HW 4 Solutions

Problem 2

FS coeffs of

$$\cos\left(\frac{4\pi n}{5} + \frac{\pi}{5}\right)\sin\left(\frac{4\pi n}{5}\right) + \sin\left(\frac{7\pi n}{6} + \frac{\pi}{6}\right)\cos\left(\frac{7\pi n}{6}\right)$$

$$\frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{2}{5} \quad \rightarrow 5$$

$$N_1 = 5$$

$$\frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{7}{12} \quad \rightarrow 12$$

$$N_2 = 12$$

$$N = \text{LCM}(N_1, N_2) = 60$$

$$\omega_0 = \frac{2\pi}{60} = \frac{\pi}{30}$$

$$\cos\left(\frac{4\pi n}{8} + \frac{\pi}{5}\right)$$

a_{24} $\quad$ a_{-24} $\quad$ $\frac{1}{2\jmath}$

$$\sin\left(\frac{4\pi n}{8}\right) \longleftrightarrow b_{24}, \; b_{-24} \leftarrow -\frac{1}{2\jmath}$$

Note

$$\frac{4\pi}{5} = \frac{24\pi}{30} = k\omega_0 \Rightarrow k = 24$$

Same ω_0 $\begin{cases} x[n] \overset{FS}{\longleftrightarrow} a_k \\ y[n] \overset{FS}{\longleftrightarrow} b_k \end{cases}$

$$x[n]\, y[n] \overset{FS}{\longleftrightarrow} c_k = \sum_{\ell = \langle N \rangle} a_\ell\, b_{k-\ell}$$

$$c_{48} = a_{24}\, b_{24}$$

$$c_{-48} = a_{-24}\, b_{-24} = c_{12}$$

$$c_0 = a_{24}\, b_{-24} + a_{-24}\, b_{24}$$

$$\sin\left(\frac{7\pi n}{6} + \frac{\pi}{6}\right) \cos\left(\frac{7\pi n}{6}\right)$$

d_{35}, d_{-35} $\qquad$ e_{35}, e_{-35}

$f_{70} = d_{35}\, e_{35} = f_{10}$ $\qquad$ $f_0 = d_{35}\, e_{-35} + d_{-35}\, e_{35}$

$f_{-70} = d_{-35}\, e_{-35} = f_{-10} = f_{50}$

$$g_0 = c_0 + f_0$$

$$g_{10} = f_{10}$$

$$g_{12} = c_{12}$$

$$g_{48} = c_{48}$$

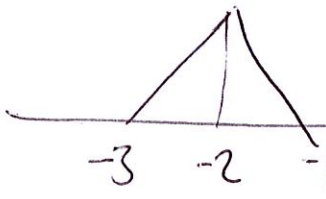
$$g_{50} = c_{50}$$

$$a_k = \frac{1}{N} \sum_{n} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n} \qquad k = \langle N \rangle$$

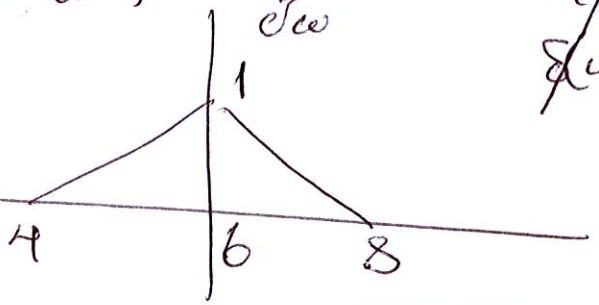
$$a_0 = \frac{1}{N} \sum_{n} x[n] \qquad k = \langle N \rangle$$

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\frac{2\pi}{N}n}$$

$x(t)$



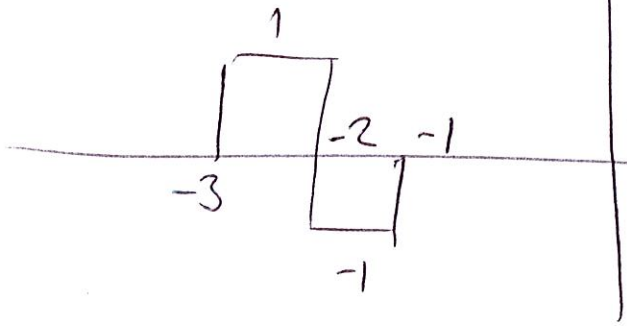
$$X(j\omega) = \frac{X'(j\omega)}{j\omega} + \pi X'(0)\,\delta(\omega) \quad (4)$$

