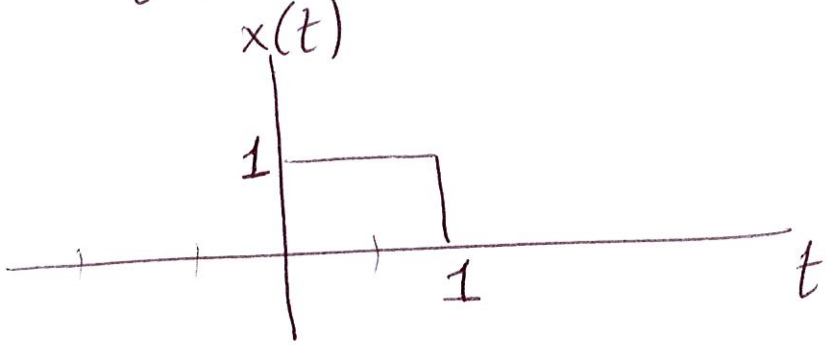
ECE 301 - Lecture #2

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

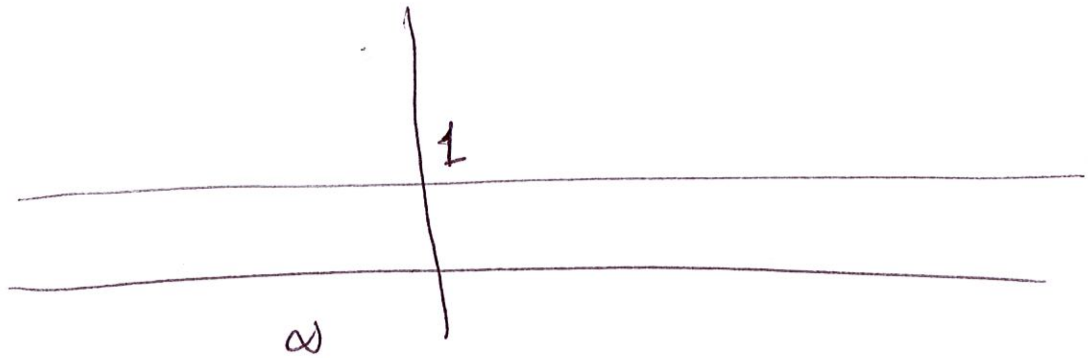


$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = 1$$

$$P_\infty = 0$$

If $E_\infty < \infty$ then $P_\infty = 0$

$x(t) = 1$ for all t

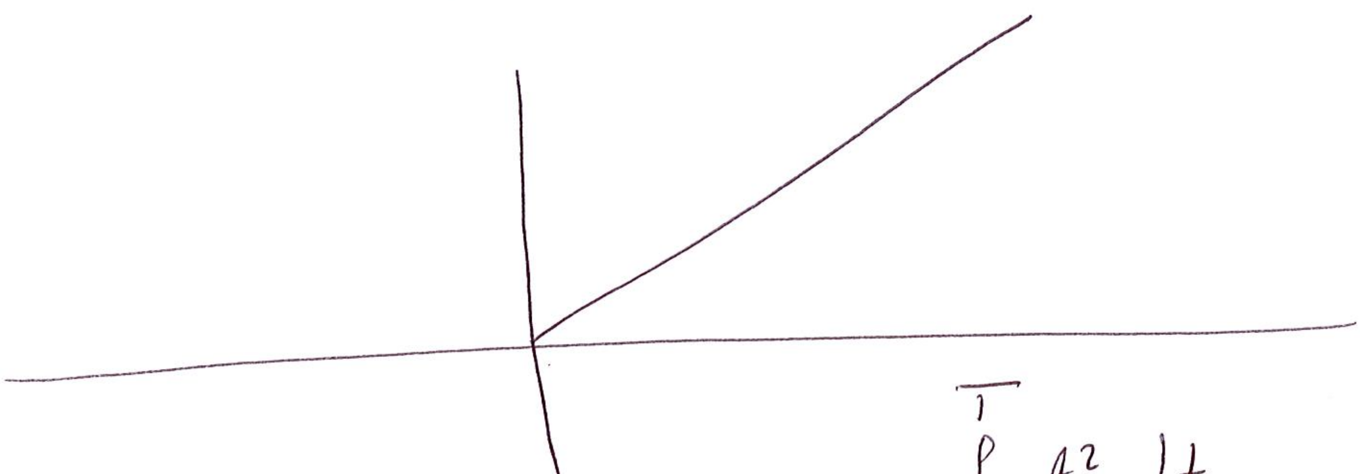


$$E_\infty = \int_{-\infty}^{\infty} 1 \, dt = \infty$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1 \, dt = \lim_{T\to\infty} \frac{2T}{2T}$$

$$= 1$$

$$x(t) = \begin{cases} t, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$



$E_\infty = \infty$ $\quad$ $P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} t^2 \, dt = \infty$

Can we have a signal with $E_\infty = \infty$ and $P_\infty = 0$?

