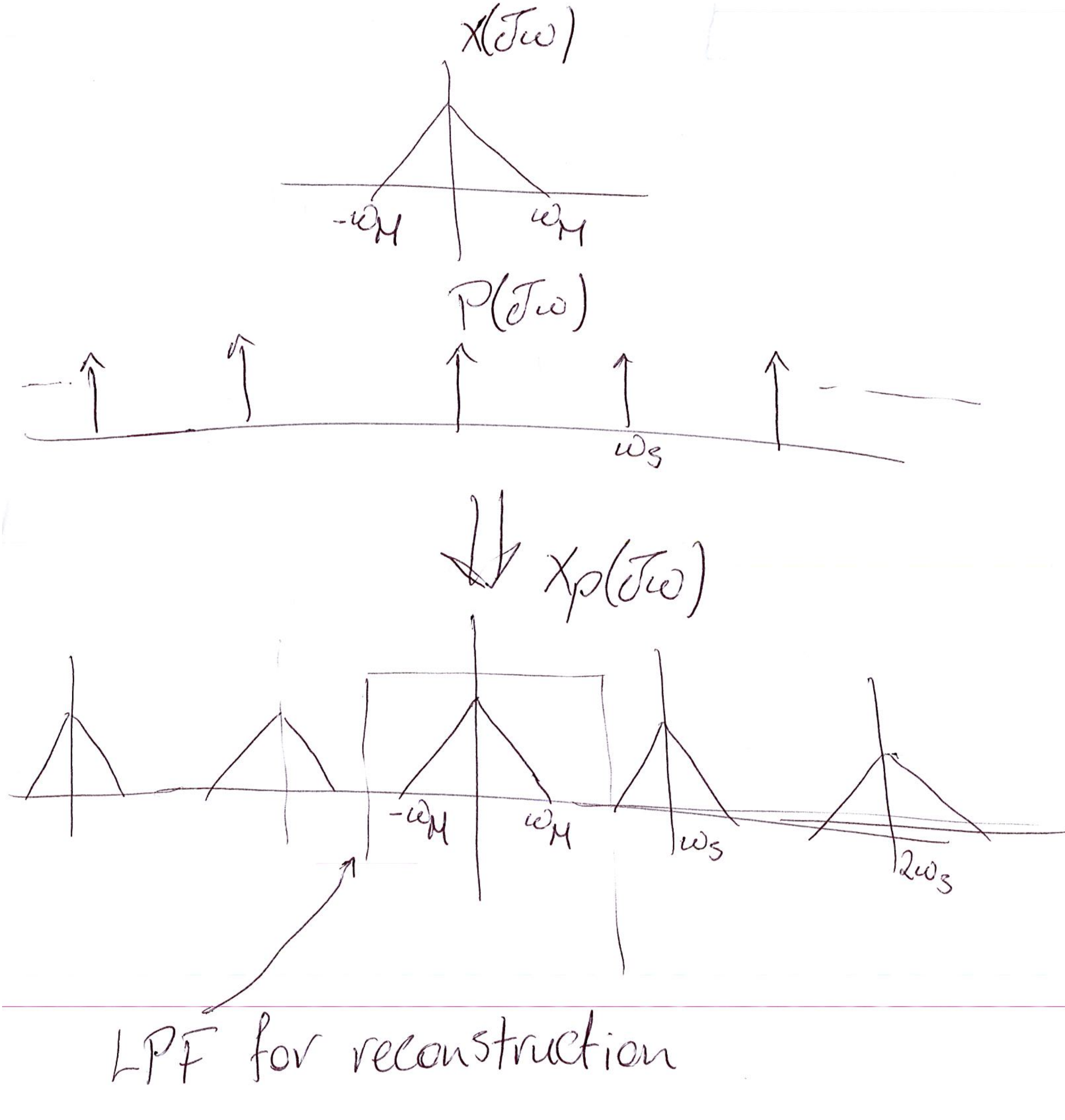
ECE 301 - Lecture # 34

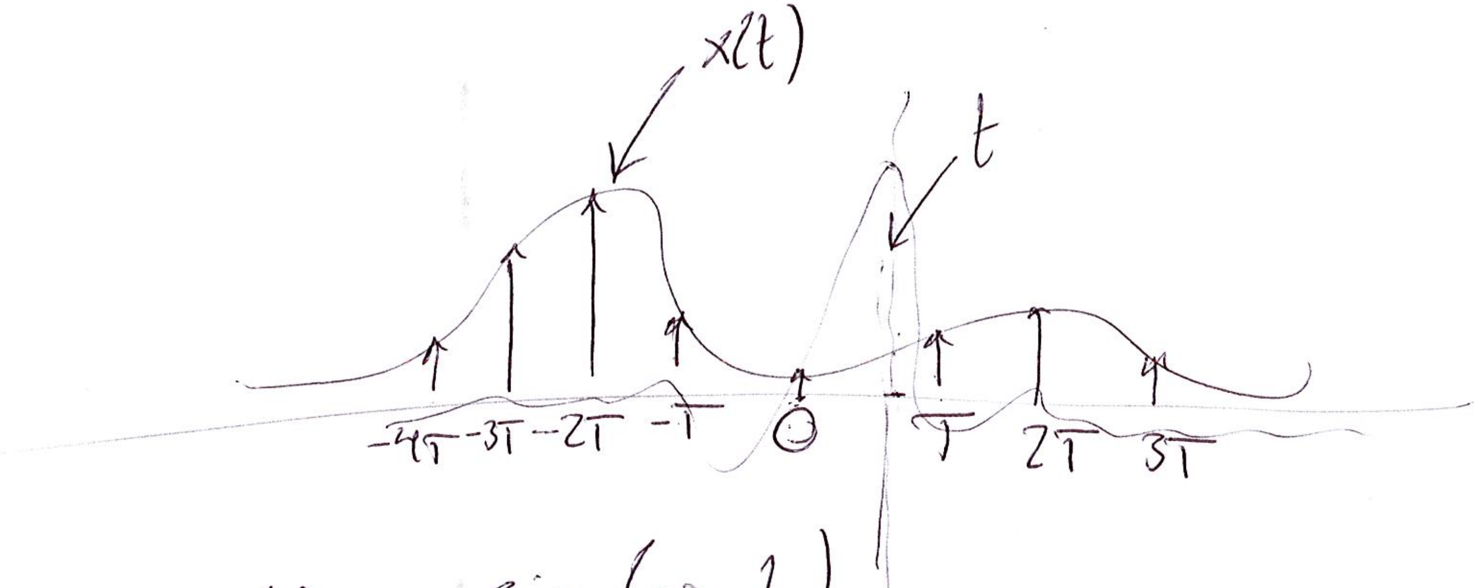
Signal Reconstruction through Interpolation



$$x_r(t) = x_p(t) * \overbrace{h(t)}^{\text{IR of LPF}}$$

$$= \int_{\tau=-\infty}^{\infty} x_p(\tau)\, h(t-\tau)$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\, h(t-nT)$$

