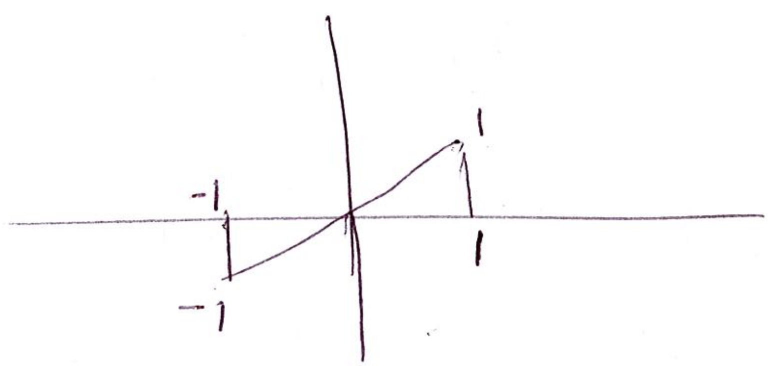
ECE 301 - Lecture # 25

Example 4.12

$$g(t) = \frac{d}{dt}\, x(t)$$

$x(t)$



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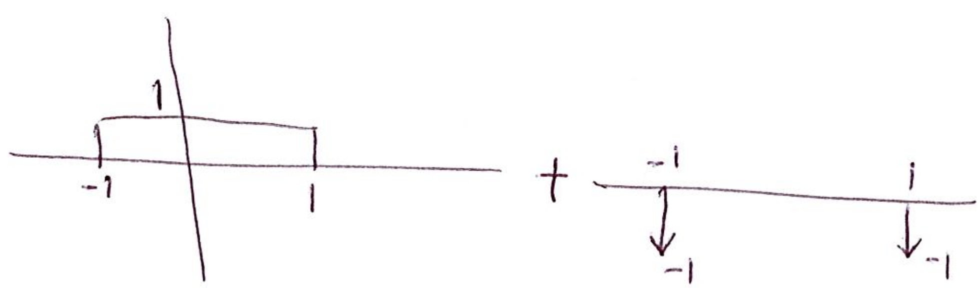

$$G(j\omega) = \left(\frac{2\sin\omega}{\omega}\right) - e^{j\omega} - e^{-j\omega}$$

$$X(j\omega) = \frac{G(j\omega)}{j\omega} + \pi\, G(0)\, \delta(\omega)$$

$$X(j\omega) = \frac{2\sin\omega}{j\omega^2} - \frac{2\cos\omega}{j\omega}$$

Because $x(t)$ is real and odd,

$X(j\omega)$ is pure imaginary and odd

Time and Frequency Scaling

If $\quad x(t) \xleftrightarrow{\mathcal{F}} X(j\omega)$

then $\quad x(at) \xleftrightarrow{\mathcal{F}} \dfrac{1}{|a|} X\left(\dfrac{j\omega}{a}\right)$

$$\mathcal{F}\{x(at)\} = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega t}\, dt$$

Let $\tau = at$

$$\mathcal{F}\{x(at)\} = \begin{cases} \dfrac{1}{a} \displaystyle\int_{-\infty}^{\infty} x(\tau)\, e^{-j(\omega/a)\tau}\, d\tau \\ -\dfrac{1}{a} \displaystyle\int_{-\infty}^{\infty} x(\tau)\, e^{-j(\omega/a)\tau}\, d\tau \end{cases}$$

Note: Compress in time $\Longleftrightarrow$

Faster Signal $\Longleftrightarrow$ Stretch in Frequency

Stretch in Time $\Longleftrightarrow$ Slower Signal

$\Longleftrightarrow$ Compress in Frequency

Duality

$$x_1(t) = \begin{cases} 1 & , \ |t| < T_1 \\ 0 & , \ |t| > T_1 \end{cases} \xleftrightarrow{\mathcal{F}} X_1(j\omega) = \frac{2\sin\omega T_1}{\omega}$$

$$x_2(t) = \frac{\sin Wt}{\pi t} \xleftrightarrow{\mathcal{F}} X_2(j\omega) = \begin{cases} 1 & , \ |\omega| < W \\ 0 & , \ |\omega| > W \end{cases}$$

Example 4.13

$$g(t) = \frac{2}{1+t^2}$$

Fact

$$\boxed{\text{If } X(j\omega) = \frac{2}{1+\omega^2} \quad \text{then } x(t) = e^{-|t|}}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega$$

$$e^{-|t|} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{1+\omega^2} e^{j\omega t} \, d\omega$$

$$2\pi e^{-|t|} = \int_{-\infty}^{\infty} \frac{2}{1+\omega^2} e^{j\omega t} \, d\omega$$

$$2\pi e^{-|t|} = \int_{-\infty}^{\infty} \frac{2}{1+\omega^2} e^{-j\omega t} \, d\omega$$

Switch ω and t

$$2\pi e^{-|\omega|} = \int_{-\infty}^{\infty} \frac{2}{1+t^2} e^{-j\omega t} dt$$

$$G(j\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$G(j\omega) = 2\pi e^{-|\omega|}$$

Duality in differentiation

$$\frac{d X(j\omega)}{d\omega} = \int_{-\infty}^{\infty} -jt\, x(t)\, e^{-j\omega t} dt$$

$$-jt\, x(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{dX(j\omega)}{d\omega}$$

Duality in Integration

$$-\frac{1}{jt}x(t) + \pi x(0)\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} \int_{-\infty}^{\omega} X(\eta)\, d\eta$$

Duality in time shifting

$$e^{j\omega_0 t} x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(j(\omega-\omega_0))$$

Parserval's Relation

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$