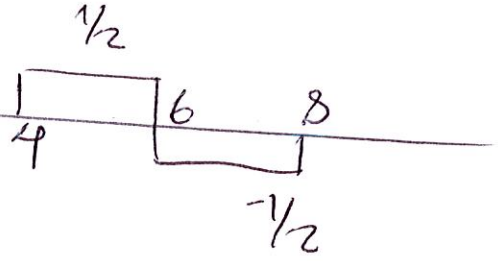


$\omega = 0 \Rightarrow$ L'hôpital

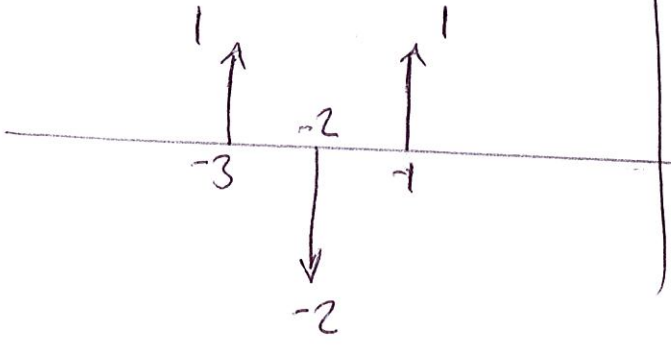
$$x'(t) = \frac{dx(t)}{dt}$$

$$X'(j\omega) = \frac{X''(j\omega)}{j\omega} + \pi X''(0)\,\delta(\omega)$$

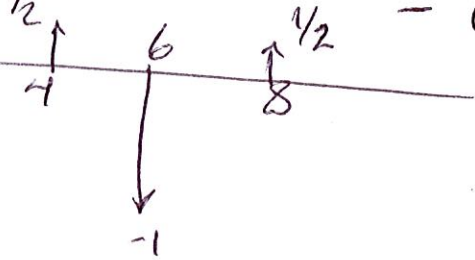


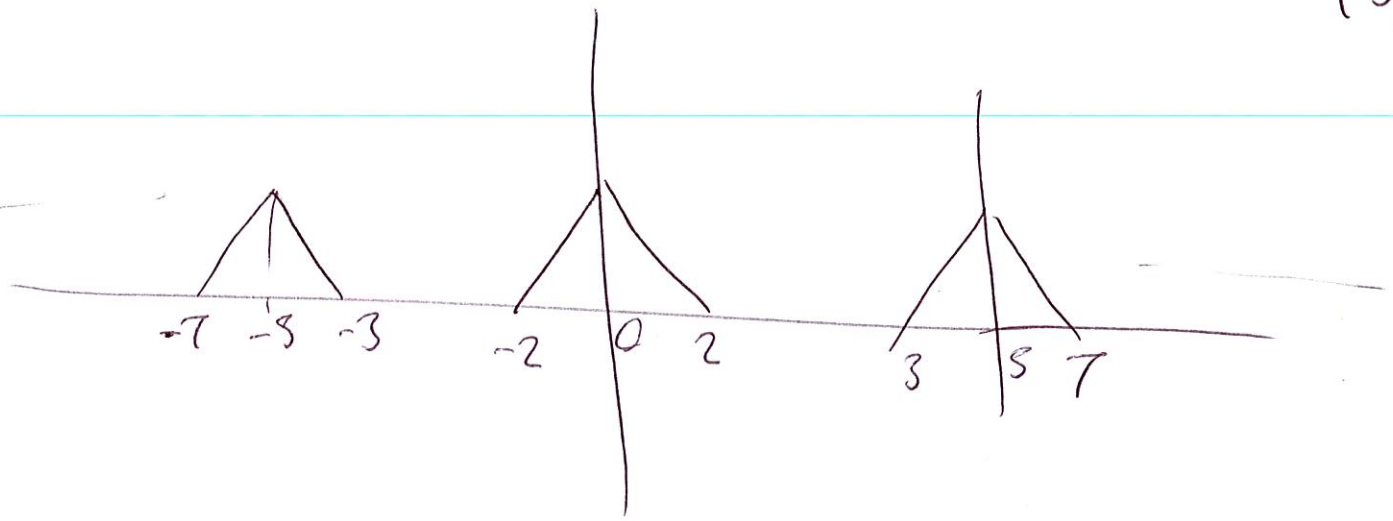


$$x''(t) = \frac{dx'(t)}{dt}$$

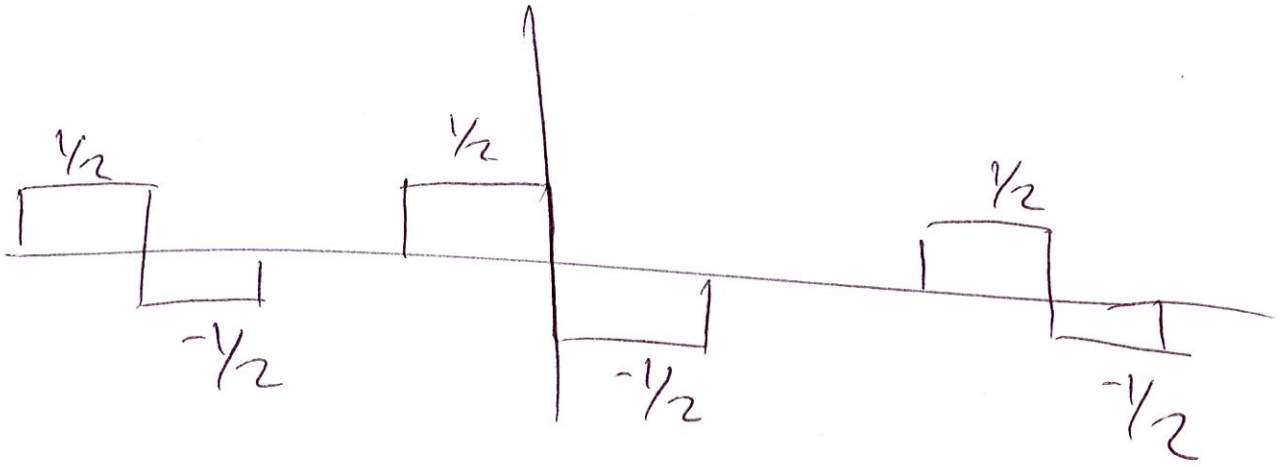


$$X''(j\omega) = e^{j\omega} - 2e^{j2\omega} + e^{j3\omega} + \frac{1}{2}e^{-j4\omega} - e^{-j6\omega} + \frac{1}{2}e^{-j8\omega}$$





$\frac{d}{dt}$ ⇓



$\frac{d}{dt}$ ⇓

$a_k = \frac{1}{2} e^{-jk\omega_0 \cdot 2} \cdot \frac{1}{5}$