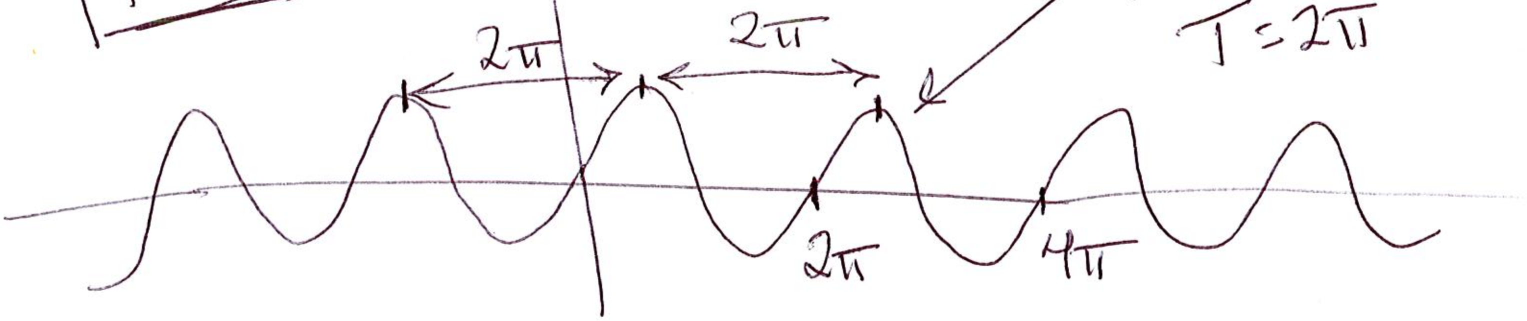
Periodic Signals

$x(t)$ is periodic if there exists a T such that $x(t) = x(t+T)$ for all values of t