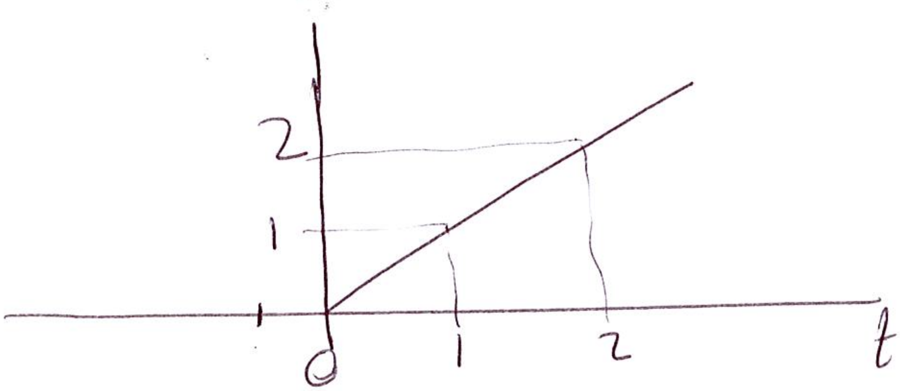
Bonus Exercise 0.2 %

Due Fri. Jan. 18

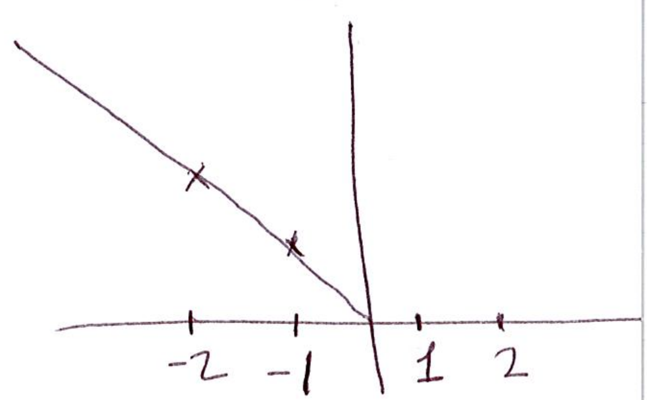
In-Class

Transformations of the independent variable

$x(t) = t, \; t \geq 0$

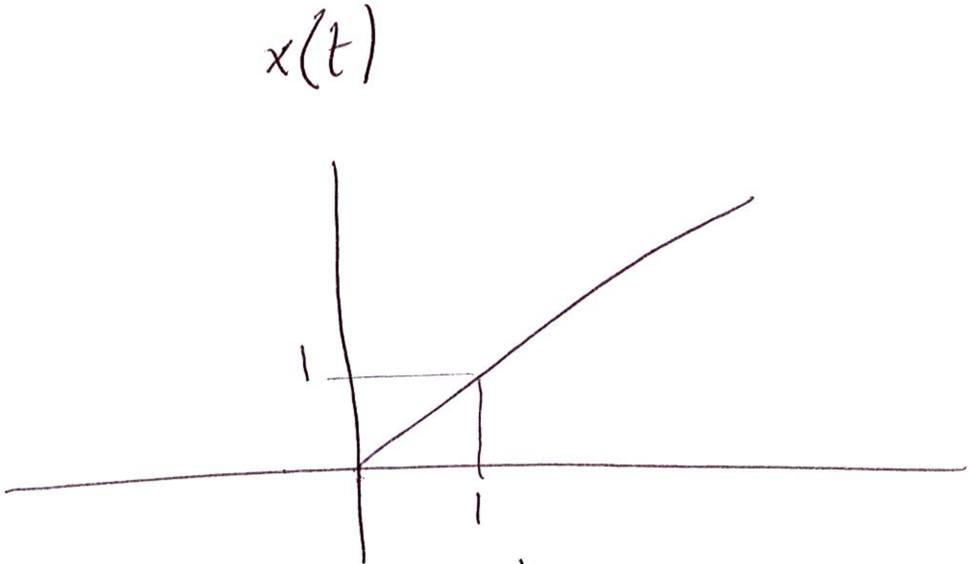


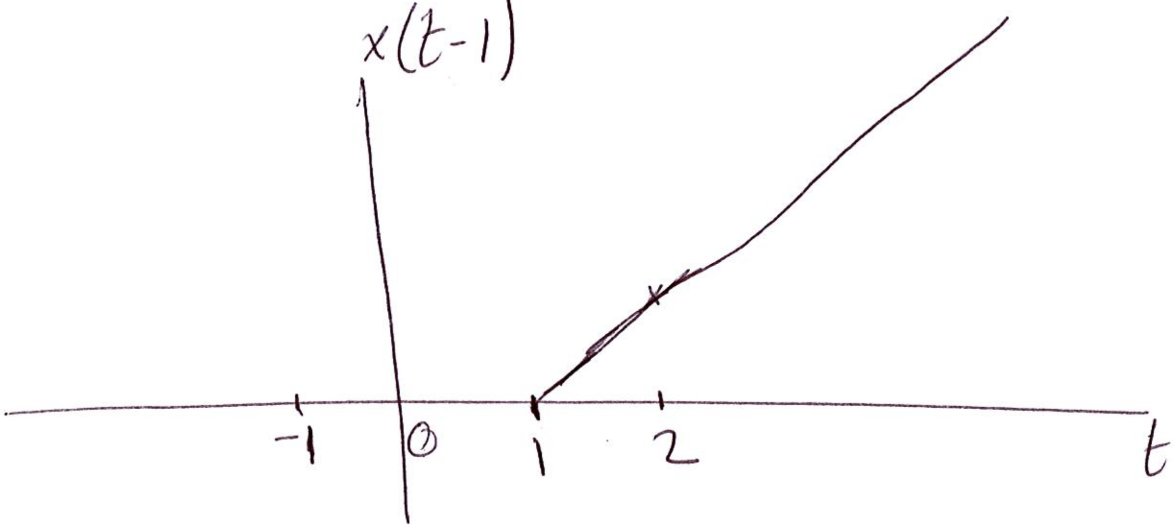
