# ECE 301 - Lecture # 32



## Duality between DTFT and CTFS

DTFT
$$\begin{cases} x[n] = \dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega. & (1) \\ X(e^{j\omega}) = \displaystyle\sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n} & (2) \end{cases}$$

CTFS
$$\begin{cases} x(t) = \displaystyle\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} & (3) \\ a_k = \dfrac{1}{T}\displaystyle\int_{T} x(t)\, e^{-jk\omega_0 t}\, dt. & (4) \end{cases}$$

(1) and (4) are similar $\boxed{\text{if } T = 2\pi}$

(2) and (3) are similar

If we think of $X(e^{j\omega})$ as a CT signal, then the FS coeffs are given by $x[-n]$

# Example 5.17

$$x[n] = \frac{\sin(\pi n/2)}{\pi n}$$

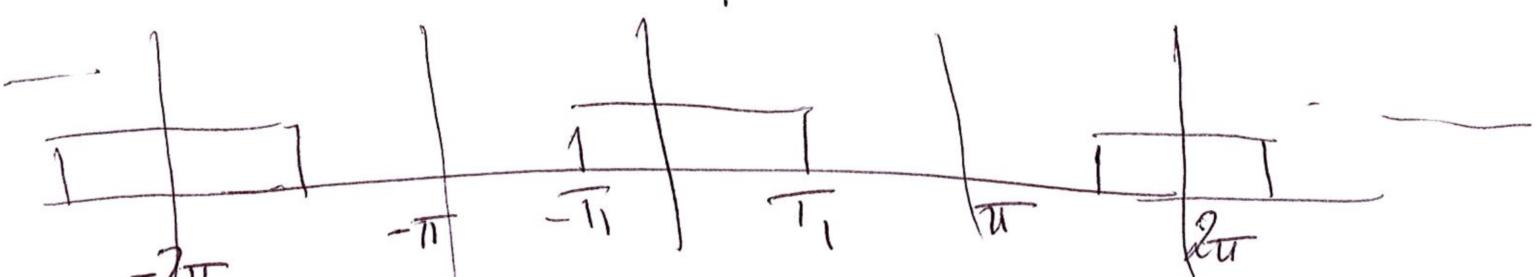
$$\updownarrow \text{DTFT}$$

$$X(e^{j\omega})\,?$$

We need to find a CT signal that has period $2\pi$ and FS coeffs. $a_k = x[k]$

$$g(t) = \begin{cases} 1 & , \ |t| \leq T_1 \\ 0 & , \ T_1 < |t| \leq \pi \end{cases}$$

with period $2\pi$




then $b_k = \dfrac{\sin(kT_1)}{k\pi}$

if $T_1 = \frac{\pi}{2}$

then $b_k = x[k] = \frac{\sin(\pi k/2)}{k\pi}$

$$b_k = \frac{1}{2\pi} \int_{2\pi} g(t)\, e^{-jk\omega_0 t}\, dt$$

$$\frac{\sin(\pi k/2)}{k\pi} = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{-jkt}\, dt$$

Replace k by n $\Big\}$ Switch time
Replace t by $\omega$ $\Big\}$ and frequency

$$\frac{\sin(\pi n/2)}{\pi n} = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{-jn\omega}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \begin{cases} 1 & , \quad |\omega| \leq \frac{\pi}{2} \\ 0 & , \quad \frac{\pi}{2} < |\omega| \leq \pi \end{cases}$$

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## Systems characterized by linear constant coefficient Difference equations

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$$\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{k=0}^{M} b_k \, x[n-k]$$

N-th order

$$\boxed{\text{LTI System}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}$$

$$\sum_{k=0}^{N} a_k e^{-jk\omega} \, Y(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-jk\omega} X(e^{j\omega})$$

Example 5.18

$$y[n] - a\,y[n-1] = x[n] \quad , \quad |a|<1$$

$$H(e^{j\omega}) = \frac{1}{1-a e^{-j\omega}}$$

$$h[n] = a^n u[n]$$

LPF Approximation

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Example 5.19

$$y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2] = 2x[n]$$

$$H(e^{j\omega}) = \frac{2}{1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-j2\omega}}$$

$$= \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})} = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

$$h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

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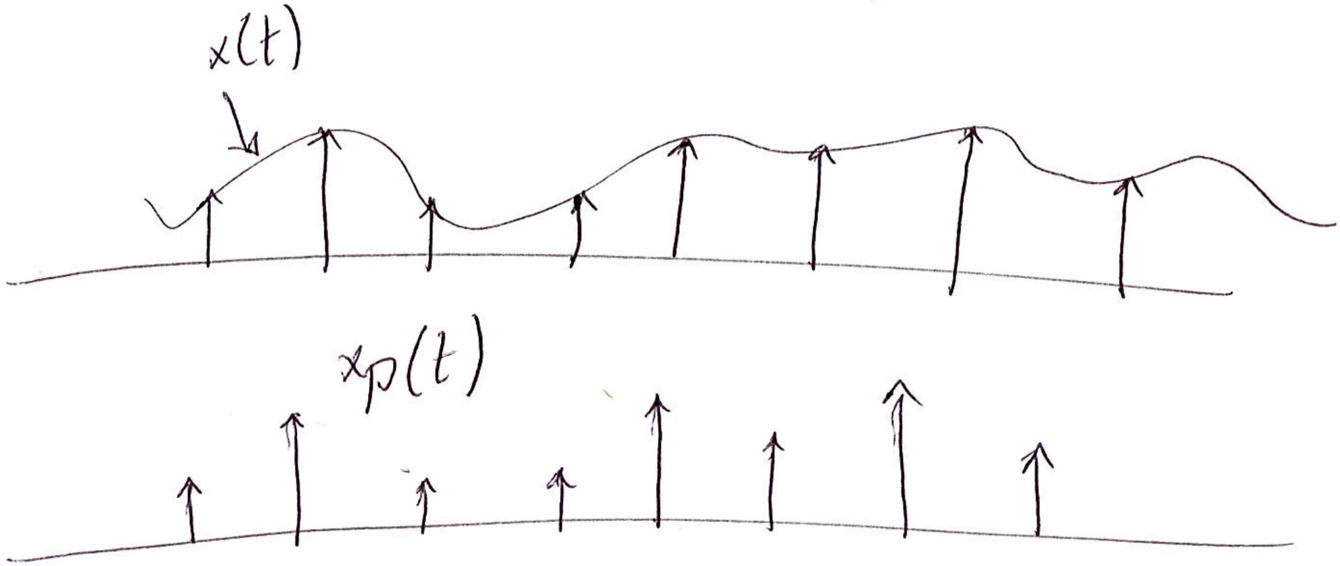
## Impulse-Train Sampling

Convert a CT signal into a discrete set of samples.

How slow can the sampling be?




We will need to reconstruct $x(t)$ from $xp(t)$: Interpolation