$\boxed{\text{Period}}$

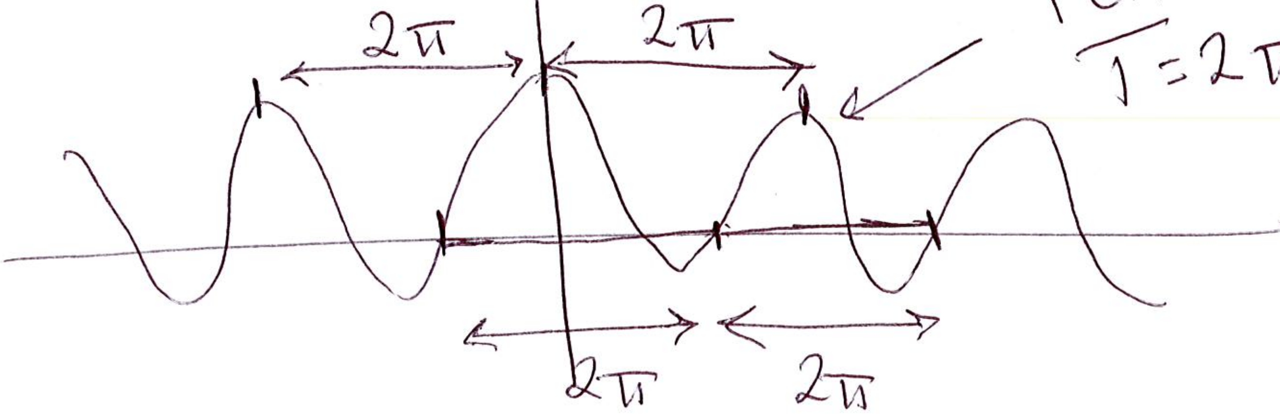
$x(t) = \sin t$

Periodic
$T = 2\pi$

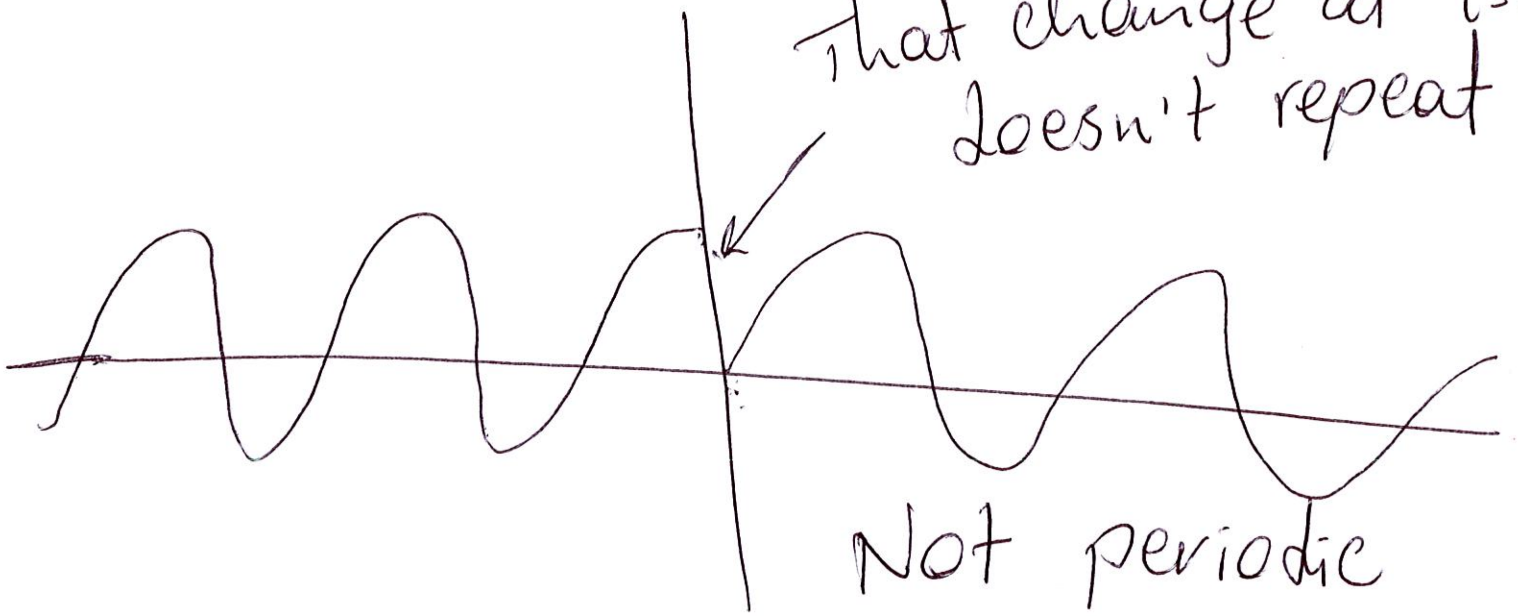


$x(t) = \cos t$

Periodic
$T = 2\pi$



$$x(t) = \begin{cases} \cos t & \text{if } t < 0 \\ \sin t & \text{if } t \geq 0 \end{cases}$$



Not periodic

If $x(t) = x(t+T)$ for all t

then T is a period

and also $2T$ is a period

and also nT is a period

for any integer n

We say that T_0 is ~~a~~ the fundamental period for $x(t)$ if T_0 is the minimum value for T such that $\boxed{x(t) = x(t+T) \text{ for all values of } t}$

Exponential signals (Continuous-time)