$x(-t)$

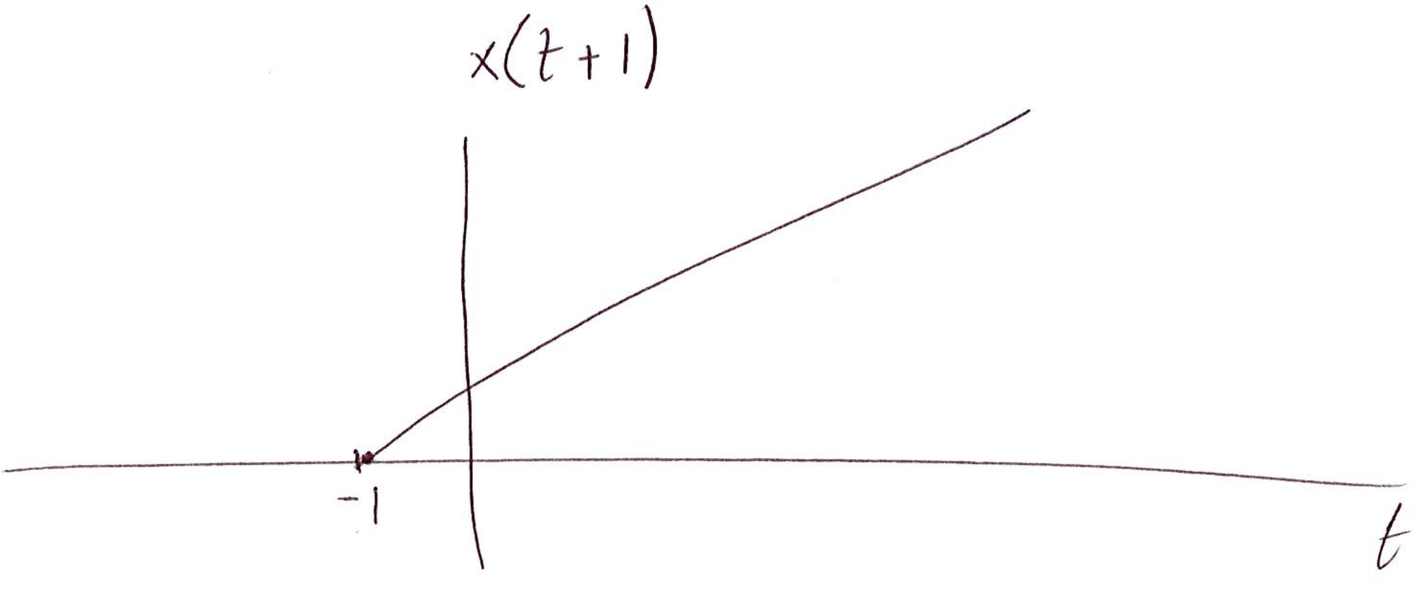


Time Reversal

Time Shifting

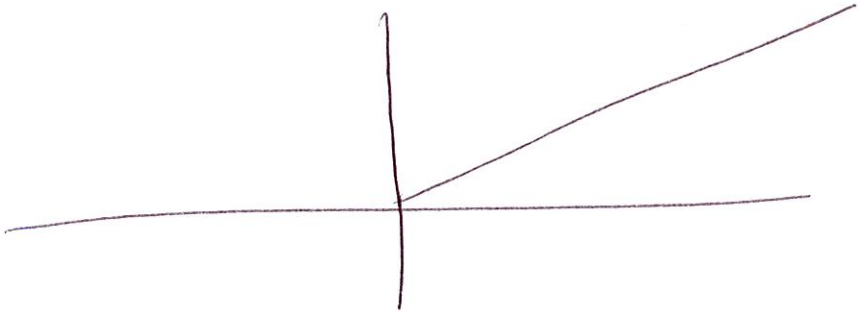




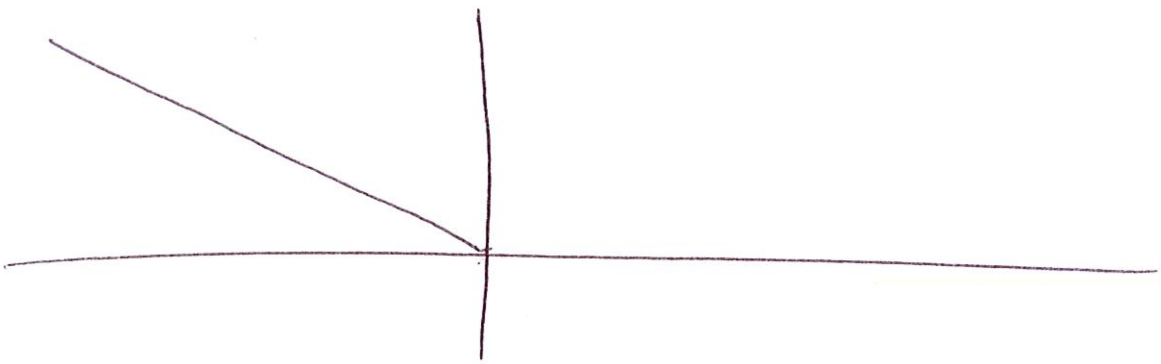


Time Reversal and Shifting

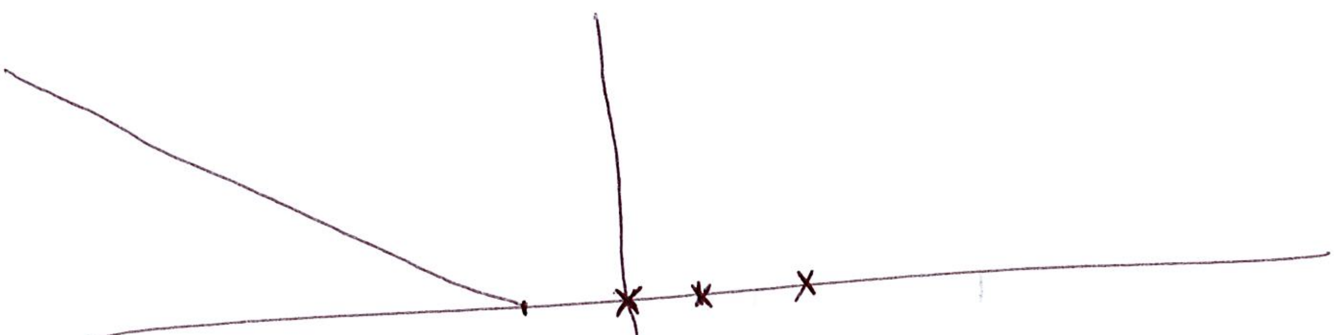
x(t)



x(-t)

