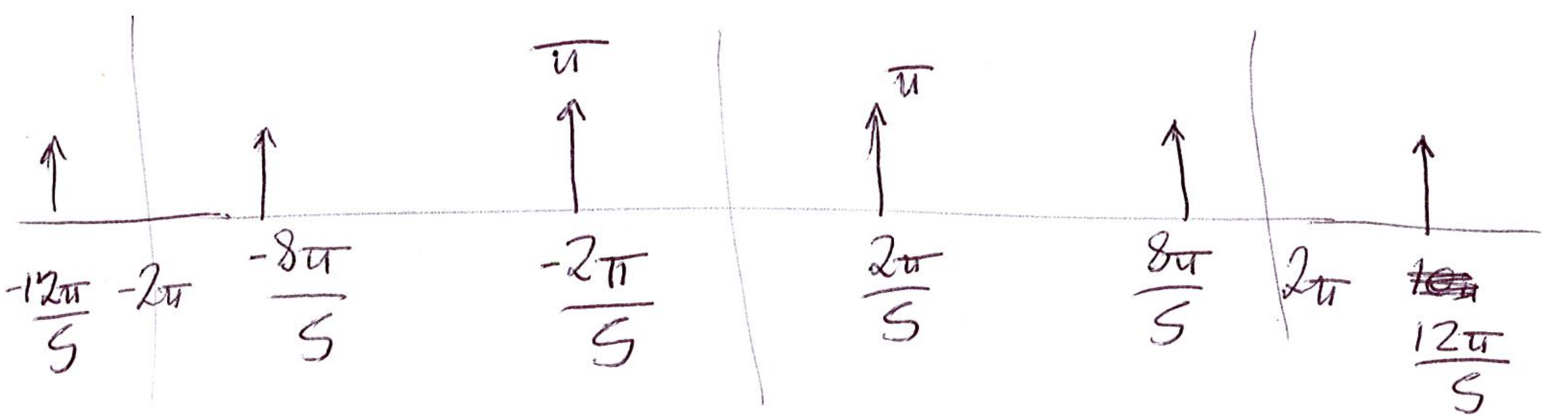
ECE 301 - Lecture #29

DTFT of Periodic Signals

$$x[n] = \cos \frac{2\pi}{5} n = \frac{1}{2} e^{j\frac{2\pi}{5}n} + \frac{1}{2} e^{-j\frac{2\pi}{5}n}$$

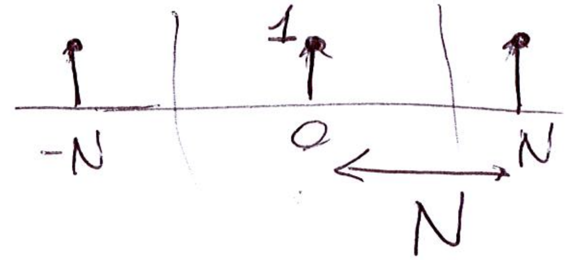
$$\frac{\omega_0}{2\pi} = \frac{m}{N} = \frac{1}{5} \longrightarrow N = 5$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k \, \delta\left(\omega - \frac{2\pi k}{N}\right)$$



