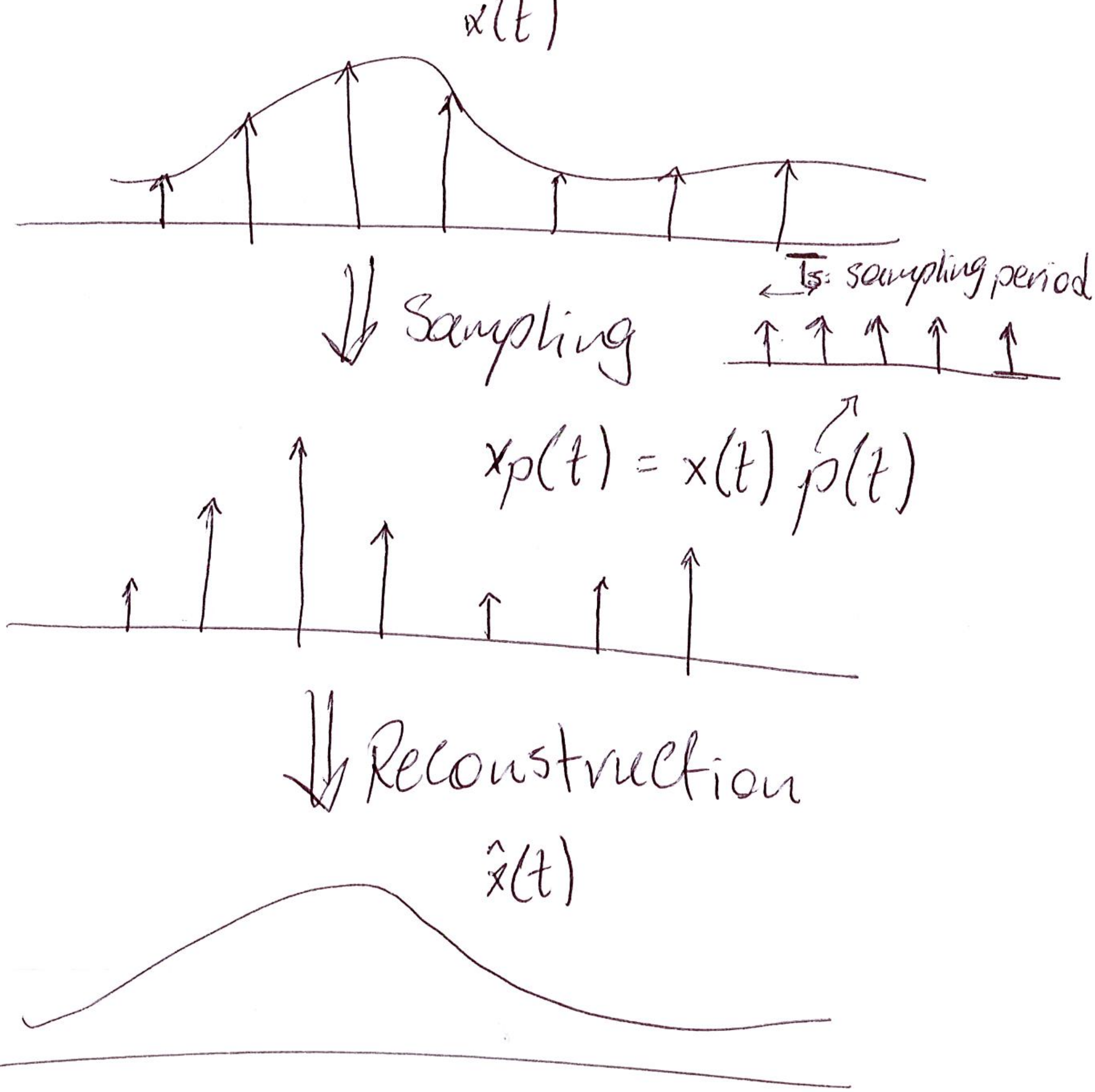
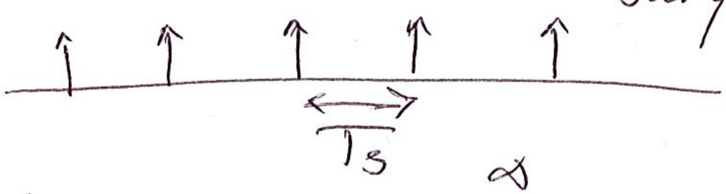
ECE 301 - Lecture #33