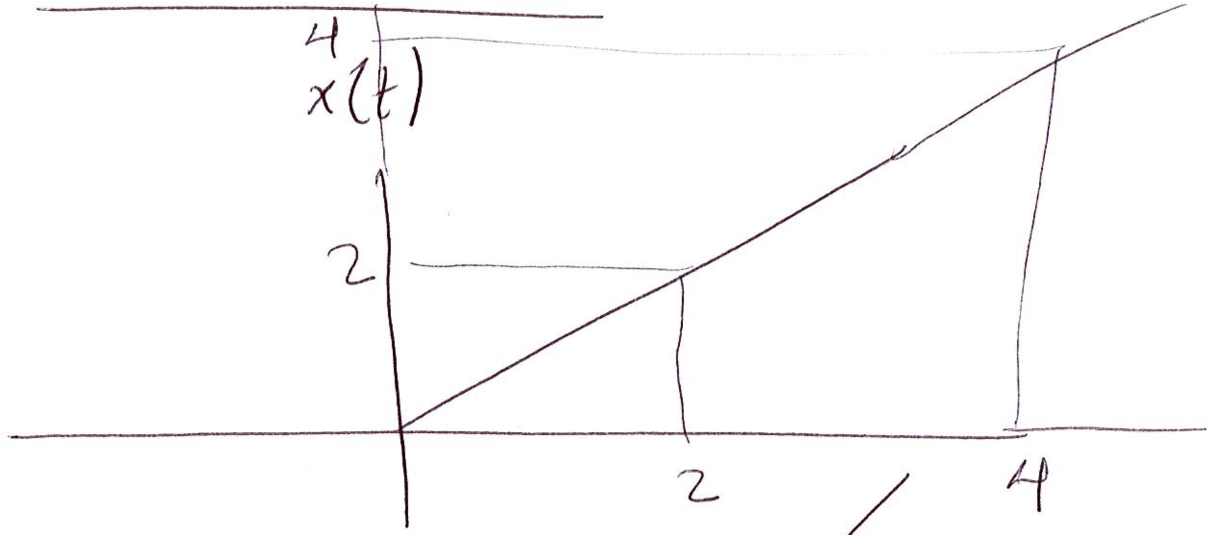


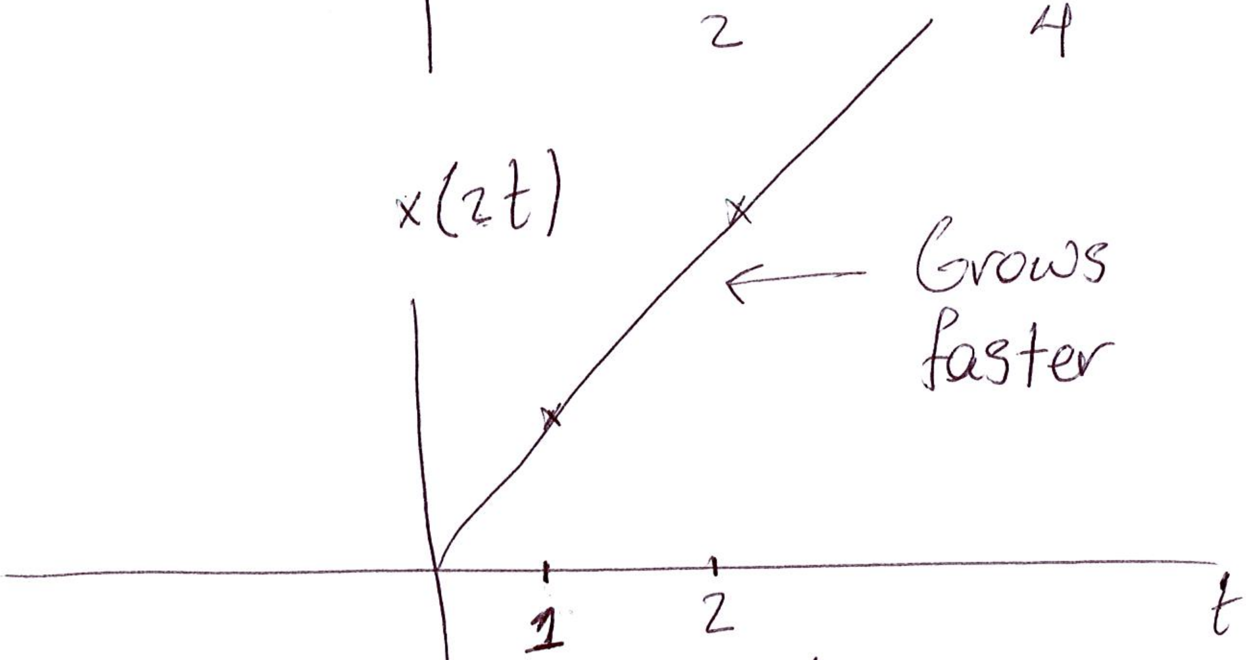
x(-t-1)



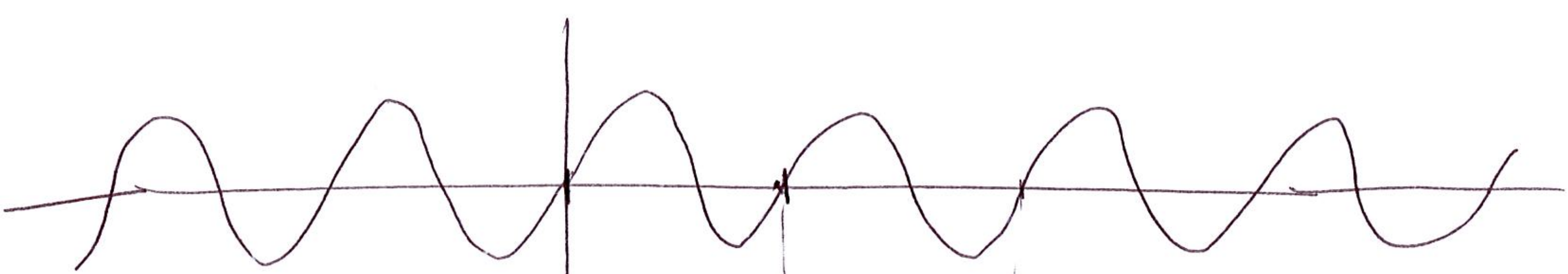
x(-t+1)

