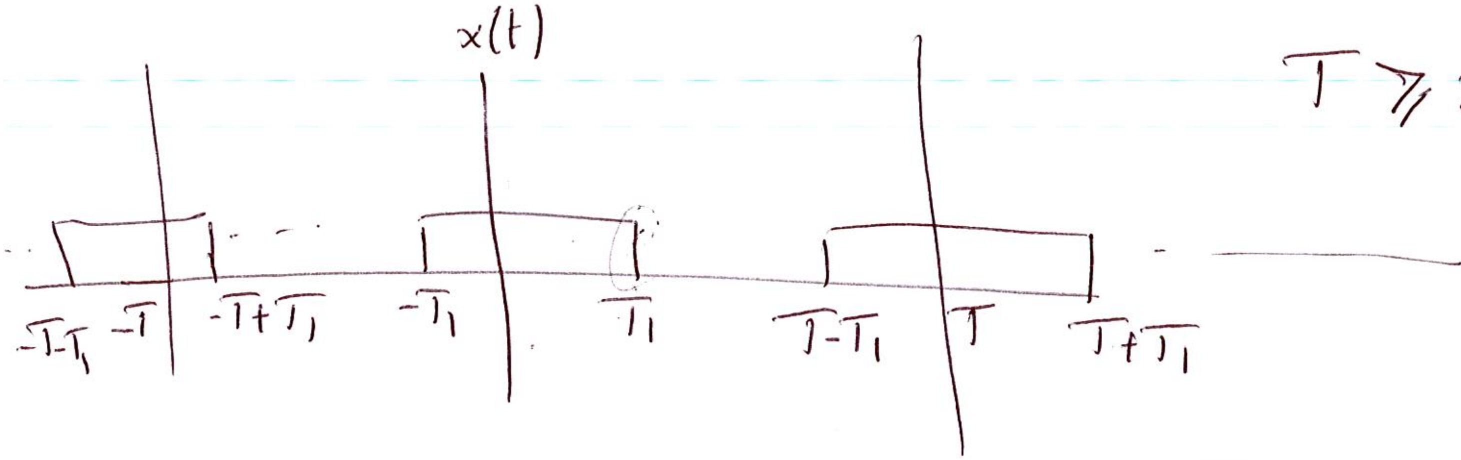
ECE 301 - Lecture #16



$T \geq 2T_1$

FS coefficients

$$\boxed{x(t) = \sum a_k e^{jk\omega_0 t}}$$

$$a_0 = \frac{2T_1}{T}$$

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}, \quad k \neq 0$$

Assume $T = 4T_1$

$$a_1 = a_{-1} = \frac{1}{\pi} \qquad a_3 = a_{-3} = -\frac{1}{3\pi}$$

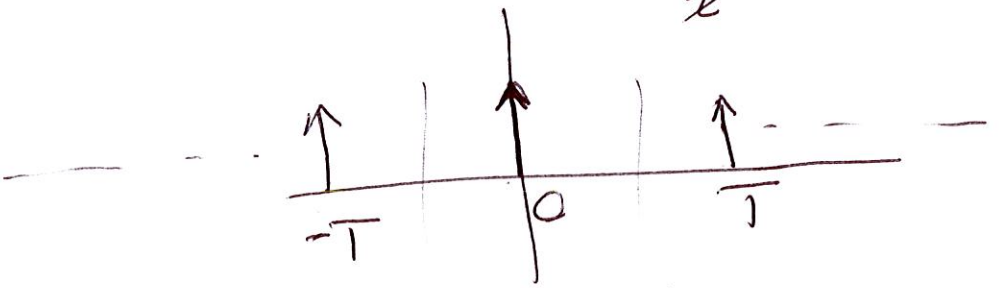
$$a_5 = a_{-5} = \frac{1}{5\pi} \qquad a_7 = a_{-7} = -\frac{1}{7\pi}$$

$$a_9 = a_{-9} = \frac{1}{9\pi} \quad \cdots$$

$a_k = 0$ for k even and $k \neq 0$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$



$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}$$

for all k

Convergence of the Fourier Series

$$x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$$

What's the limit as $N \to \infty$

Will, as $N \to \infty$, the Fourier Series converge to the true signal $x(t)$? and if no, will it approximate $x(t)$?

Finite Energy Periodic Signal

Over one period

$$\int_T |x(t)|^2 \, dt < \infty$$

$$e_N(t) = x(t) - x_N(t)$$

If $x(t)$ has finite energy over one period, then the energy of the error goes to zero as $N \to \infty$

over one period

If $\int_T |x(t)|^2 \, dt < \infty$

then $\int_T |e_N(t)|^2 \, dt \xrightarrow{N \to \infty} 0$

$$x(t) - \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$$

Does not imply that $x_N(t) \xrightarrow{N \to \infty} x(t)$

can look like this

Dirichlet Conditions

Set of Three Conditions. If satisfied, then the Fourier series representation converges to the true signal, except at points where the true signal is not continuous. At these points, the Fourier Series representation converges to the average value taken by the true signal.

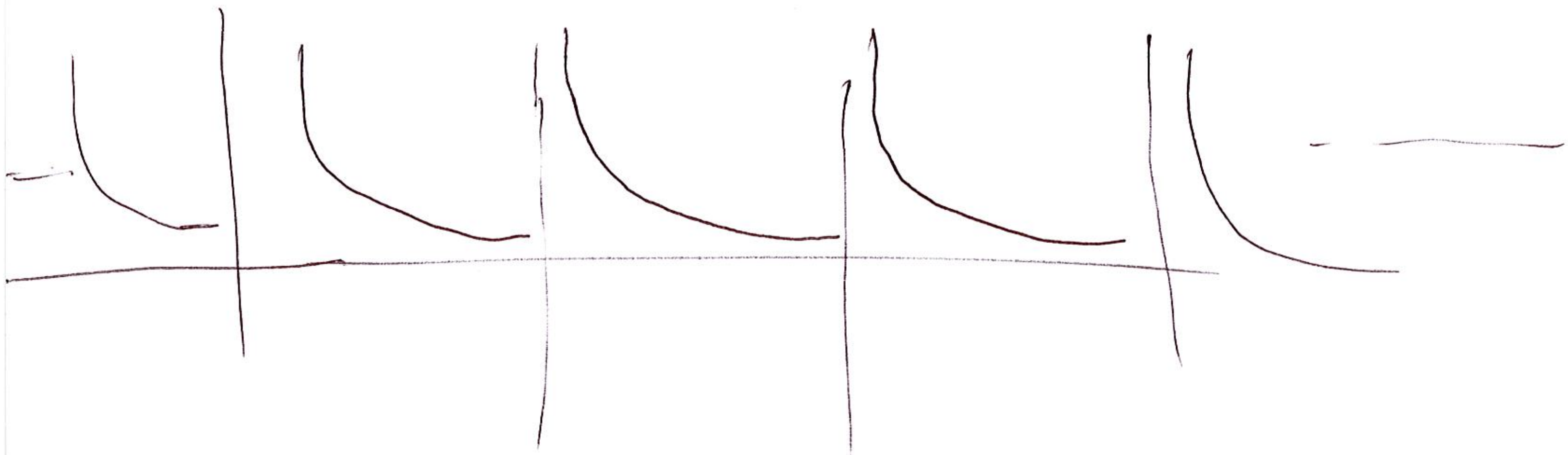
Condition 1

$$\int_T |x(t)| < \infty$$

$x(t)$ is absolutely integrable over one period

Counter Example

$$x(t) = \frac{1}{t}, \quad 0 < t \leq 1$$



Does not satisfy Condition 1

Condition 2