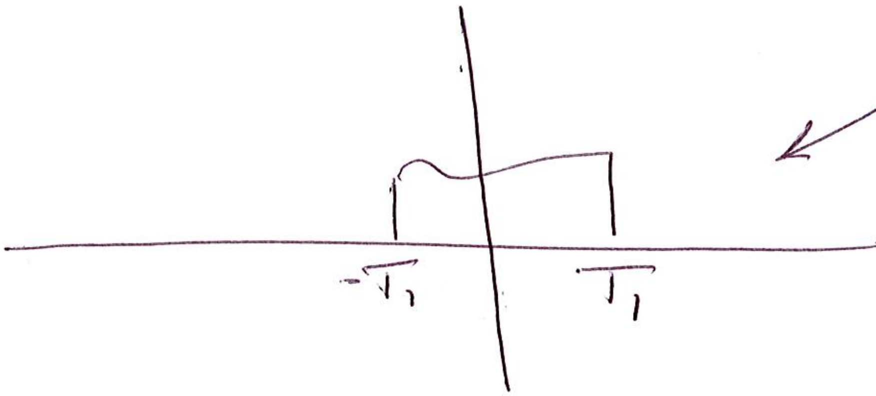
ECE 301 - Lecture #22

Fourier Transform (Continued)

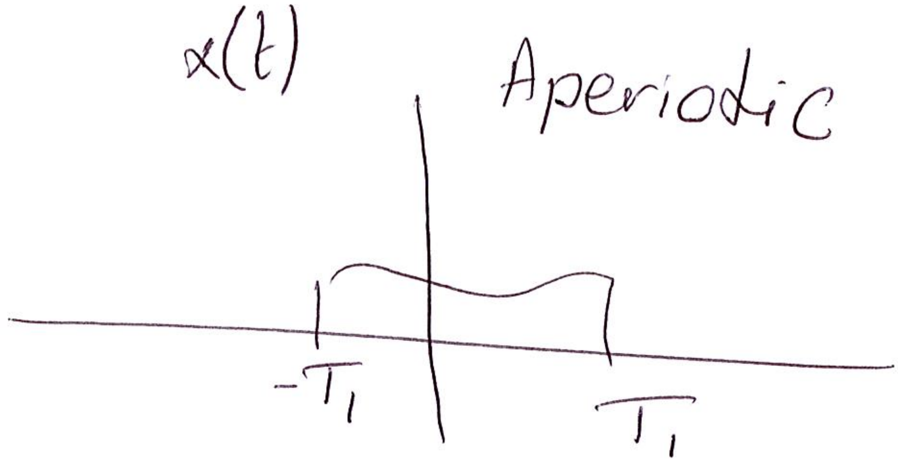


we want to find Fourier Transform

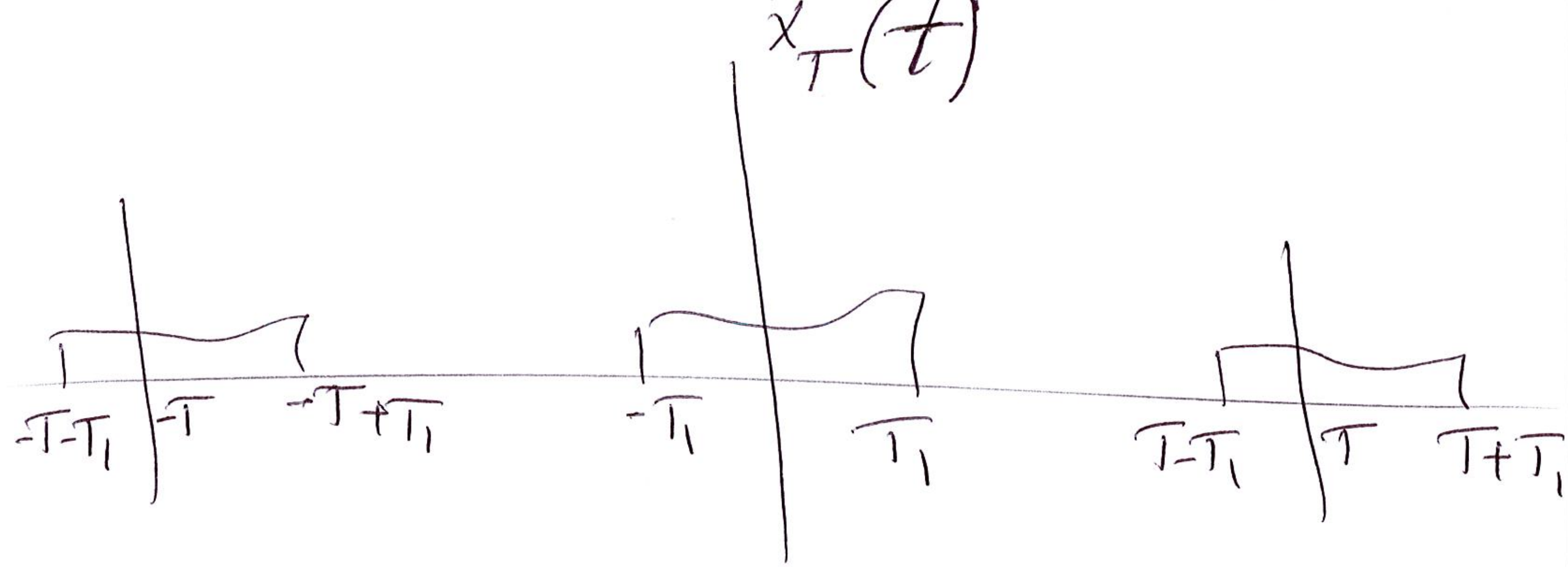
- We know how can we represent <u>periodic</u> signals as linear combinations of complex exponentials (sinusoids).