And Sinusoidal

$$x(t) = C\, e^{at}$$

C: complex constant

a: complex constant

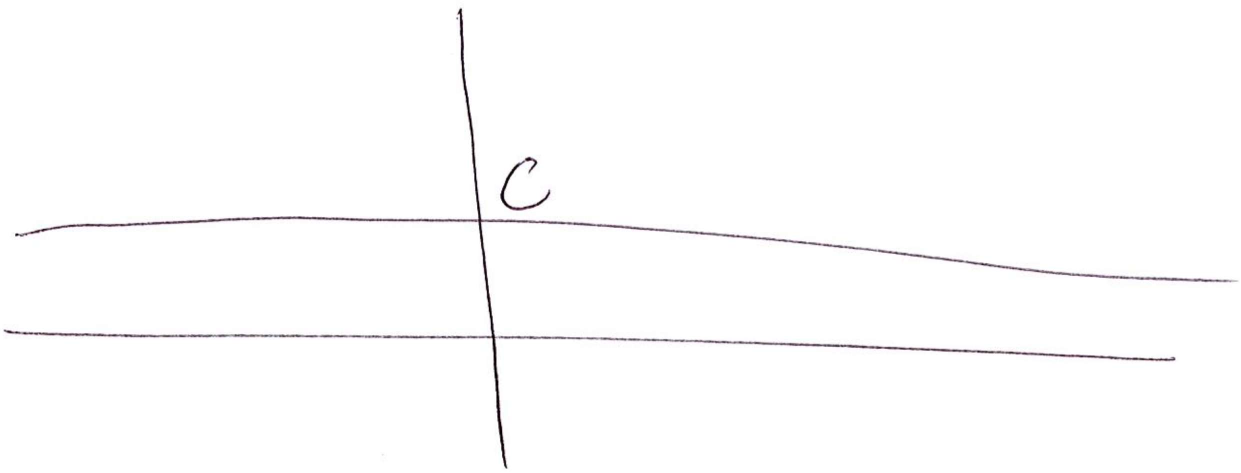
Let's say C is real

Consider cases for the exponent a

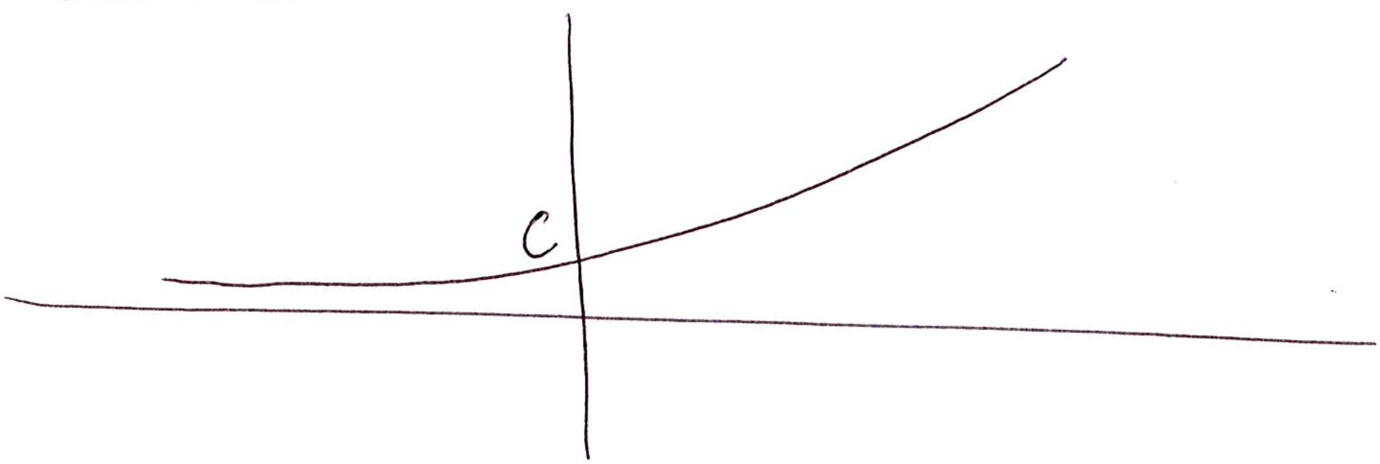
Case 1: a is real

$$C e^{at}$$

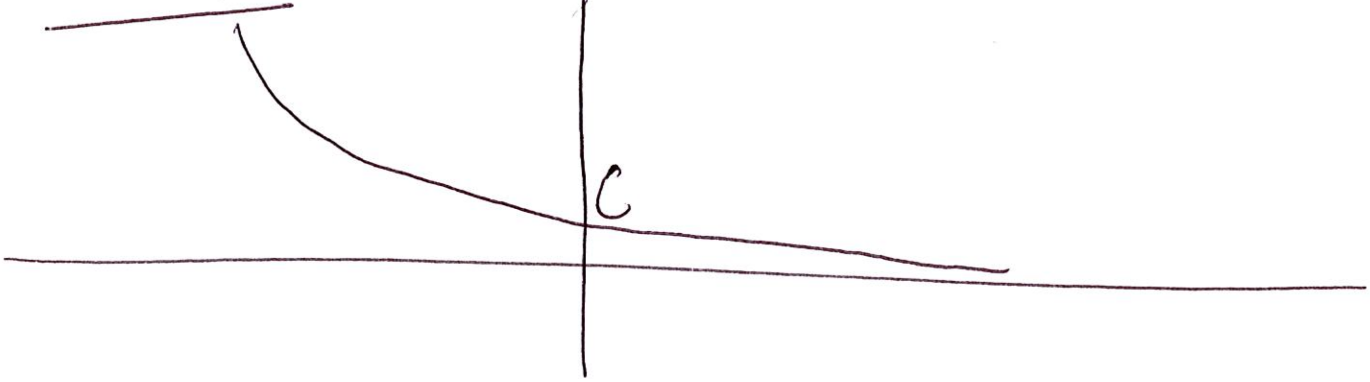
$a = 0$



$a > 0$



$a < 0$



If a is purely imaginary

Let $a = j\omega_0 t$ and $C = 1$

ω_0 real

$$x(t) = e^{j\omega_0 t} = e^{j\omega_0 (t+T_0)} = e^{j\omega_0 t} e^{j\omega_0 T_0}$$

$$= e^{j\omega_0 t} \underbrace{\cancel{e^{j2\pi}}}_{1}$$

Euler's formula

$$= \underbrace{\cos \omega_0 t}_{} + j \underbrace{\sin \omega_0 t}_{}$$

Periodic

Fundamental Period

$$T_0 = \frac{2\pi}{|\omega_0|}$$

Periodic

$$T_0 = \frac{2\pi}{|\omega_0|}$$

Periodic , $T_0 = \dfrac{2\pi}{|\omega_0|}$

$$x(t) = e^{j\omega_0 t}$$

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} (\cos^2 \omega_0 t + \sin^2 \omega_0 t) \, dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} 1 \, dt = \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = 1$$

