ECE 301 - Lecture #27

Integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{Integrator}$$

$$H(j\omega) = \frac{1}{j\omega} + \pi\,\delta(\omega)$$

why?

$$y(t) = x(t) * h(t)$$

$$Y(j\omega) = X(j\omega)\, H(j\omega)$$

$$= \frac{1}{j\omega} X(j\omega) + \pi\, X(j\omega)\, \delta(\omega)$$

$$= \frac{1}{j\omega} X(j\omega) + \pi\, X(0)\, \delta(\omega)$$

Consistent with Integration Property

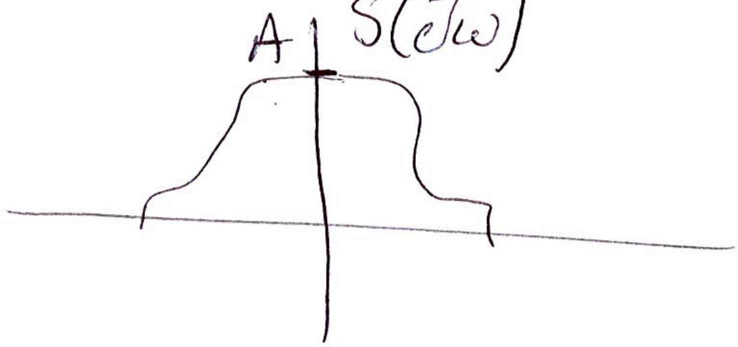
The Multiplication Property

$$r(t) = s(t)\,p(t) \xleftrightarrow{\mathcal{F}} R(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(j\omega)\,P(j(\omega-\theta))\,d\theta$$

Amplitude Modulation

Wireless Communication System

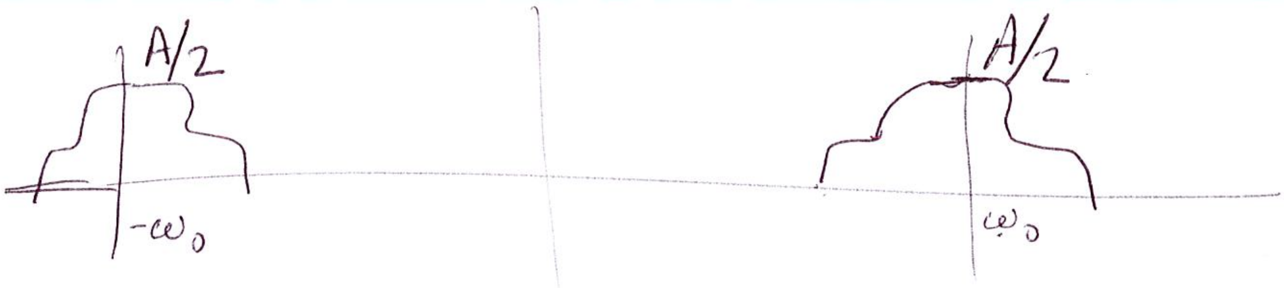
Example 4.21



$$p(t) = \cos \omega_0 t$$

$$P(j\omega) = \pi\,\delta(\omega-\omega_0) + \pi\,\delta(\omega+\omega_0)$$

$R(j\omega)$



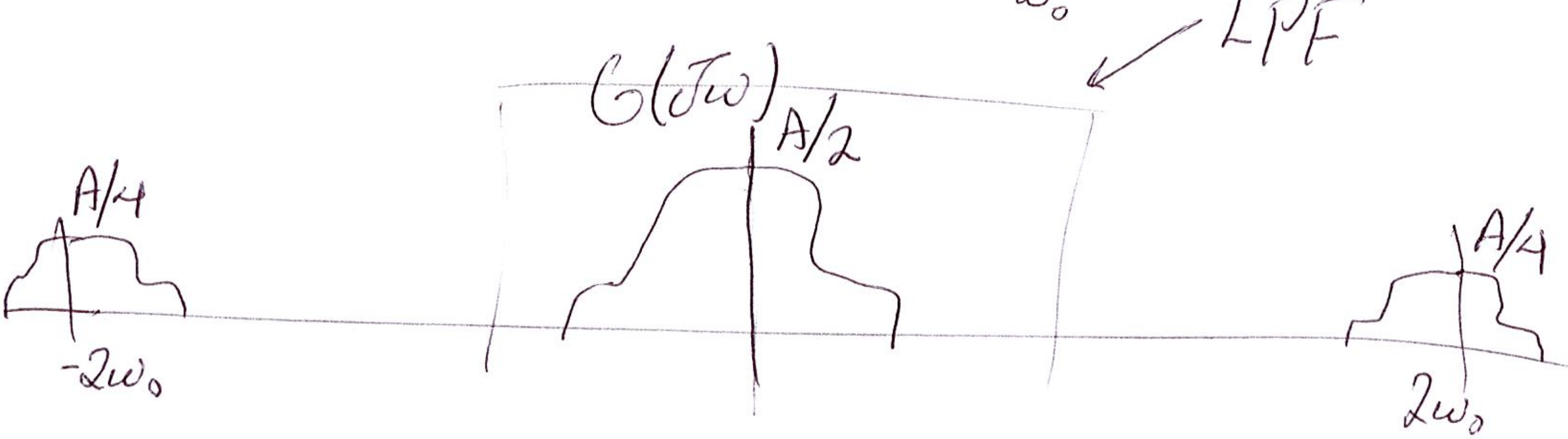
How to process at receiver?