Example 5.6

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - kN]$$

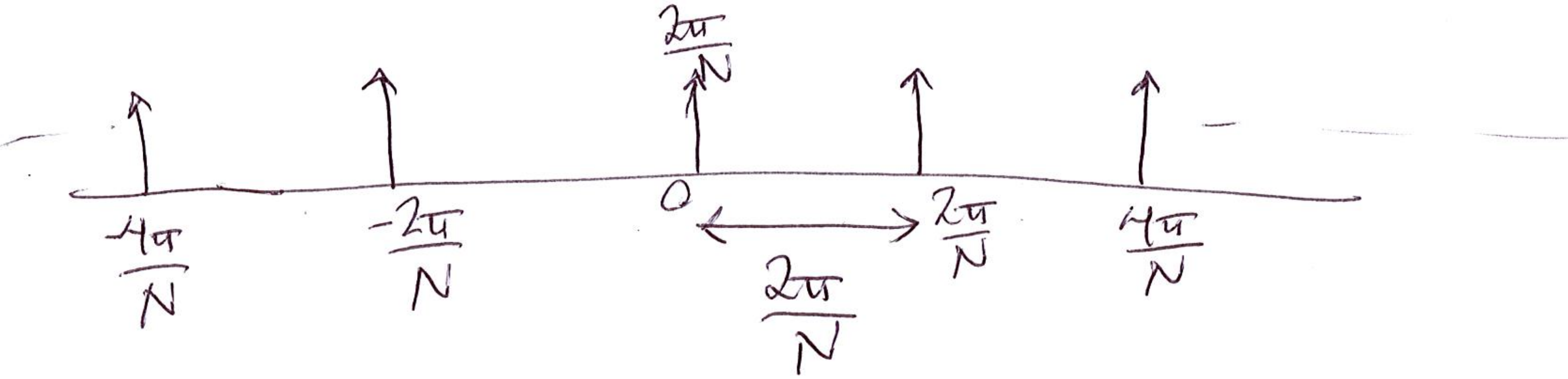


$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n]\, e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$= \frac{1}{N}$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta\left(\omega - \frac{2\pi k}{N}\right)$$

$$= \frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$$



Differencing and Accumulation

$$x[n] - x[n-1] \xrightarrow{\mathcal{F}} \left(1 - e^{-j\omega}\right) X\left(e^{j\omega}\right)$$

$$x[n] \overset{\mathcal{F}}{\longleftrightarrow} X\left(e^{j\omega}\right)$$

$$y[n] = \sum_{m=-\infty}^{n} x[m] \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{1-e^{-j\omega}} X\left(e^{j\omega}\right)$$

$$+ \pi X\left(e^{j0}\right) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

Example 5.8

$$g[n] = \delta[n] \overset{\mathcal{F}}{\longleftrightarrow} G\left(e^{j\omega}\right) = 1$$

$$x[n] = \sum_{m=-\infty}^{n} g[m] = u[n]$$

$$X\left(e^{j\omega}\right) = \frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

