} = \underline{e^{-j6}\, e^{j2t}}$$

$\rightarrow$ we want to compute $H(s) = \int_{\tau=-\infty}^{\infty} h(\tau) e^{-s\tau} d\tau$

$$h(t) = \delta(t-3)$$

$$H(s) = \int_{\tau=-\infty}^{\infty} \delta(\tau-3)\, e^{-s\tau}\, d\tau$$

$$= e^{-3s}$$

If $x(t) = e^{j2t}$, then $s = 2j$

then $H(s) = e^{-6j}$, then $\underline{y(t) = e^{-j6}\, e^{j2t}}$

Set of Harmonically related sinusoids

$$\{\phi_k(t)\}_{k=-\infty}^{\infty}$$

Fundamental Frequency

$$\phi_k(t) = e^{jk\omega_0 t}$$

Linear combination

Scaling complex coefficient

$$x(t) = \underbrace{\sum_{k=-\infty}^{\infty} a_k \, e^{jk\omega_0 t}}$$

Any periodic signal

Linear combination of harmonically related sinusoids

Example

$$x(t) = \sum_{k=-3}^{3} a_k \, e^{jk\overbrace{2\pi}^{\omega_0} t}$$

$a_0 = 1$

Average value of $x(t)$ over one period

Also called DC component $\quad T = \dfrac{2\pi}{\omega_0}$

$a_1 = a_{-1} = \dfrac{1}{4} \qquad a_2 = a_{-2} = \dfrac{1}{2}$

$a_3 = a_{-3} = \dfrac{1}{3}$

$$x(t) = 1 + \frac{1}{4}\left(e^{j2\pi t} + e^{-j2\pi t}\right)$$

$$+ \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right)$$

$$+ \frac{1}{3}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

Note If $a_k = a_{-k}^*$, for any k, then $x(t)$ is real

$$\cos x t = \frac{1}{2} e^{jxt} + \frac{1}{2} e^{-jxt}$$

$\cos xt + j\sin xt$ $\qquad\qquad$ $\cos xt - j\sin xt$

* Derive an altenate representation for the FS for real periodic signals

* Determination the FS coefficients

* What do we mean by approximation, and what are the conditions.