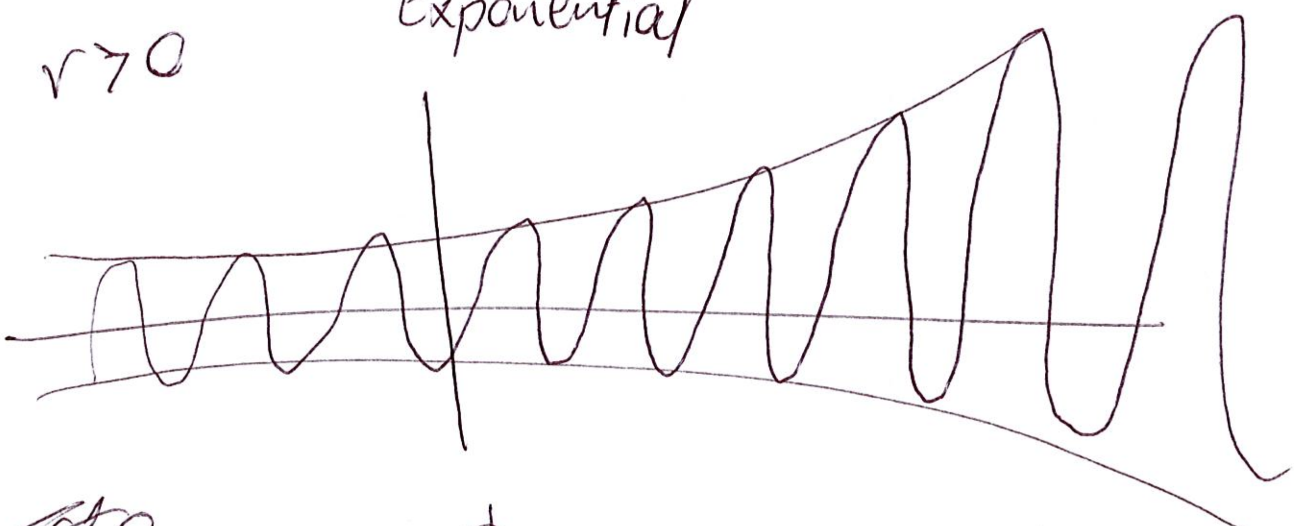
$x(t) = C\, e^{at}$

$C = |C|\, e^{j\theta}$ polar form

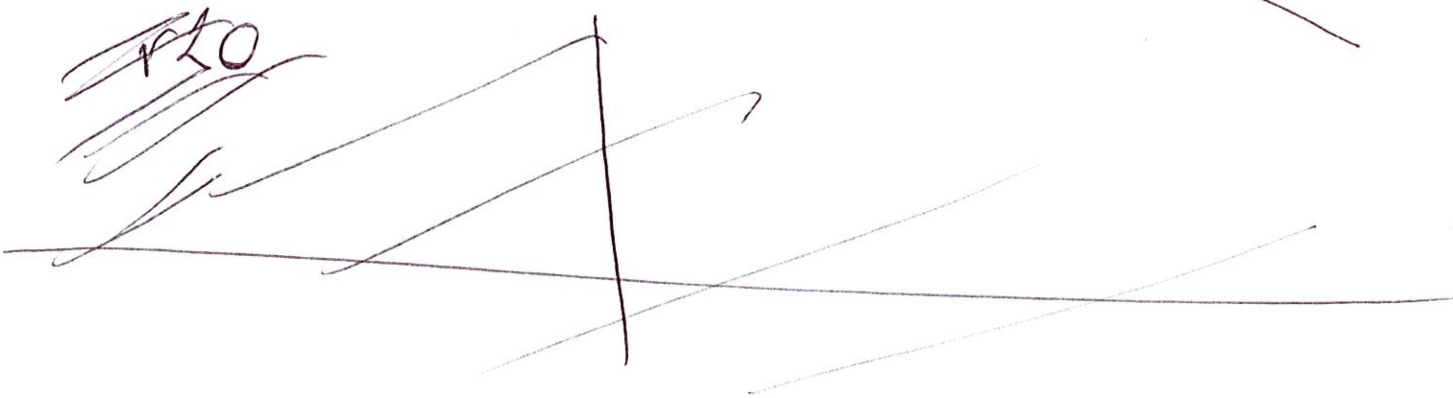
$a = r + j\omega_0$ rectangular form

$$x(t) = |C|\, e^{j\theta}\, e^{(r + j\omega_0)t}$$
$$= |C|\, e^{j\theta}\, e^{(rt + j\omega_0 t)}$$
$$= |C|\, \underset{\substack{\uparrow \\ \text{Exponential}}}{e^{rt}}\, \underbrace{e^{j(\omega_0 t + \theta)}}_{\text{Sinusoid}}$$