Impulse - Train Sampling



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

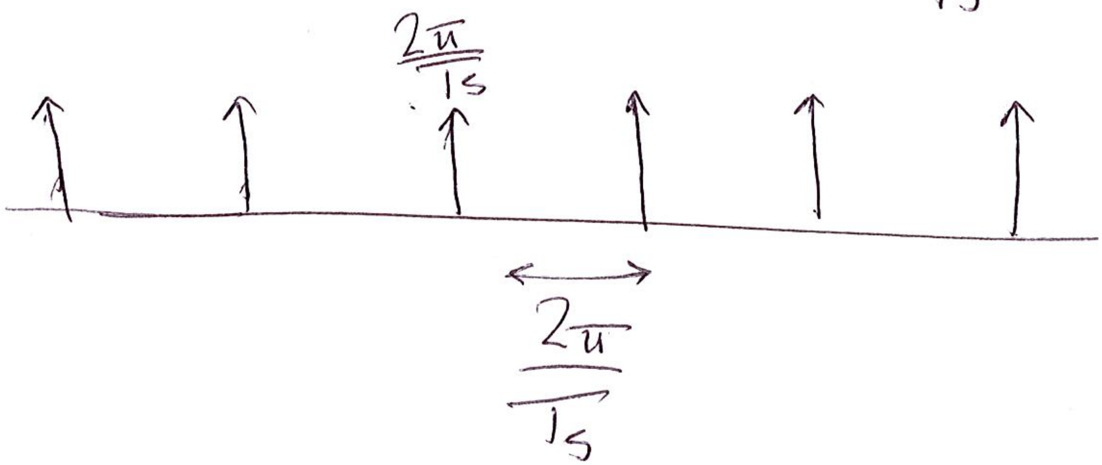
Sampling Period



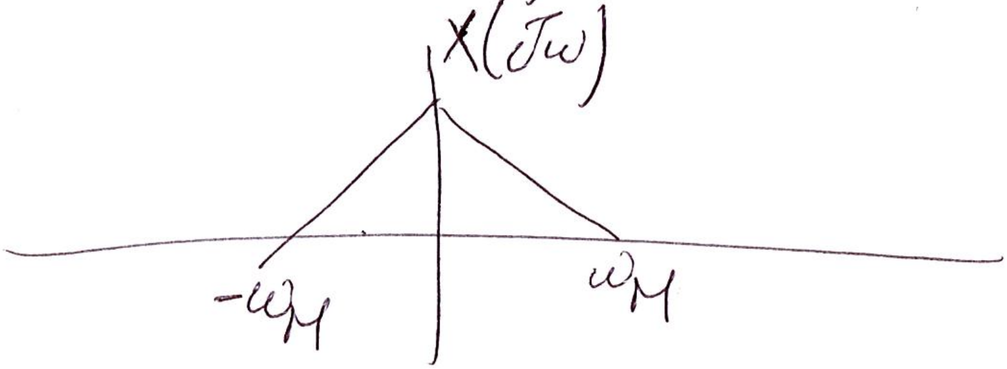
$$P(j\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