Time Scaling

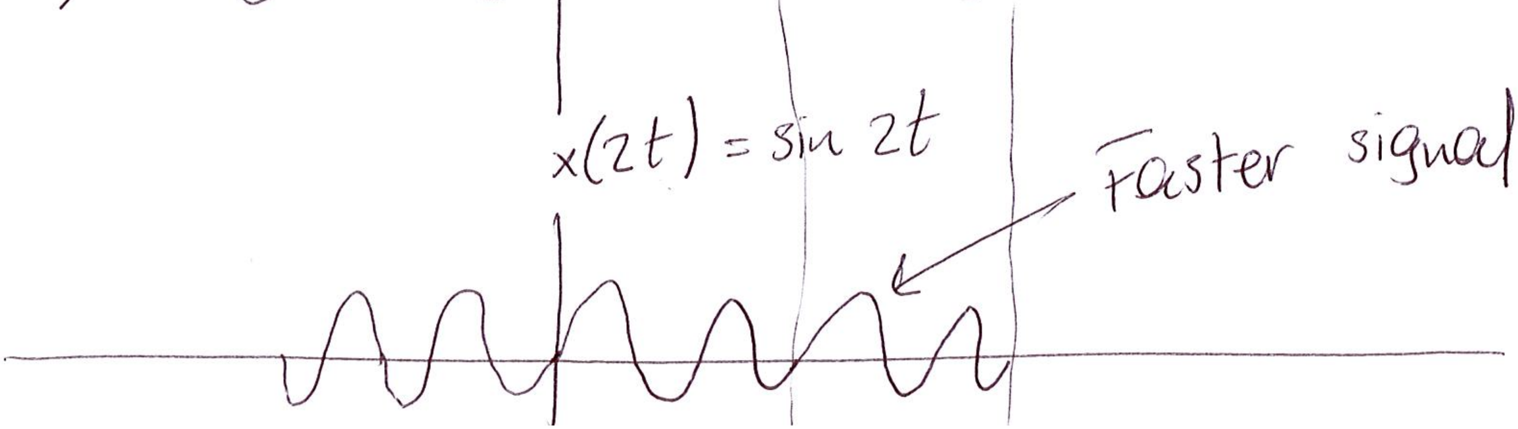




$x(t) = \sin t$

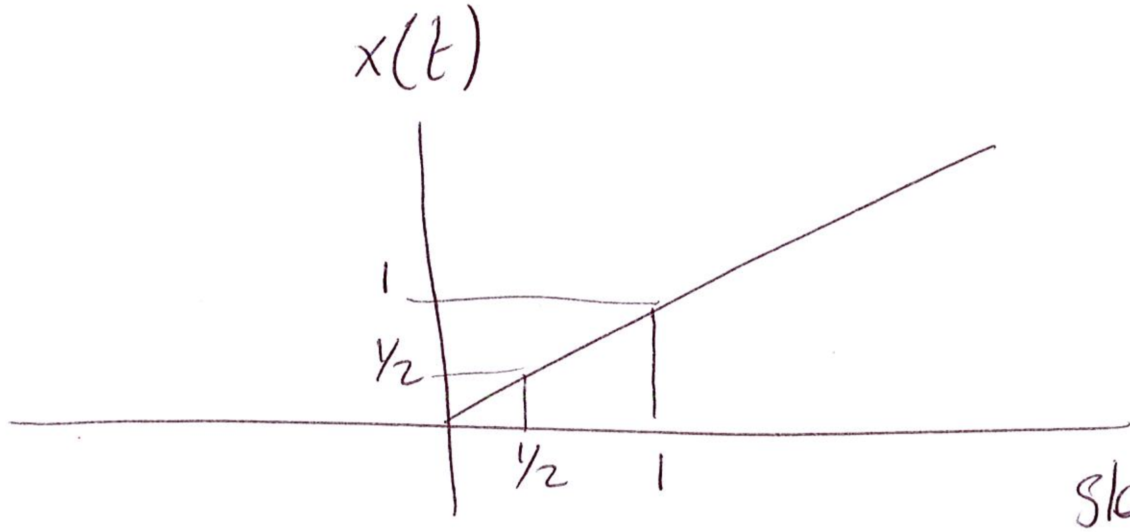


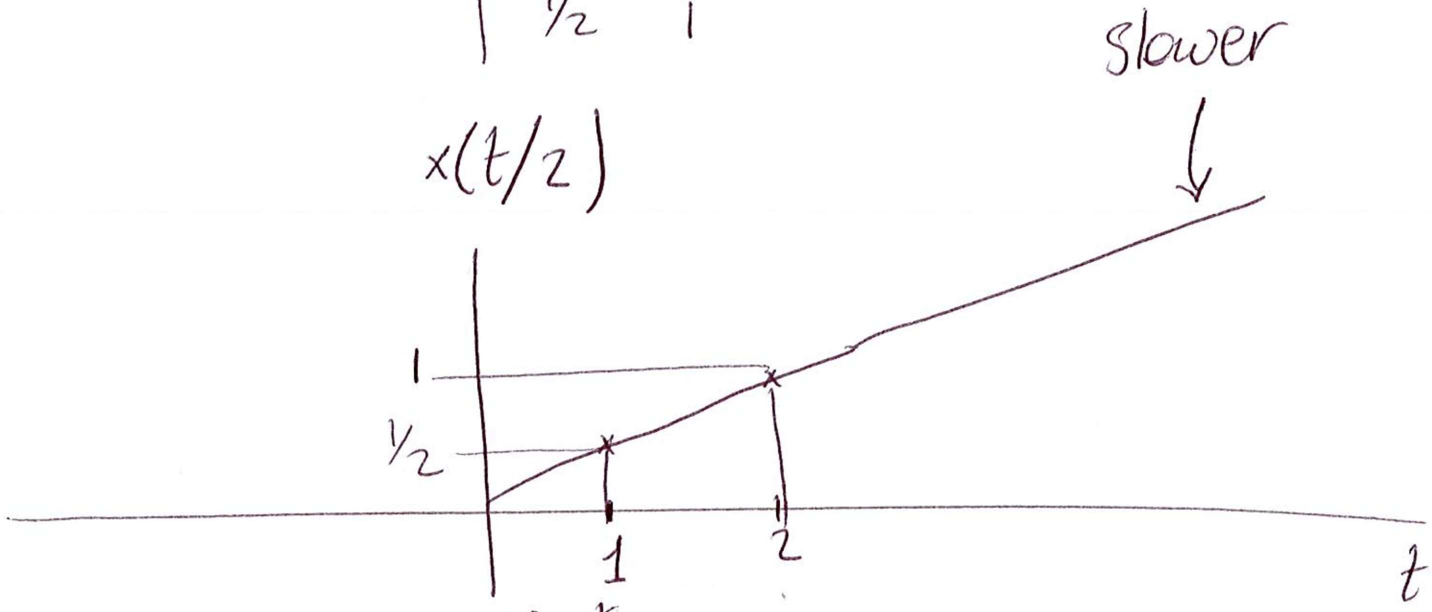
$x(2t) = \sin 2t$

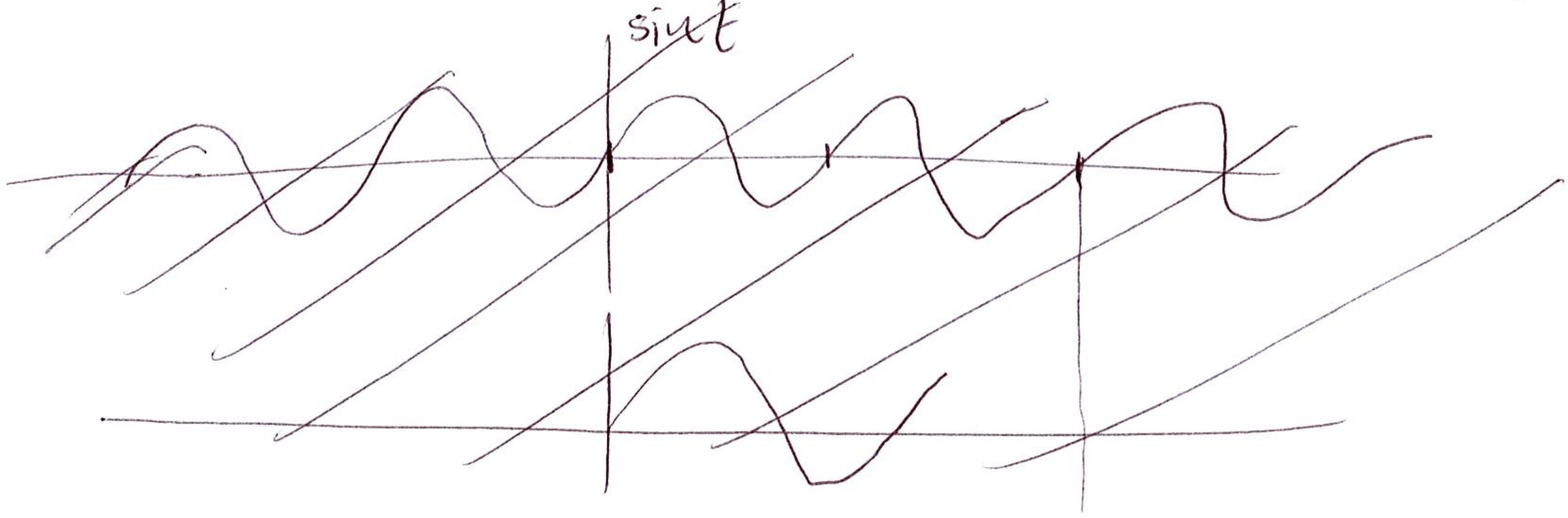


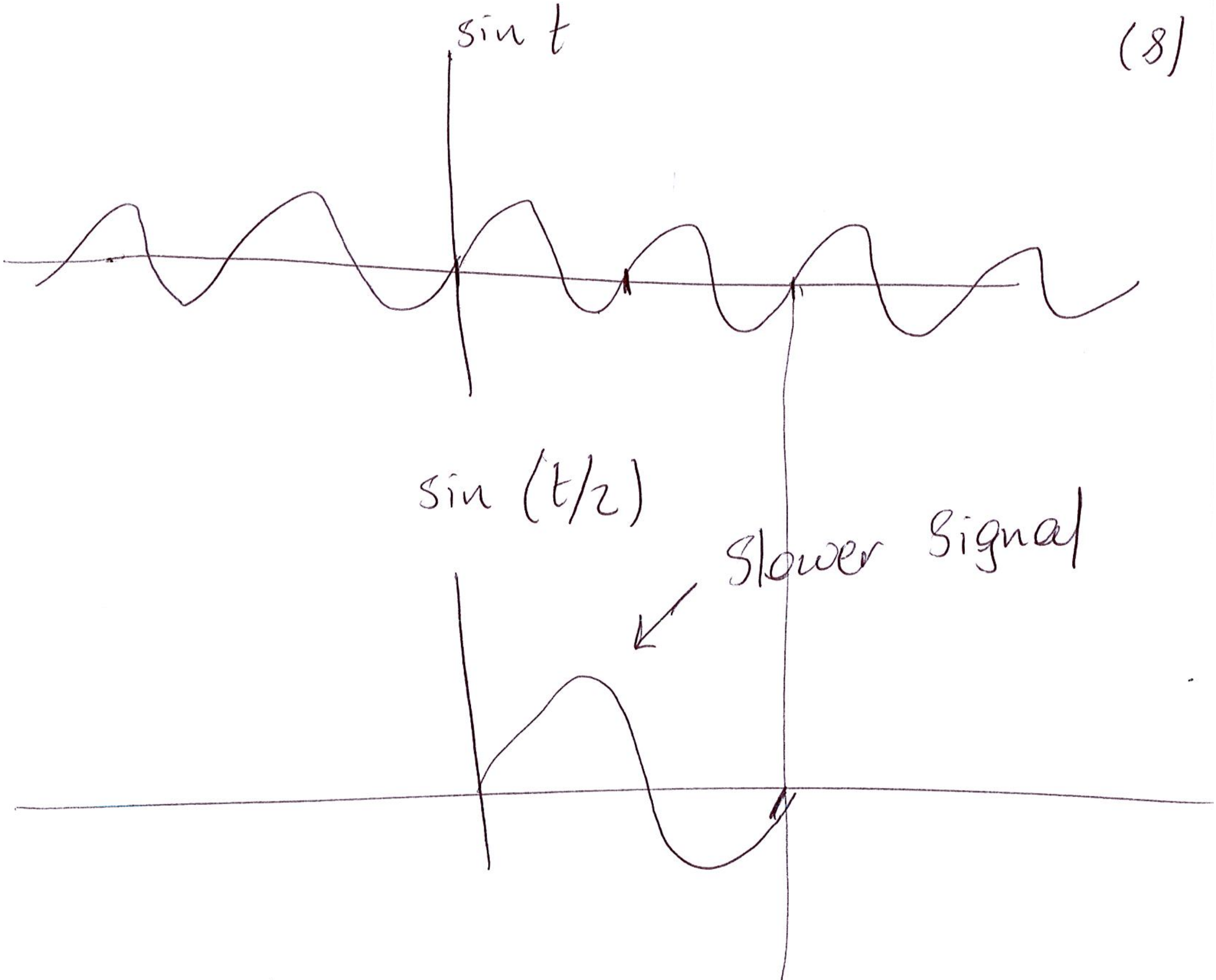
$x(\alpha t)$, where α is a ~~red~~ real number, $|\alpha| > 1$.

then the signal is "compressed"









$x(\alpha t)$, $\alpha \in \mathbb{R}$, $|\alpha| < 1$,

then the signal is "stretched"

