ECE 301- Lecture # 23

For periodic signals

$$x(t) \xleftrightarrow{FS} a_k$$

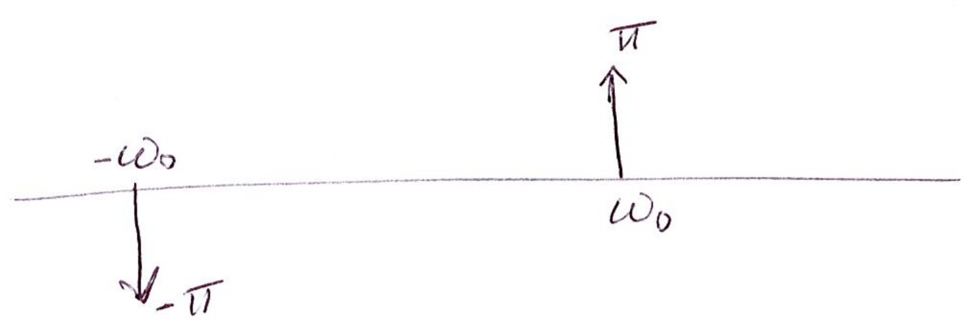
then $x(t) \xleftrightarrow{FT} X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \,\delta(\omega - k\omega_0)$

Examples

$x(t) = \sin \omega_0 t$

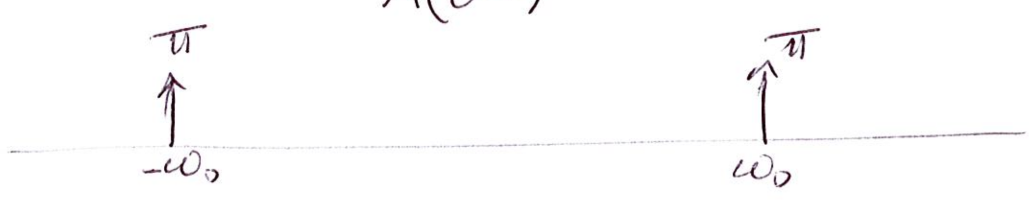
$$a_1 = \frac{1}{2j} \qquad a_{-1} = -\frac{1}{2j}$$

$$|X(j\omega)|$$



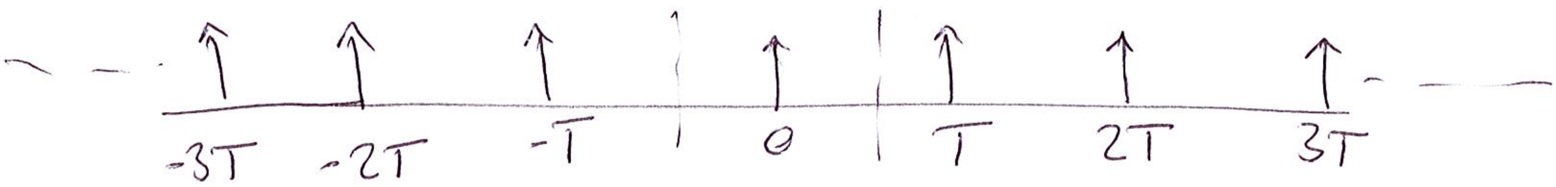
$x(t) = \cos \omega_0 t \qquad a_1 = \frac{1}{2} \qquad a_{-1} = \frac{1}{2}$