Sampling Frequency

$$\frac{2\pi}{T_s}$$

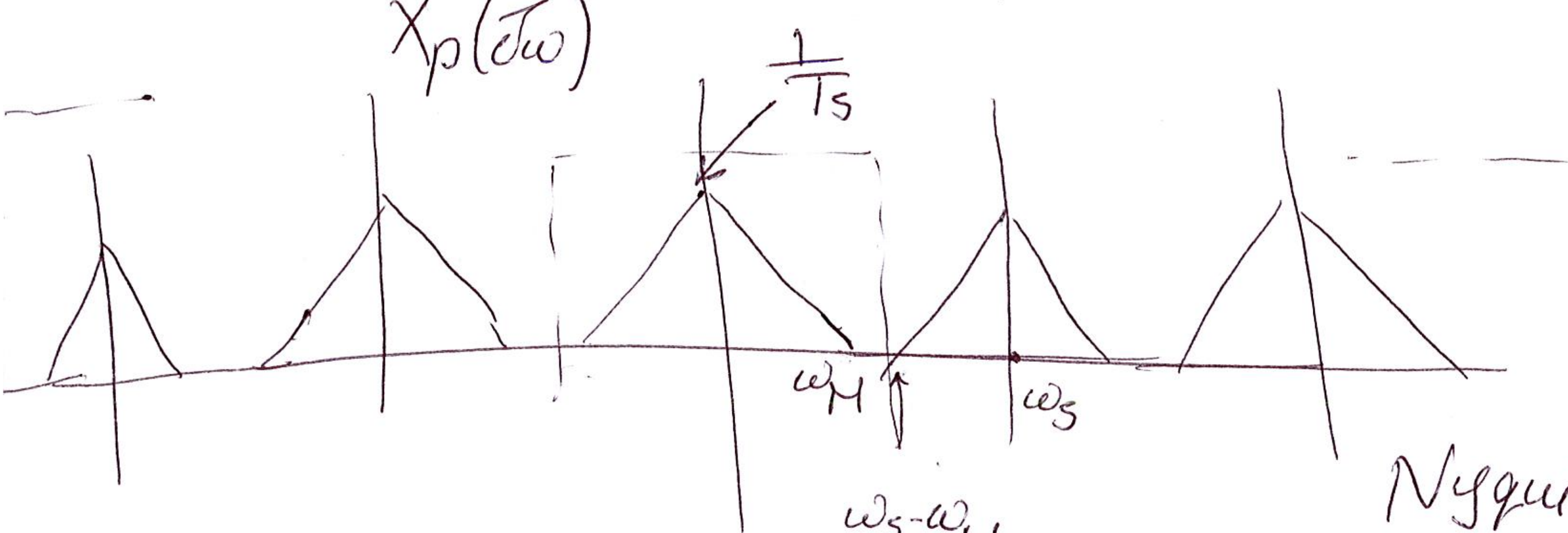


Assume that $X(j\omega)$ is band-limited



$$x_p(t) = x(t)\,p(t)$$

$$X_p(e^{j\omega}) = \frac{1}{2\pi}\underbrace{\int_{-\infty}^{\infty} X(j\theta)\,P(j(\omega-\theta))\,d\theta}_{\text{Convolution}}$$



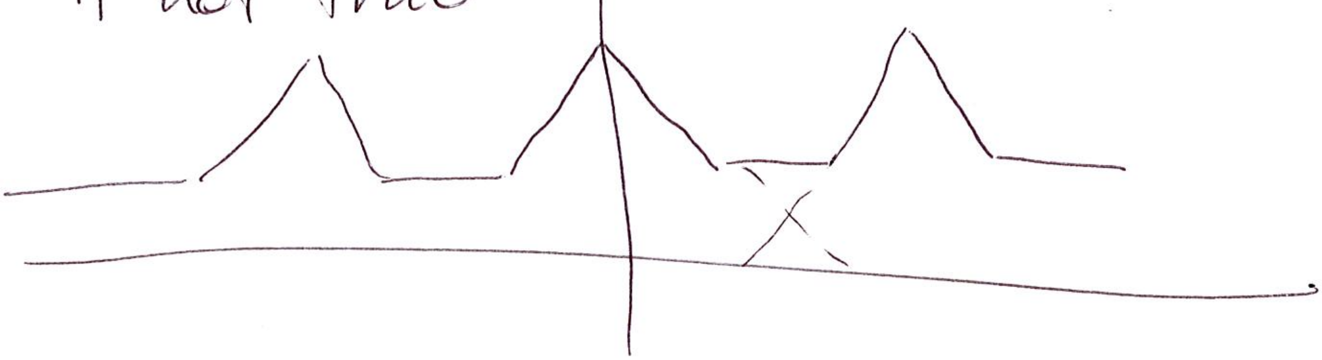
We assumed that $\omega_s - \omega_M > \omega_M$