Time Expansion

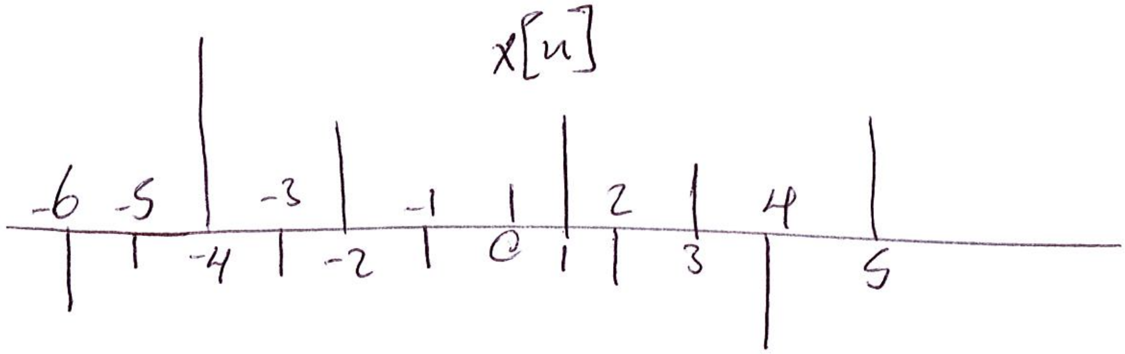
Continuous-time

$$x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$

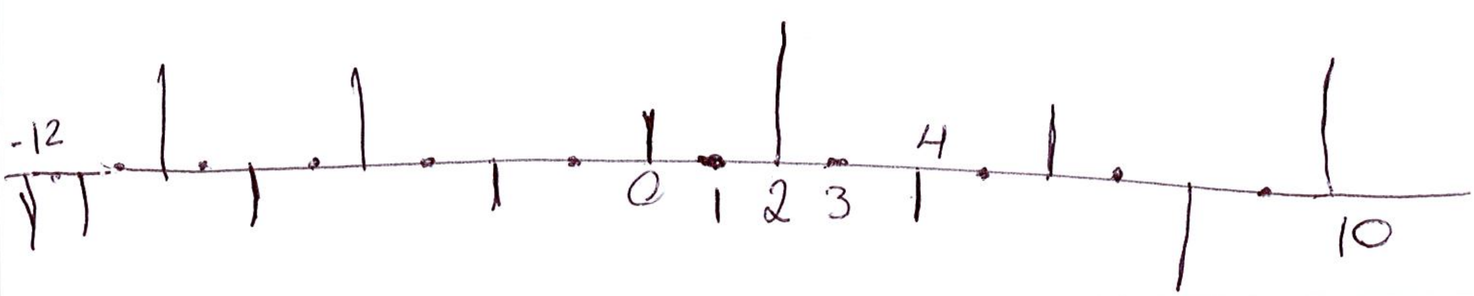
Discrete-time

$$x_{(k)}[n] = \begin{cases} x\left[\frac{n}{k}\right] & \text{if } n \text{ is a multiple of } k \\ 0 & \text{otherwise} \end{cases}$$

$x[n]$



$x_{(2)}[n]$



$$X_{(k)}\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x_{(k)}[n]\, e^{-j\omega n} \qquad (5)$$

$$= \sum_{r=-\infty}^{\infty} x_{(k)}[rk]\, e^{-j\omega rk}$$

$$= \sum_{r=-\infty}^{\infty} x[r]\, e^{-j\omega rk}$$

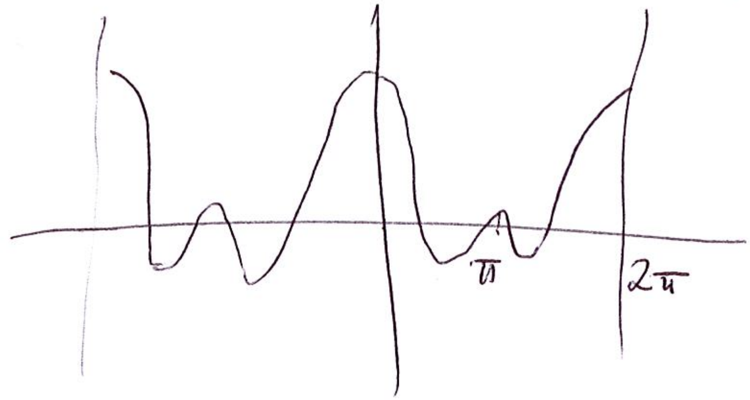
$$= \sum_{r=-\infty}^{\infty} x[r]\, e^{-j(\omega k)r}$$

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} = \sum_{r=-\infty}^{\infty} x[r]\, e^{-j\omega r}$$