$$X(j\omega)$$



Example 4.8

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$



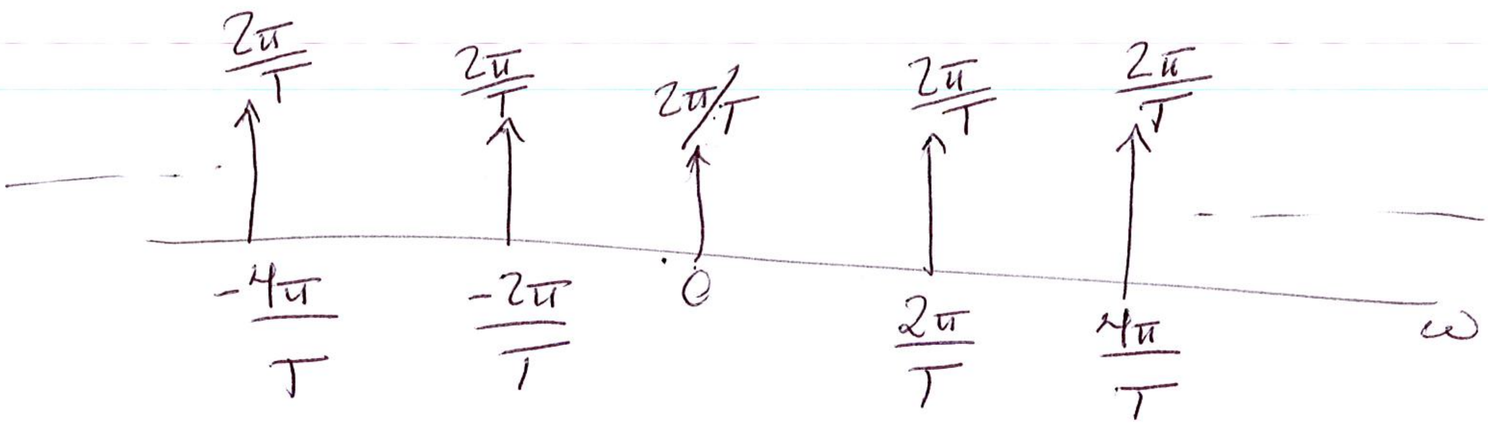
$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-jk\overbrace{\omega_0}^{\frac{2\pi}{T}} t}\, dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} \delta(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}$$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta(\omega - k\omega_0)$$

$$= \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\, \delta\left(\omega - \frac{2\pi k}{T}\right)$$

$X(j\omega)$



Properties of the Continuous-Time Fourier Transform

Linearity

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$y(t) \xleftrightarrow{FT} Y(j\omega)$$

$$\cancel{axy} \quad a\,x(t) + b\,y(t) \xleftrightarrow{FT} a\,X(j\omega) + b\,Y(j\omega)$$

Time Shifting

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t-t_0) \xleftrightarrow{FT} ? \quad \tilde{X}(j\omega) = e^{-j\omega t_0} X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\tilde{X}(j\omega) = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\omega t} dt$$

Let $\tilde{t} = t - t_0$

$$= \int_{-\infty}^{\infty} x(\tilde{t}) e^{-j\omega(\tilde{t}+t_0)} d\tilde{t} \quad (1)$$

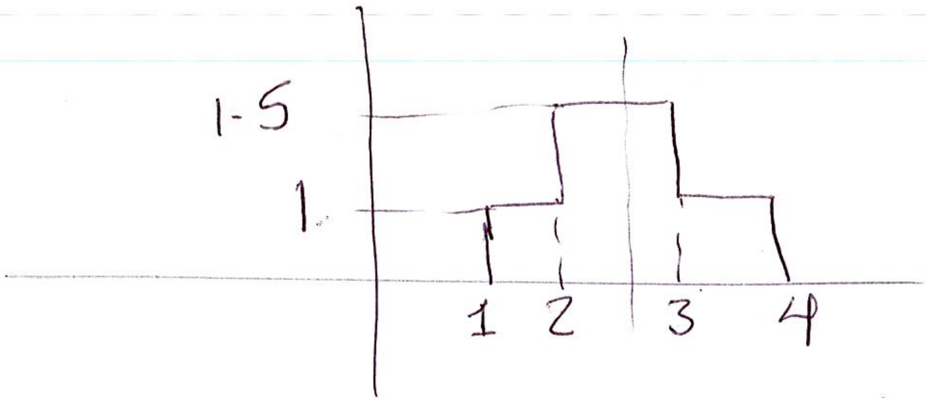
$$e^{-j\omega t_0} X(j\omega) = e^{-j\omega t_0} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} x(t) e^{-j\omega(t+t_0)} dt \quad (2)$$

(1) and (2) are identical

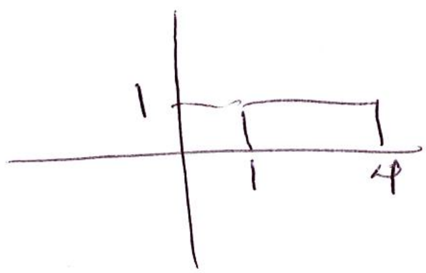
Example 4.9

$x(t)$



$\tilde{x}_1(t) = x_1\left(t - \frac{5}{2}\right)$ $\qquad$ $\tilde{x}_2(t) = \frac{1}{2} x_2\left(t - \frac{5}{2}\right)$