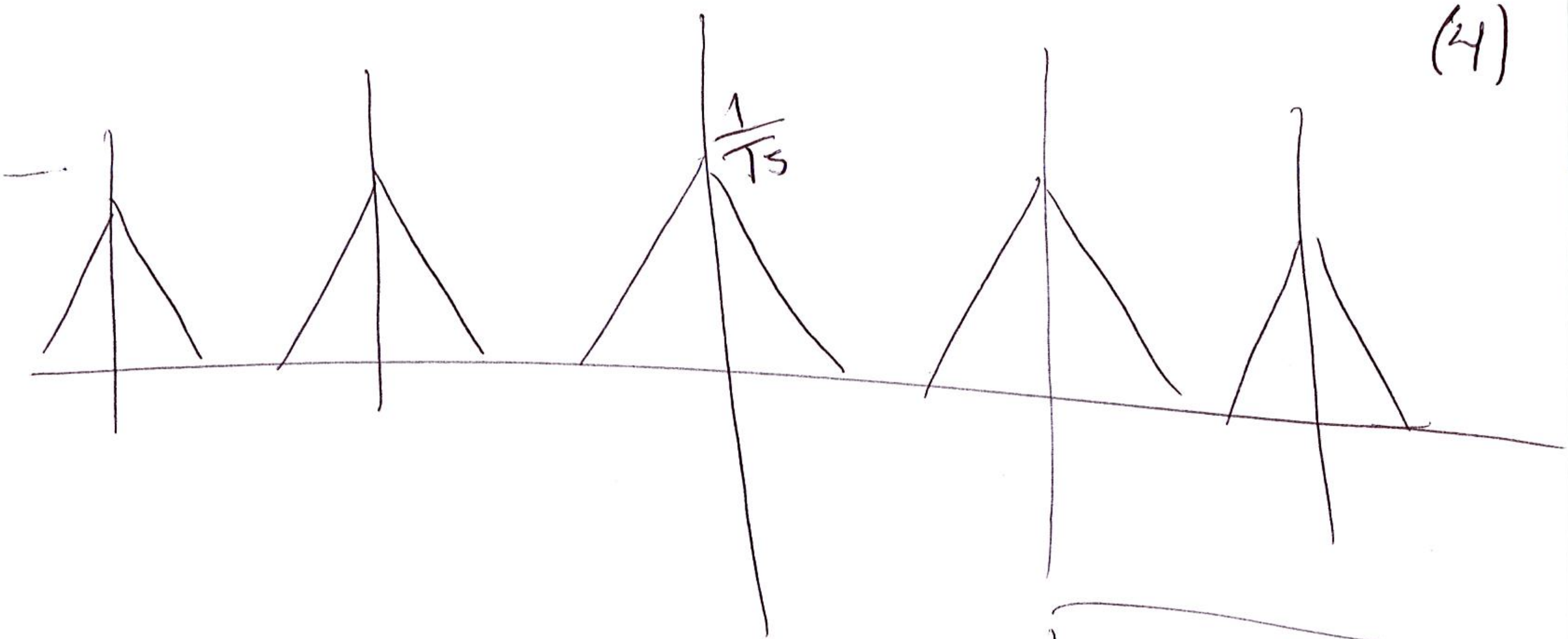
Nyquist Rate

$$\boxed{\omega_s > 2\omega_M}$$

Sampling Theory Condition

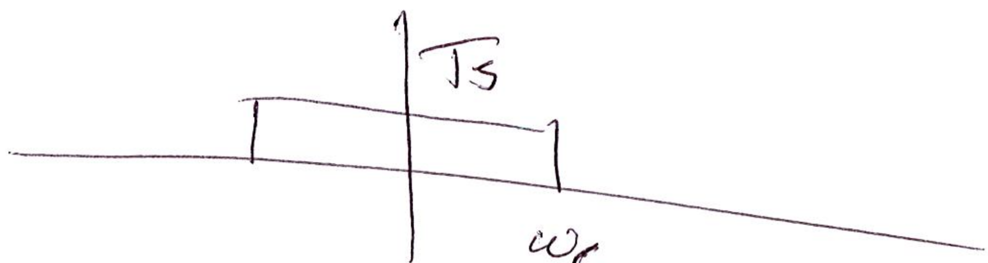
If not true





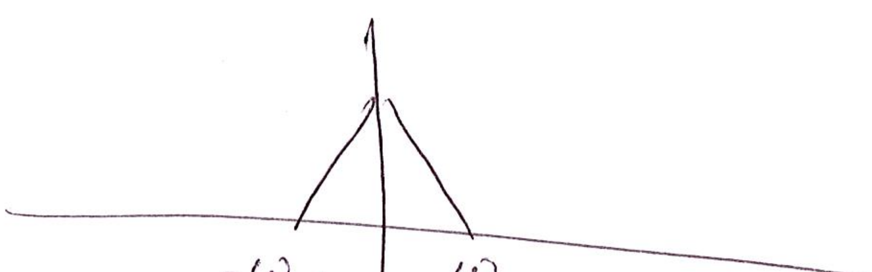
Reconstruction

$\Downarrow$ LPF