$$X_{(k)}\left(e^{j\omega}\right) = X\left(e^{j\omega k}\right)$$

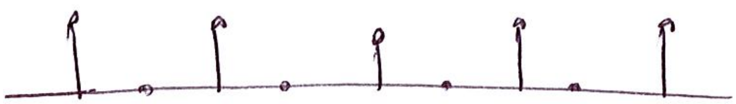
$X(e^{j\omega})$

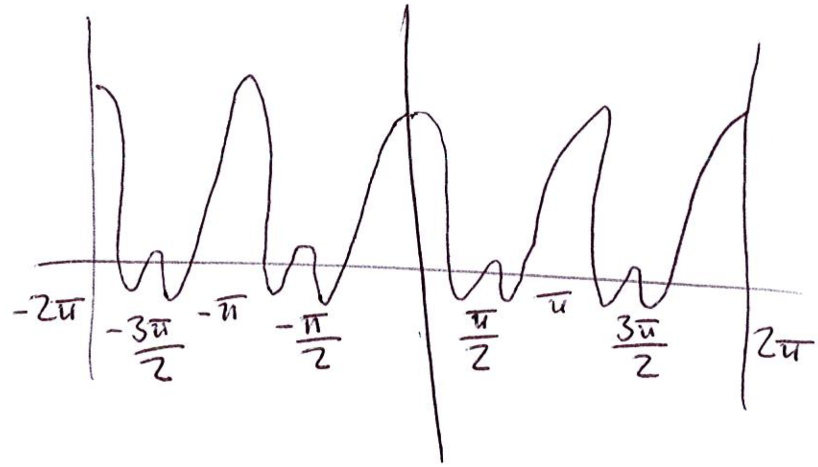
$x[n]$



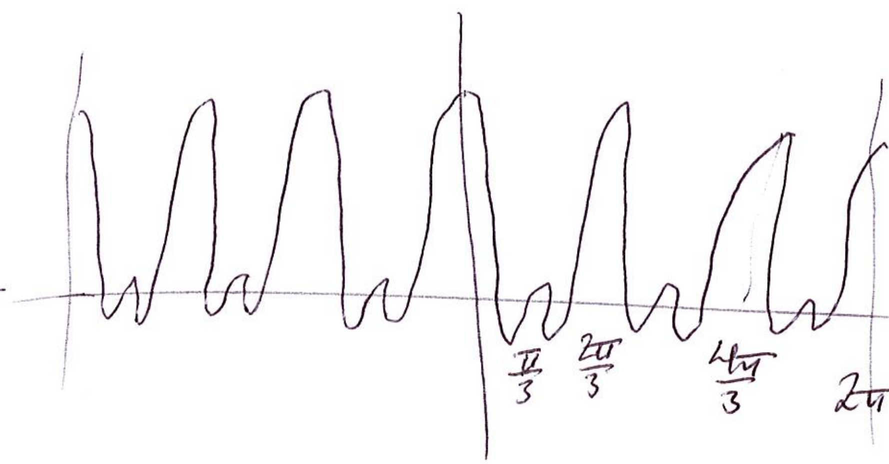
$x_{(2)}[n]$

$X_{(2)}(e^{j\omega})$



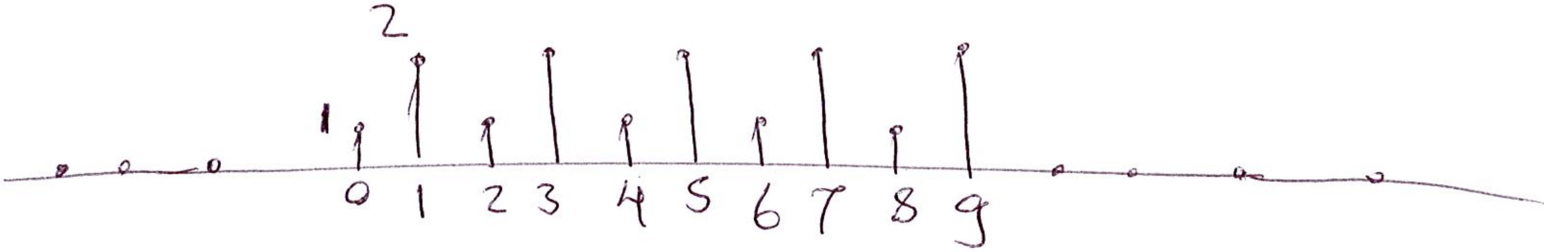


$x_{(3)}[n]$

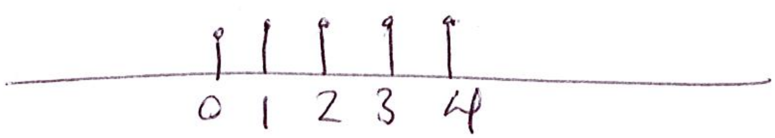


Example 5.9

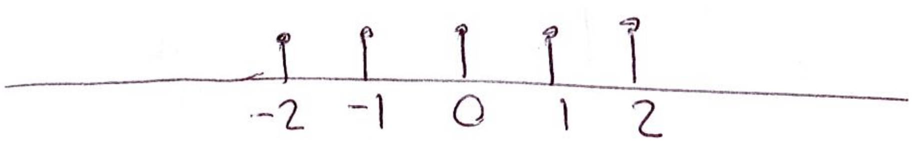
$x[n]$



$$y[n] = g[n-2]$$



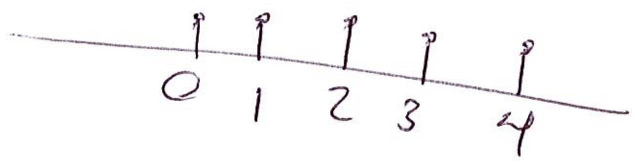