$=$



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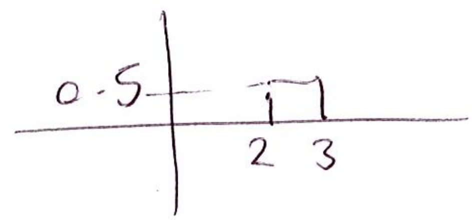
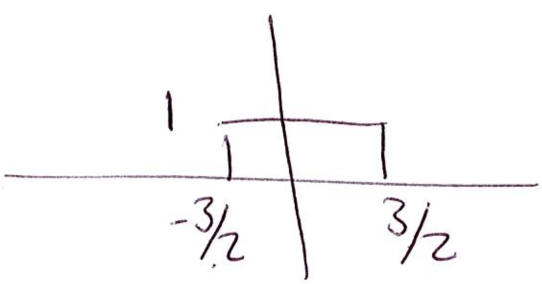
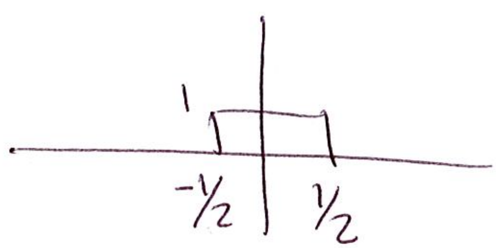


$\tilde{X}_1(j\omega) = X_1(j\omega)\, e^{-j\omega\frac{5}{2}}$ $\qquad$ $\tilde{X}_2(j\omega) = \frac{1}{2} X_2(j\omega)\, e^{-j\omega\frac{5}{2}}$

$x_1(t)$



$x_2(t)$



$$X_1(j\omega) = \frac{2\sin\left(\frac{3\omega}{2}\right)}{\omega} \qquad X_2(j\omega) = \frac{2\sin\left(\frac{\omega}{2}\right)}{\omega}$$

(6)

$$X(j\omega) = e^{-j\frac{5\omega}{2}} \left[\frac{\sin(\omega/2) + 2\sin\left(\frac{3\omega}{2}\right)}{\omega} \right]$$

Differentiation

$$x(t) \xleftrightarrow{FT} X(j\omega)$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$x'(t) = \frac{d\,x(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underbrace{j\omega\, X(j\omega)}_{=}\, e^{j\omega t}\, d\omega$$

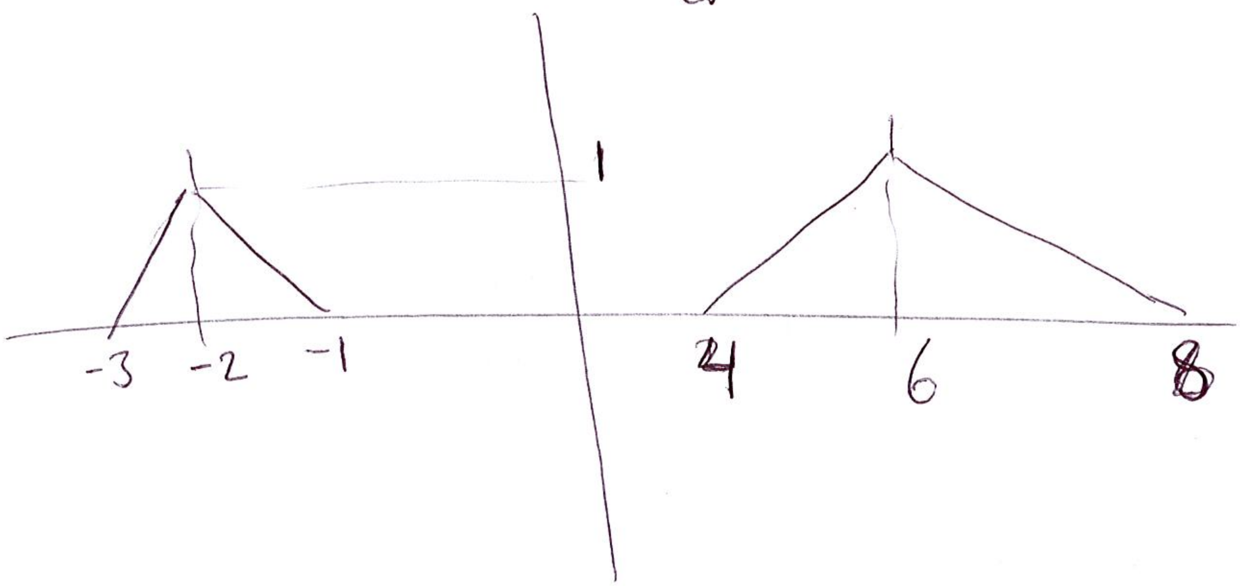
$$x'(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overbrace{X'(j\omega)}\, e^{j\omega t}\, d\omega$$