- We have seen in last class how to reperesent <u>aperiodic</u> signals as linear combinations (over a continuum) of ~~ex~~ complex exponentials (sinusoids)

Aperiodic



Assume "virtual" period



$$x(t) = \lim_{T \to \infty} \tilde{x}_T(t)$$

Start with a FS representation of $\tilde{x}_T(t)$, take the limit as $T \to \infty$

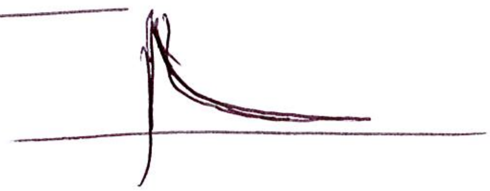
$\Longrightarrow$ FT representation for $x(t)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

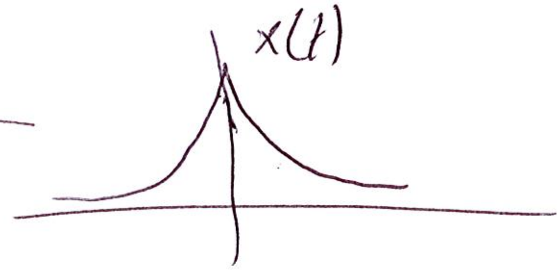
We then did two examples

1. $x(t) = e^{-at} u(t), \; a > 0$

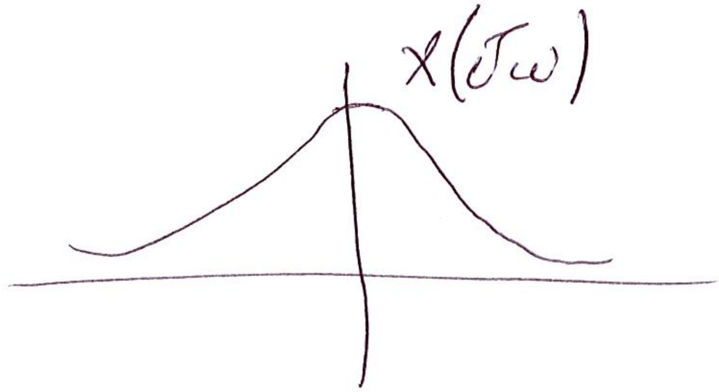


$$X(j\omega) = \frac{1}{a + j\omega}$$

2. $x(t) = e^{-a|t|}$



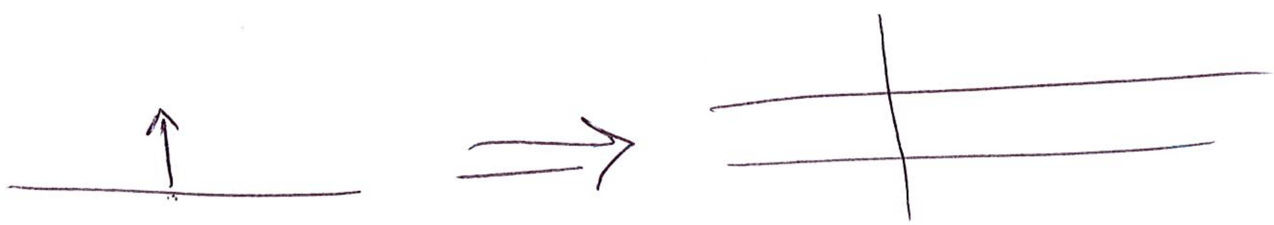
$$X(j\omega) = \frac{2a}{a^2 + \omega^2}$$