<u>Example 4.22</u>

$$g(t) = r(t)\, p(t)$$

$$p(t) = \cos \omega_0 t$$

$$G(j\omega) = \frac{1}{2\pi} R(j\omega) * P(j\omega)$$



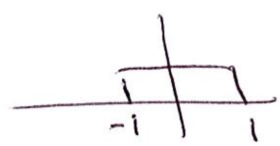
Example 4.23

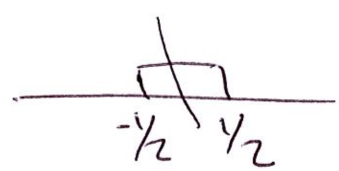
$$x(t) = \frac{\sin(t)\,\sin(t/2)}{\pi t^2}$$

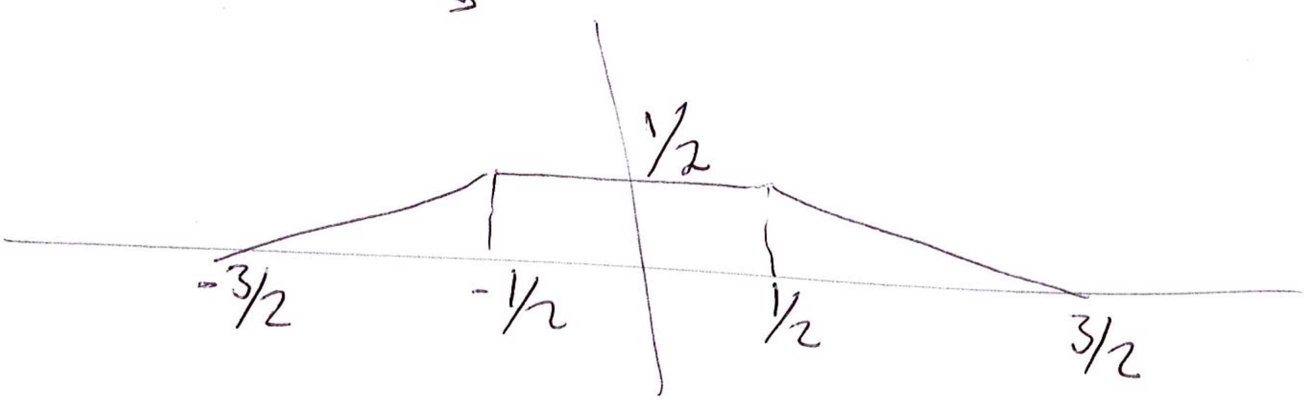
$X(j\omega)$?

$$x(t) = \pi \left(\frac{\sin t}{\pi t}\right)\left(\frac{\sin(t/2)}{\pi t}\right)$$

$$X(j\omega) = \frac{1}{2}\; \underbrace{\mathcal{F}\left\{\frac{\sin t}{\pi t}\right\}} * \underbrace{\mathcal{F}\left\{\frac{\sin t/2}{\pi t}\right\}}$$







Systems Characterized by linear Constant ~~differ~~ Coefficient Differential Equations

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

N-th order

$$\mathcal{F}\left\{ \sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} \right\} = \mathcal{F}\left\{ \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k} \right\}$$

$$\sum_{k=0}^{N} a_k \, \mathcal{F}\left\{ \frac{d^k y(t)}{dt^k} \right\} = \sum_{k=0}^{M} b_k \, \mathcal{F}\left\{ \frac{d^k x(t)}{dt^k} \right\}$$

$$\sum_{k=0}^{N} a_k (j\omega)^k Y(j\omega) = \sum_{k=0}^{M} b_k (j\omega)^k X(j\omega)$$

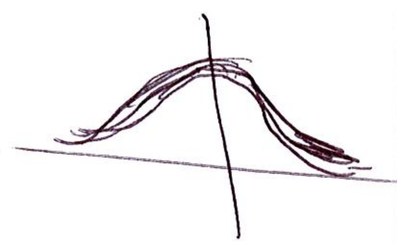
$$Y(j\omega) \sum_{k=0}^{N} a_k (j\omega)^k = X(j\omega) \sum_{k=0}^{M} b_k (j\omega)^k \quad (6)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k}$$

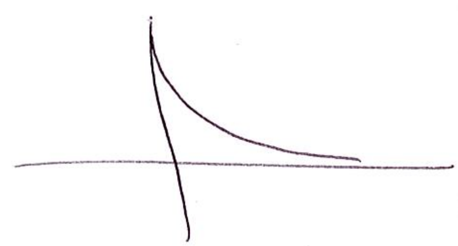
Example 4-24

$$\frac{d\,y(t)}{dt} + a\,y(t) = x(t)$$

$$H(j\omega) = \frac{1}{a + j\omega}$$



$$h(t) = e^{-at}\,u(t)$$



LPF Approximation

Example 4.25

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

$$H(j\omega) = \frac{2 + j\omega}{3 + 4j\omega + (j\omega)^2}$$

$$= \frac{2 + j\omega}{(1 + j\omega)(3 + j\omega)}$$

Partial Fraction Expansion

$$= \frac{1/2}{1 + j\omega} + \frac{\frac{1}{2}}{3 + j\omega}$$

$$h(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)$$

$$x(t) = e^{-t} u(t)$$

$$y(t) \,?$$

$$Y(j\omega) = H(j\omega)\, X(j\omega)$$

$$= \left[\frac{2+j\omega}{(1+j\omega)(3+j\omega)}\right]\left(\frac{1}{1+j\omega}\right)$$

$$= \frac{(2+j\omega)}{(1+j\omega)^2\,(3+j\omega)}$$

$$= \frac{1/4}{1+j\omega} + \frac{1/2}{(1+j\omega)^2} - \frac{1/4}{3+j\omega}$$

$$y(t) = \left[\tfrac{1}{4} e^{-t} + \frac{1}{2} t e^{-t} - \frac{1}{4} e^{-3t}\right] u(t)$$