$$X'(j\omega) = j\omega\, X(j\omega)$$

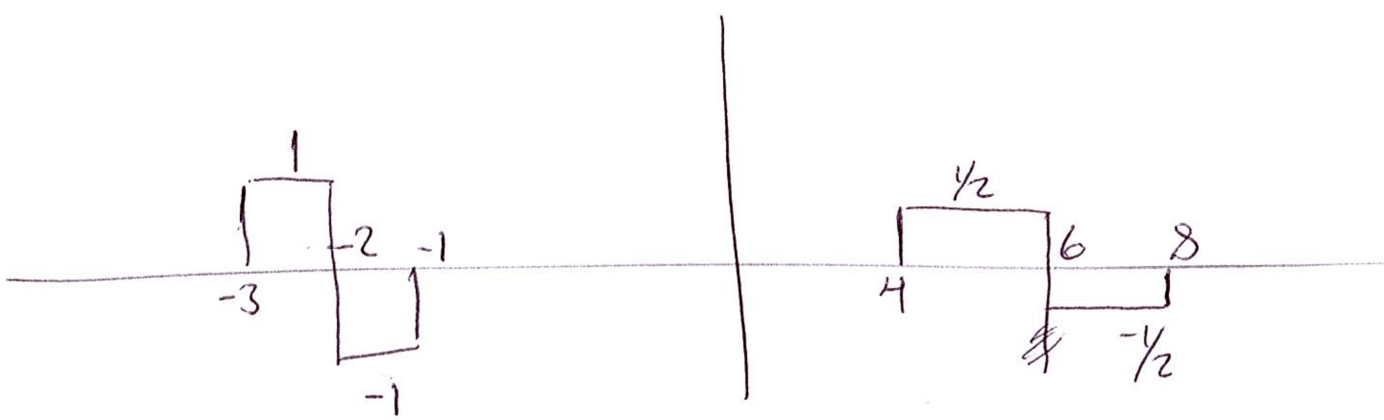
Integration

$$\int_{-\infty}^{t} x(\tau)\, d\tau \xleftrightarrow{FT} \frac{1}{j\omega} X(j\omega) + \pi X(0)\,\delta(\omega)$$

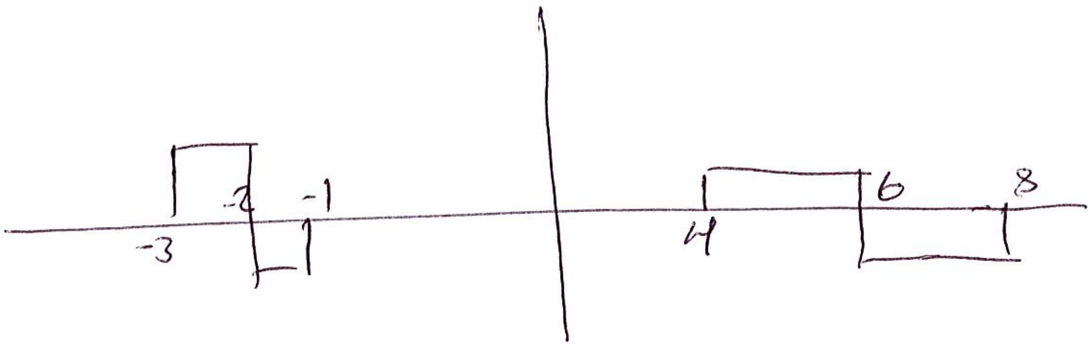
$$x(t) = \int_{-\infty}^{t} x'(\tau)\, d\tau$$



$$x'(t) = \frac{d\,x(t)}{dt}$$



$x'(t)$



$$x''(t) = \frac{d\,x'(t)}{dt}$$