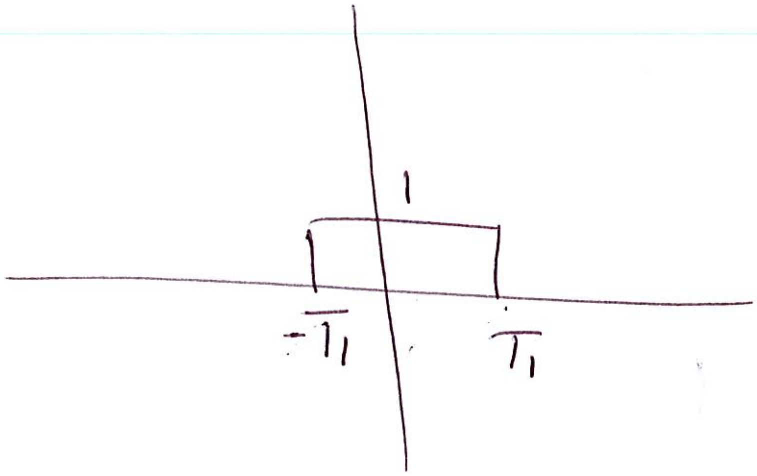
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \delta(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt = 1$$



Example 4.4



$$x(t) = \begin{cases} 1 & , \ |t| \leq T_1 \\ 0 & , \ |t| > T_1 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

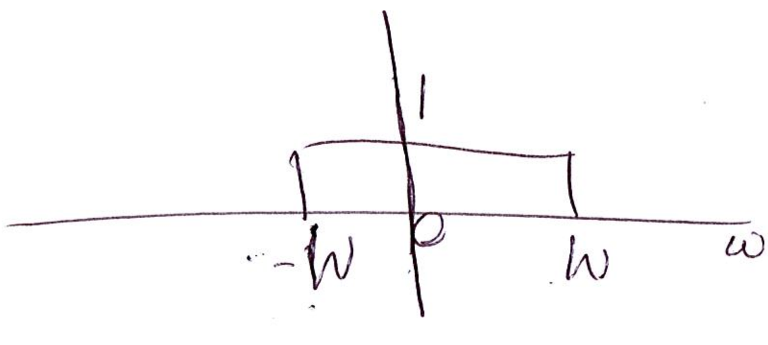
$$= \int_{-T_1}^{T_1} e^{-j\omega t}\, dt \quad = -\frac{1}{j\omega}\, e^{-j\omega t}\Big|_{-T_1}^{T_1}$$

$$= 2\,\frac{\sin \omega T_1}{\omega}$$

Sinc signal

(6)

$$X(j\omega) = \begin{cases} 1, & |\omega| \leq W \\ 0, & |\omega| > W \end{cases}$$

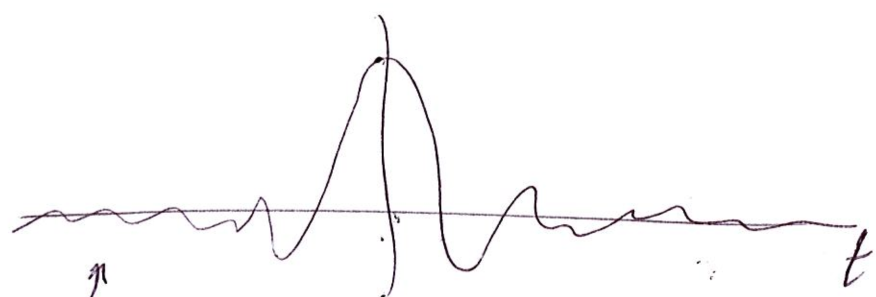


Low-Pass filter
"Frequency response of an LTI System"

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\, d\omega$$

$$= \frac{\sin Wt}{\pi t}$$

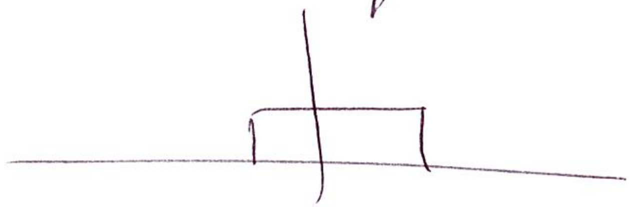


Time domain Impulse response of a LPF "Non-Causal"
"Oscillatory behavior"