$\omega_M < \omega_c < \omega_s - \omega_M$

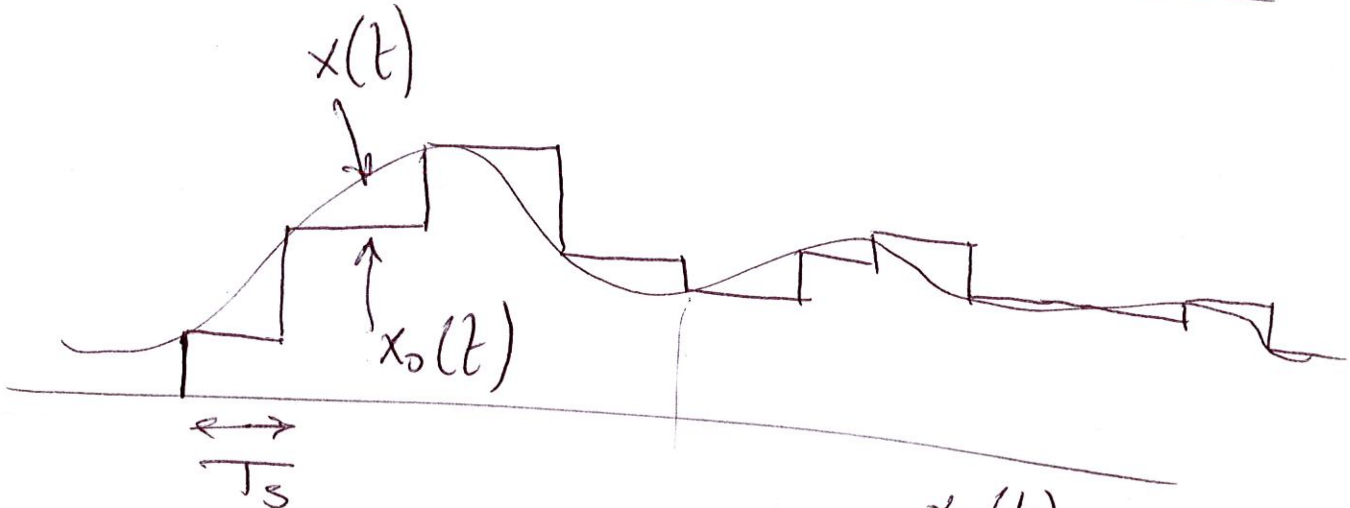
$\Downarrow$

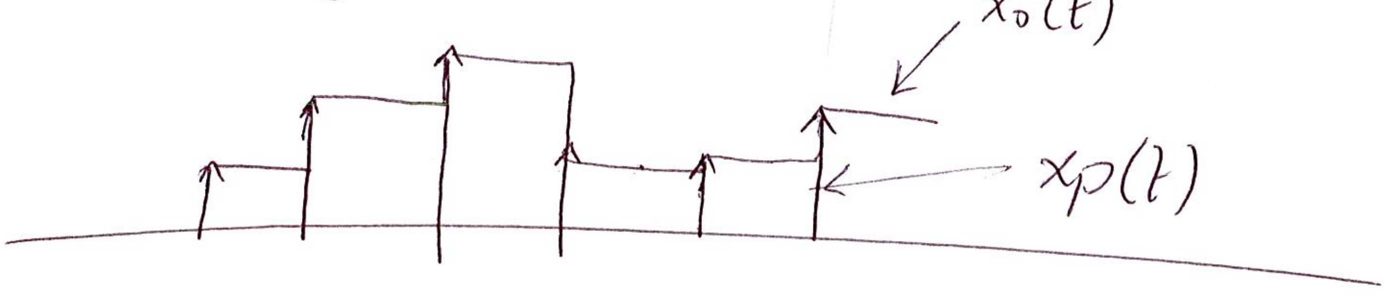


Impulse-Train Sampling is hard to implement in many applications

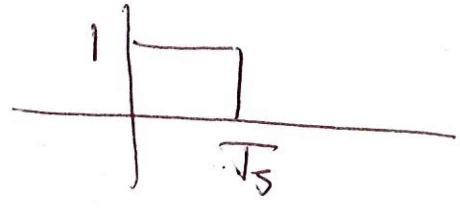
Alternatives (Approximations)