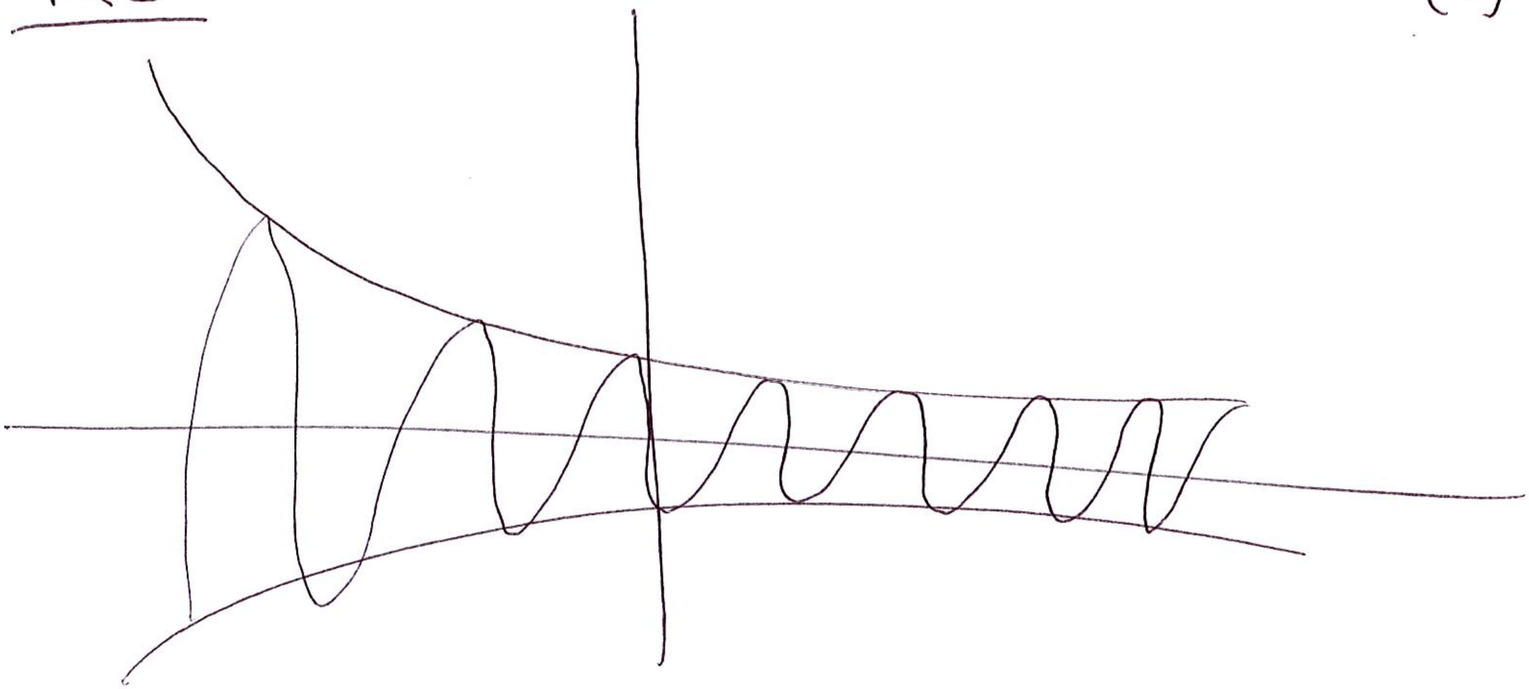
$r > 0$



$\cancel{r < 0}$



$r<0$



$x(t) = e^{j\omega_0 t}$ Periodic, $T_0 = \frac{2\pi}{|\omega_0|}$

ω_0: fundamental frequency

Has a single tone

we will be interested in decomposing any signal $x(t)$ into a linear combination of "single-tone" signals

Each of these single-tone signals (9)
will have a fundamental frequency.

The coefficient of that single-tone signal in ~~that~~ the linear combination is the value of the original signal at the corresponding frequency