Ideal LPF

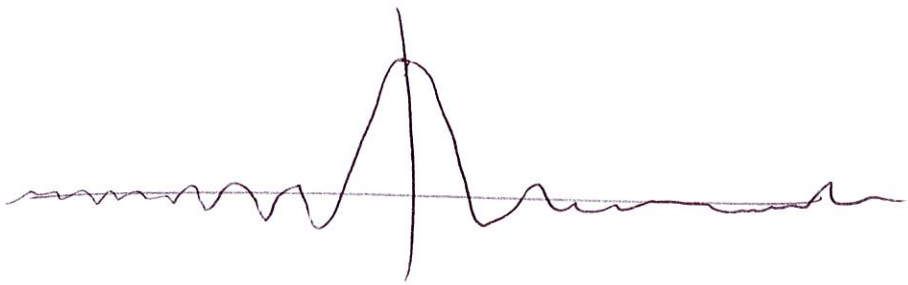
Freq-Domain

$H(e^{j\omega})$



Time-Domain

$h(t)$



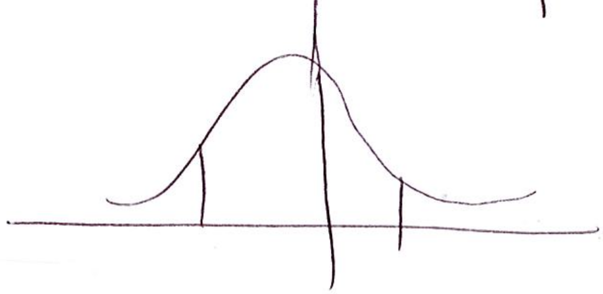
Two Issues in practice

- Non-Causal
- Oscillatory behavior

Approximation to LPF

$h(t) = e^{-at}\, u(t), \quad a > 0$

$X(e^{j\omega}) = \frac{1}{a + j\omega}$



Fourier Transform for Periodic Signals

$$\boxed{\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega \\ X(j\omega) &= \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \end{aligned}}$$

Example

$$X(j\omega) = 2\pi\,\delta(\omega-\omega_0)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\,\delta(\omega-\omega_0)\, e^{j\omega t}\, d\omega$$

$$= e^{j\omega_0 t}$$

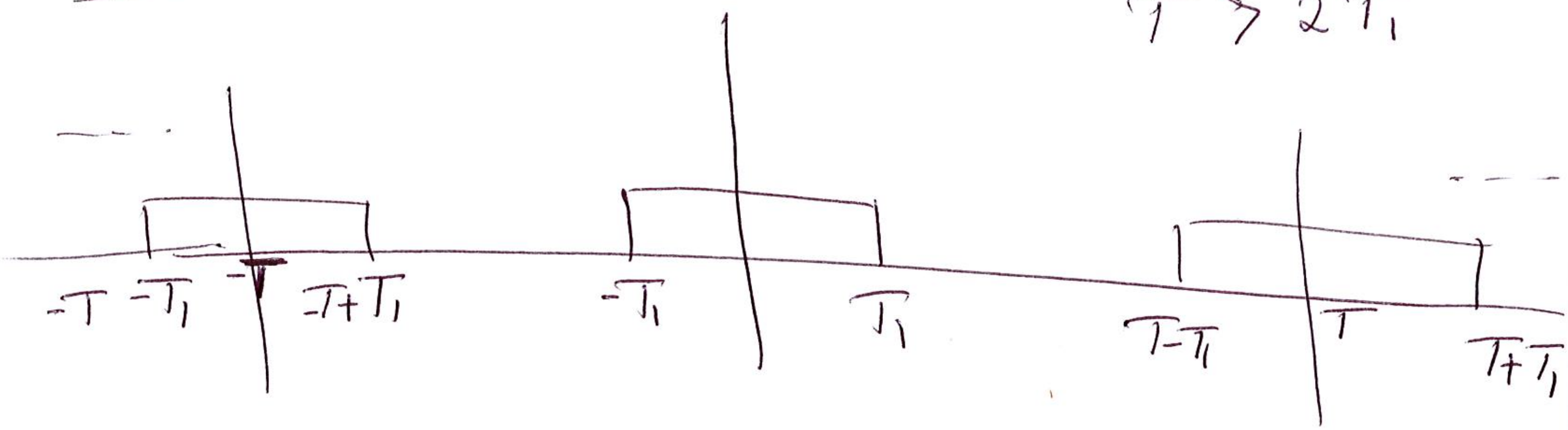
FS representation for periodic signals

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{jk\omega_0 t}$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \, \delta(\omega - k\omega_0)$$

Example 4.6

$T > 2T_1$



$$a_k = \frac{\sin k\omega_0 T_1}{k\pi}$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2 \frac{\sin k\omega_0 T_1}{k} \, \delta(\omega - k\omega_0)$$