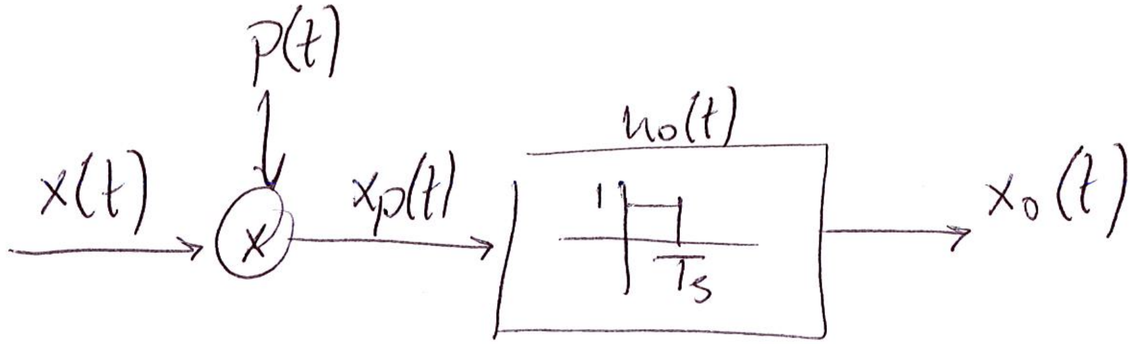
Sampling with Zero-order Hold

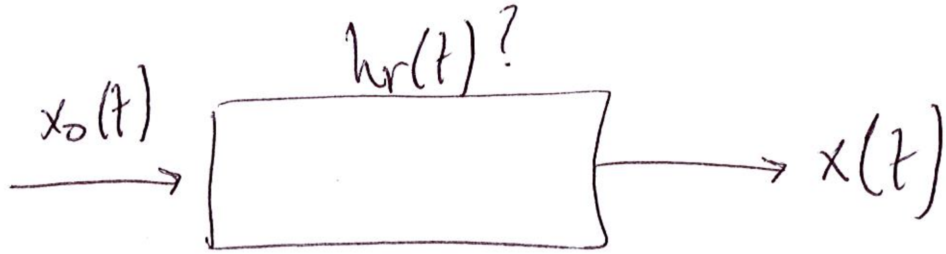


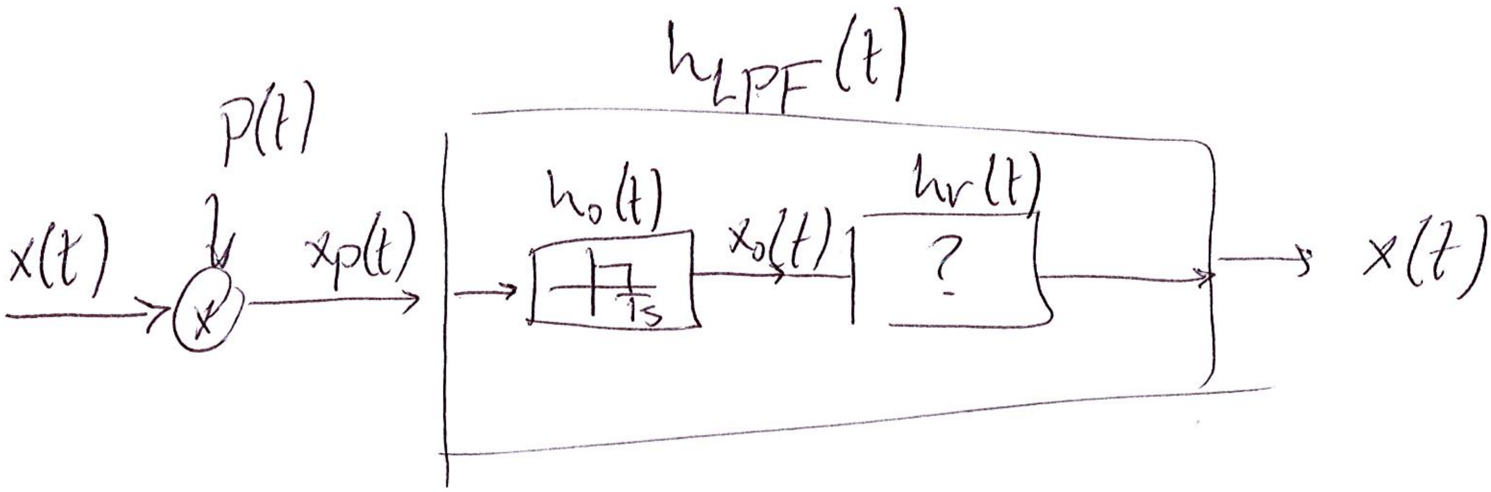


$x_0(t) = x_p(t) *$









$$H_r(j\omega) = \frac{H_{LPF}(j\omega)}{H_0(j\omega)}$$

$$H_0(j\omega) = e^{-j\omega \frac{T_s}{2}} \left[\frac{2 \sin(\omega T_s/2)}{\omega} \right]$$