$$g[n] \xleftrightarrow{\mathcal{F}} G(e^{j\omega})$$



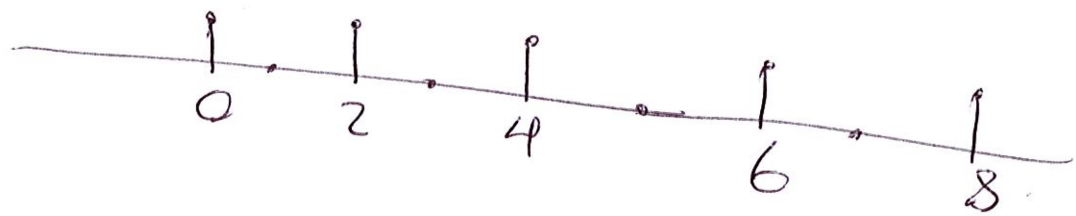
Bonus Exercise

$$Y(e^{j\omega}) = e^{-j2\omega} G(e^{j\omega})$$

$y[n]$

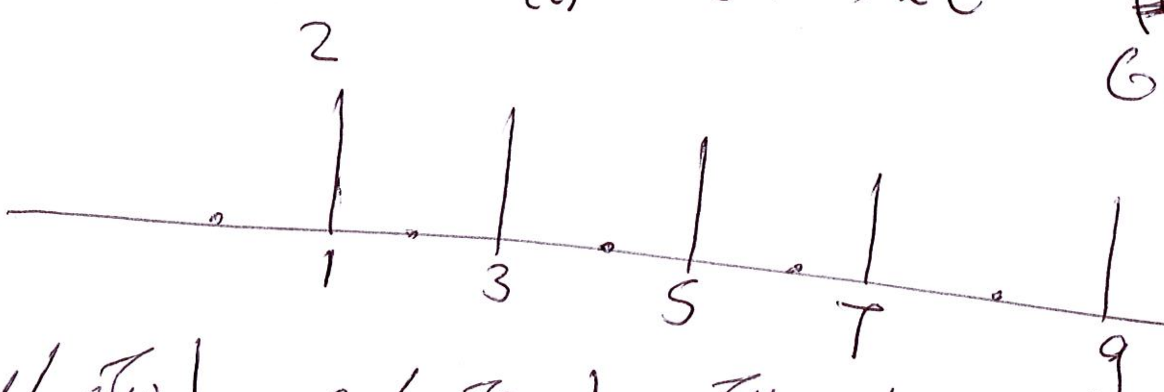


$y_{(2)}[n]$



$$Y_{(2)}(e^{j\omega}) = Y(e^{j2\omega}) = e^{-j4\omega}\, G(e^{j2\omega})$$

$$2\,y_{(2)}[n-1] \xleftrightarrow{\mathcal{F}} 2\,e^{-j5\omega}\, G(e^{j2\omega})$$



$$X(e^{j\omega}) = G(e^{j2\omega})\, e^{-j4\omega}\left(1 + 2e^{-j\omega}\right)$$