$$x(t) \longrightarrow x(\alpha t + \beta), \quad \alpha, \beta \in \mathbb{R}$$

Linearly stretched if $|\alpha| < 1$

linearly compressed if $|\alpha| > 1$

Reversed if $\alpha < 0$

shifted to the right if $\alpha > 0$ and $\beta < 0$
OR $\alpha < 0$ and $\beta > 0$

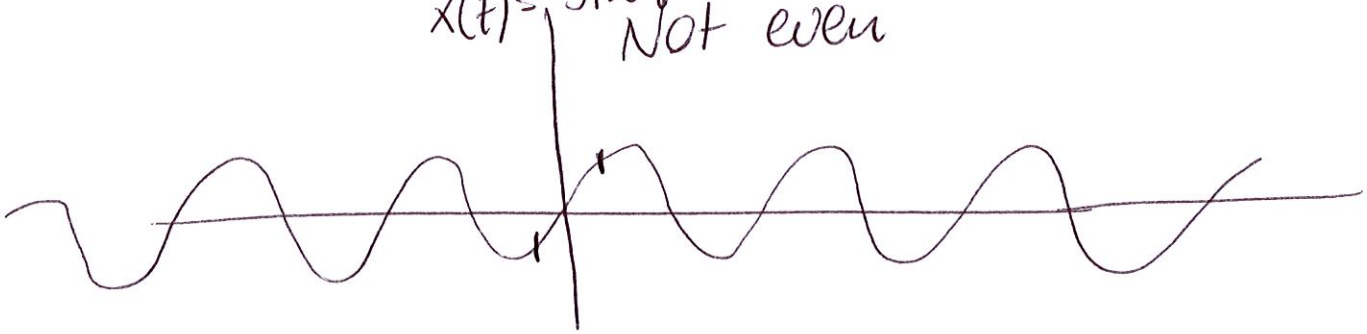
Shifted to the left if $\alpha > 0$ and $\beta > 0$
OR $\alpha < 0$ ~~and~~ and $\beta < 0$

Even and Odd Signals

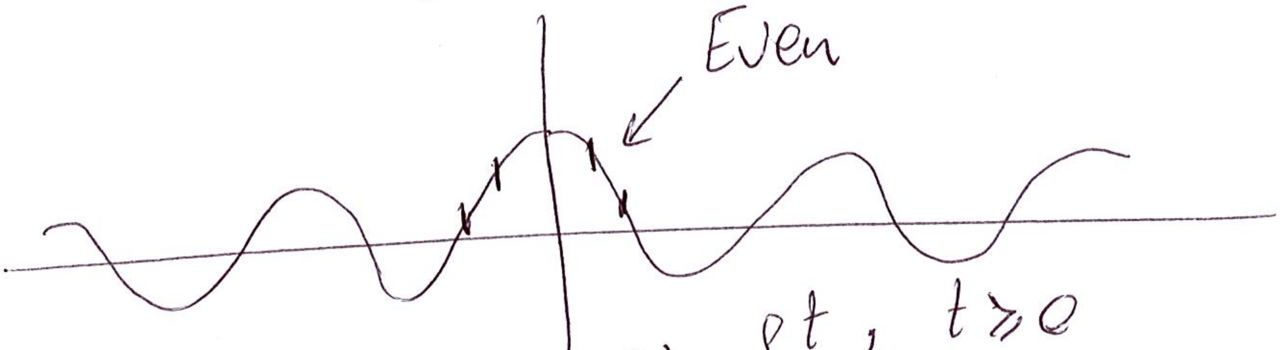
An even signal is a signal satisfying:

$$x(t) = x(-t) \quad \text{for all } t$$

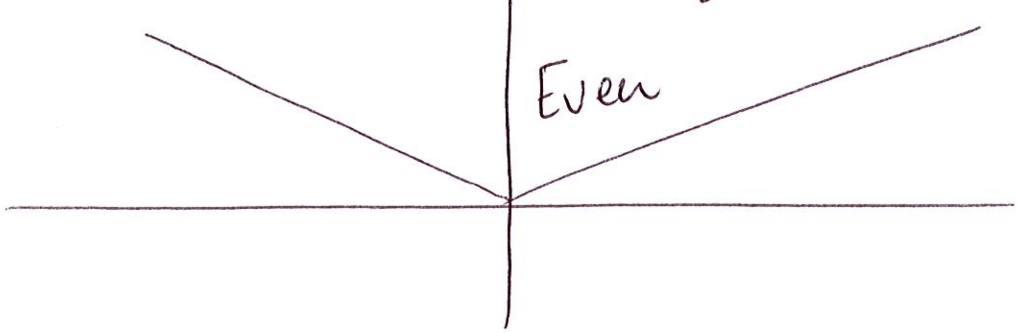
$x(t) = \sin t$



$x(t) = \cos t$

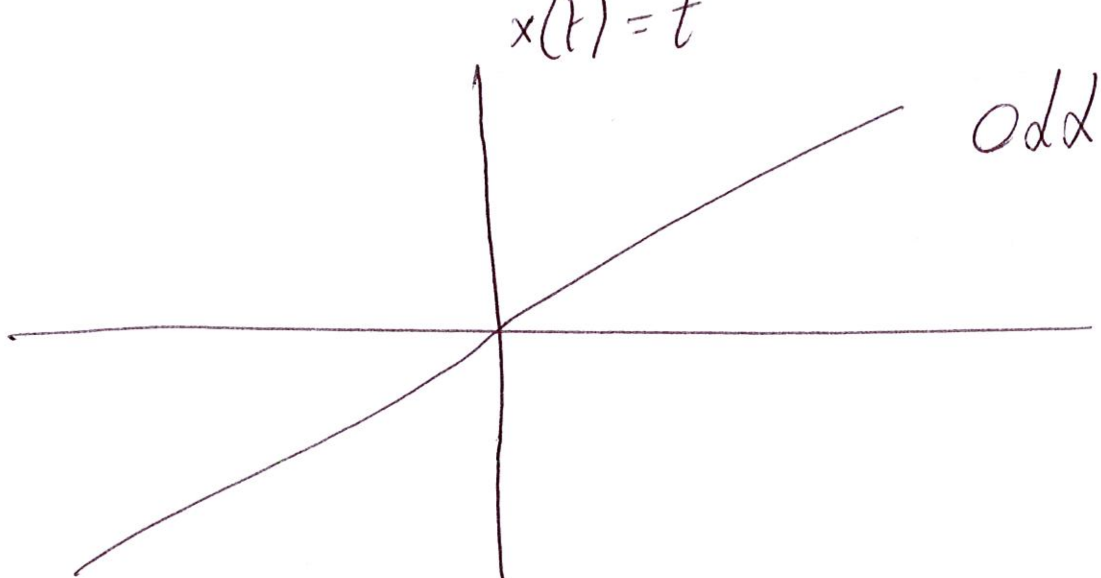


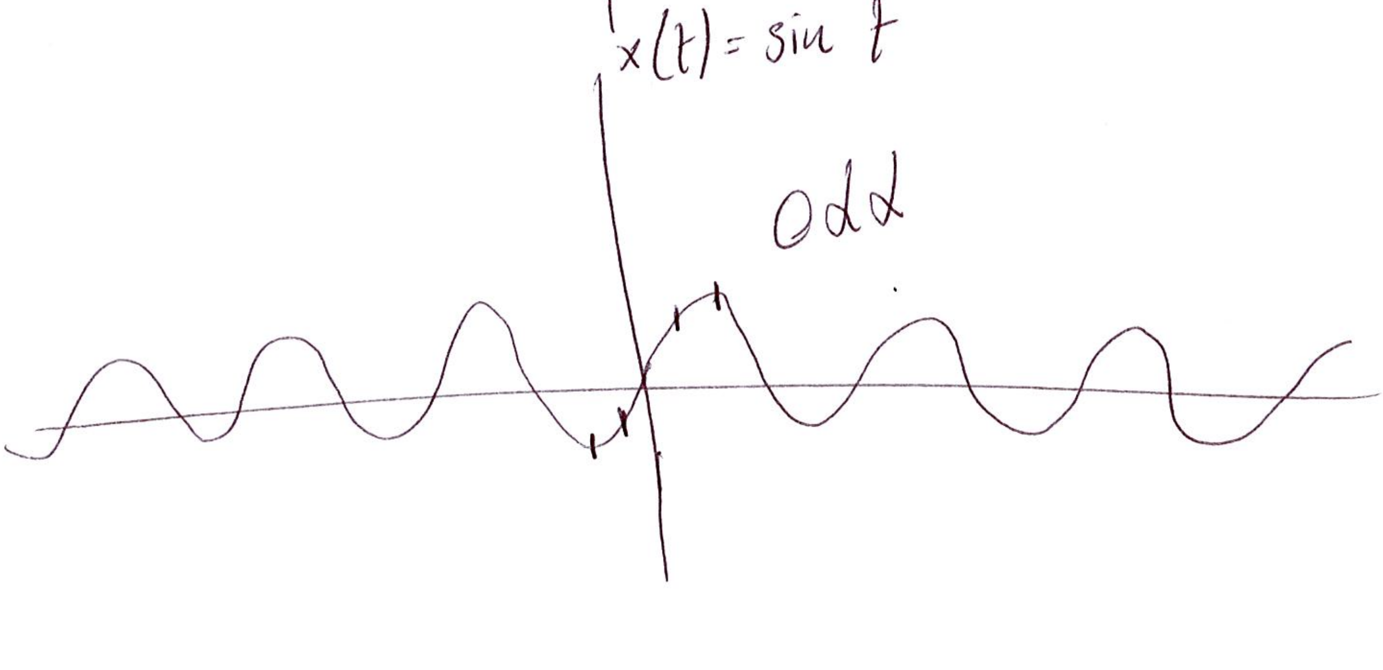
$$x(t) = \begin{cases} t, & t \geq 0 \\ -t, & t < 0 \end{cases}$$



An odd signal is a signal satisfying:

$$x(t) = -x(-t) \text{ for all } t$$





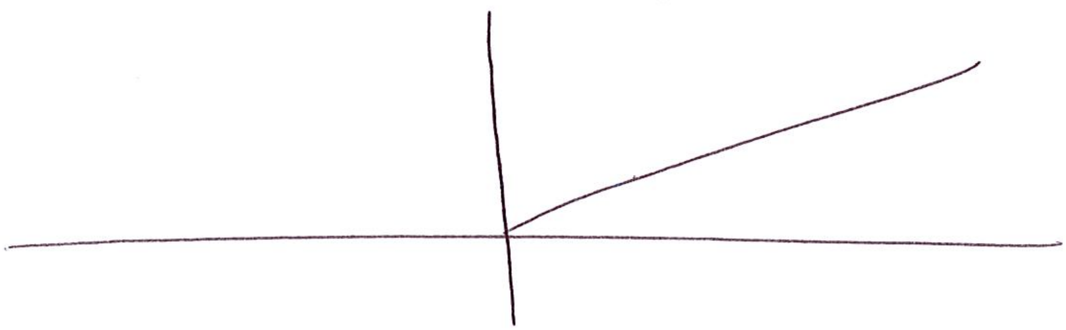
For any signal $x(t)$:

Even Part

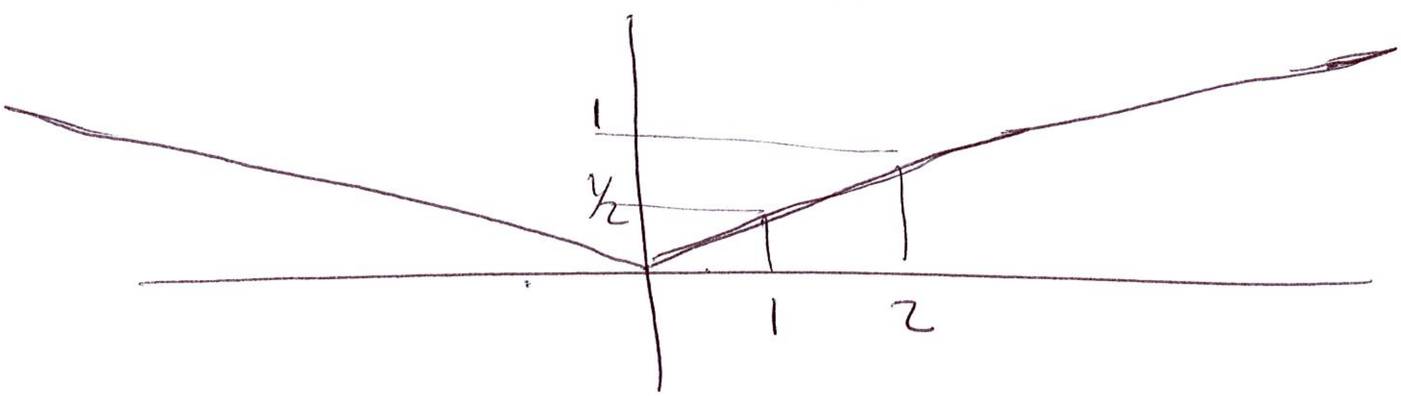
$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right]$$

Also a signal that is even

$$x(t) = t, \quad t \geq 0$$



$$\mathcal{E}v\{x(t)\}$$

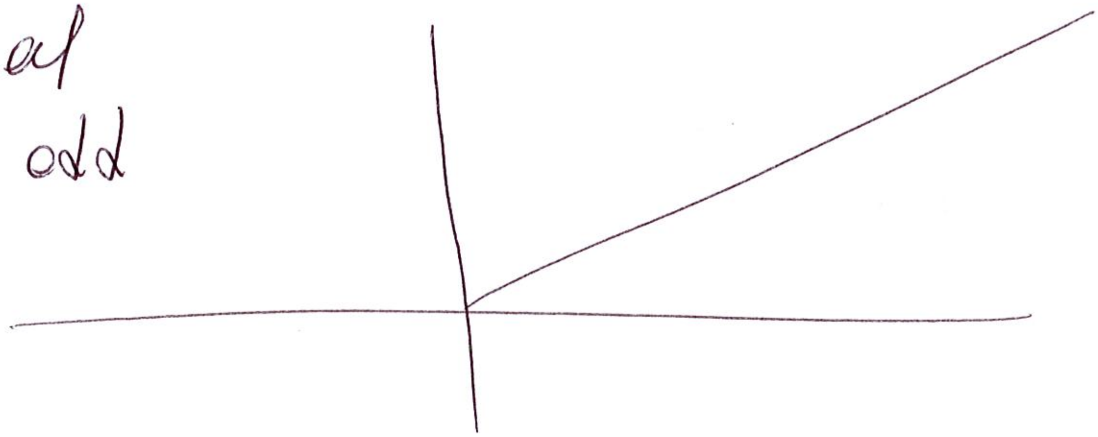


Odd Part

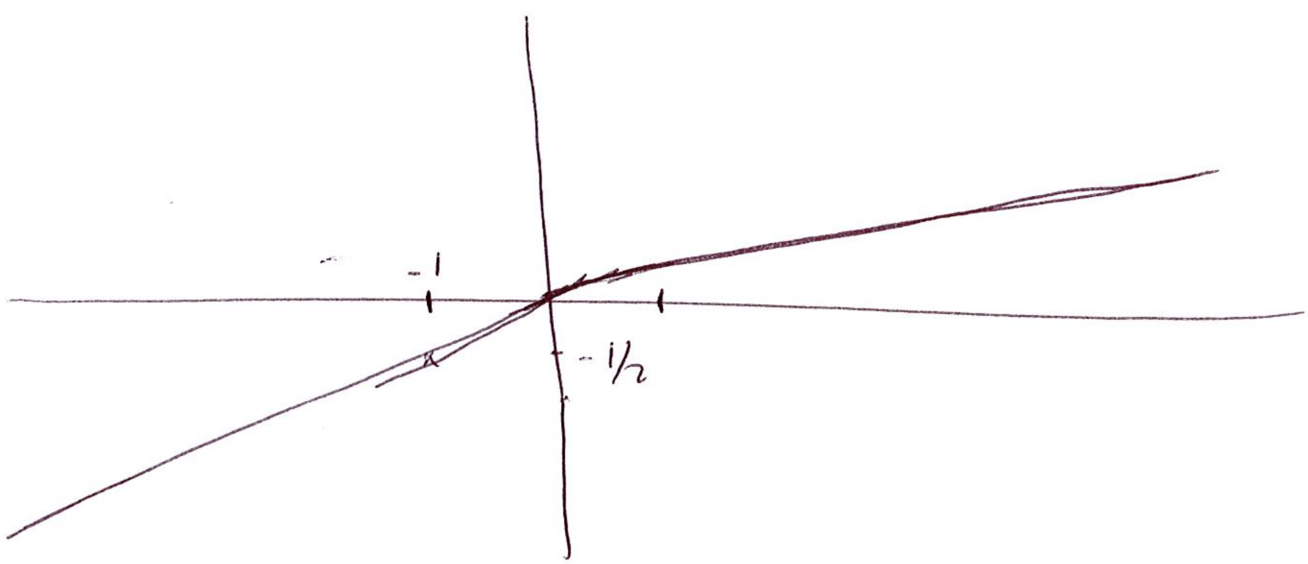
$$Od\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right]$$

Also a signal that is odd

$x(t) = t, \quad t \geq 0$



$Od\{x(t)\}$



For any $x(t)$

$$x(t) = \mathcal{E}v\{x(t)\} + \mathcal{O}d\{x(t)\}$$

$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\left[x(t) + x(-t)\right] \qquad \mathcal{O}d\{x(t)\} = \frac{1}{2}\left[x(t) - x(-t)\right]$$