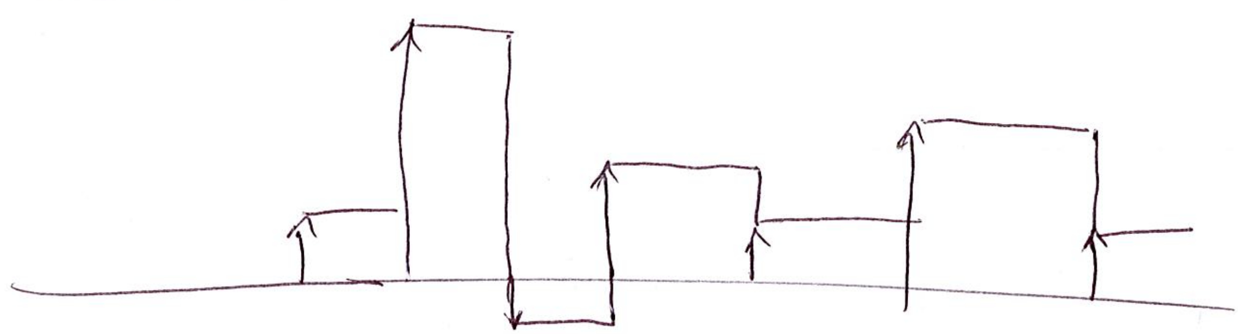


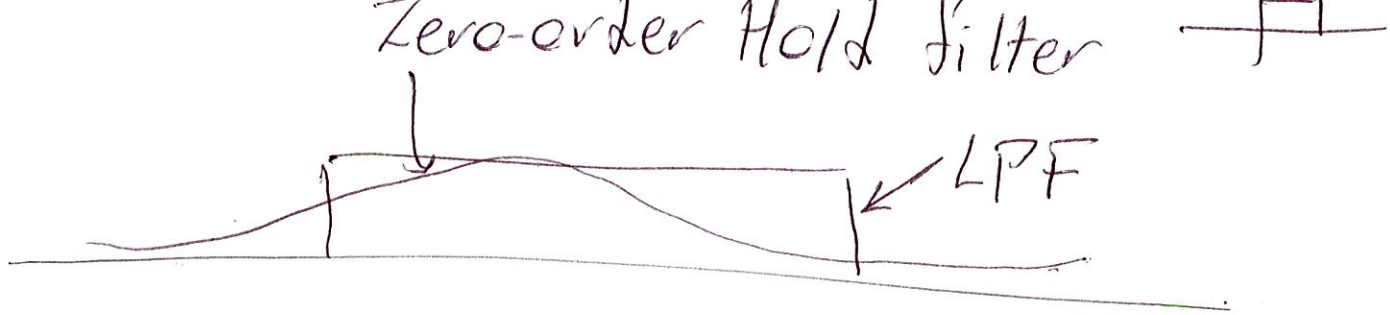
$$h(t) = \frac{\sin(\omega_c t)}{\pi}$$

- One problem with using LPF for reconstruction (Interpolation) is that we need all the samples to find $x_r(t)$ at any t.

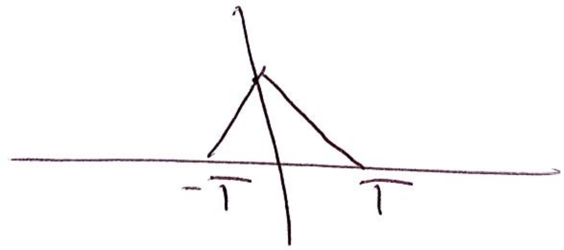
Approximations

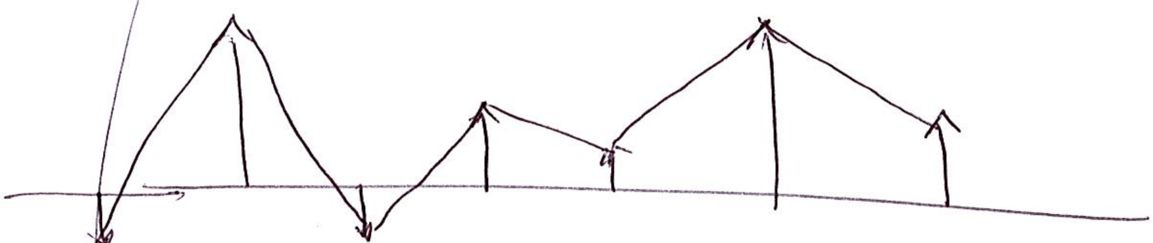
Zero-order Hold





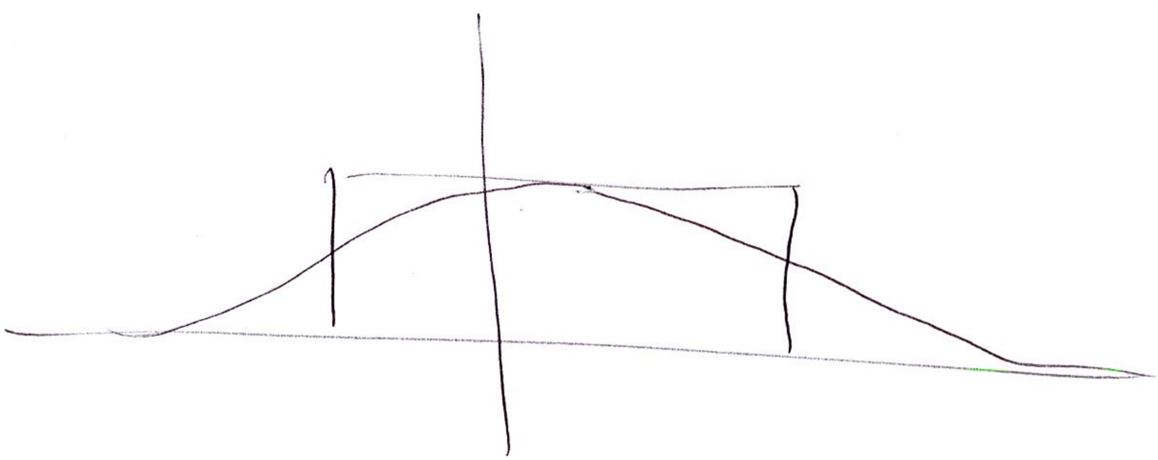
First-order Hold

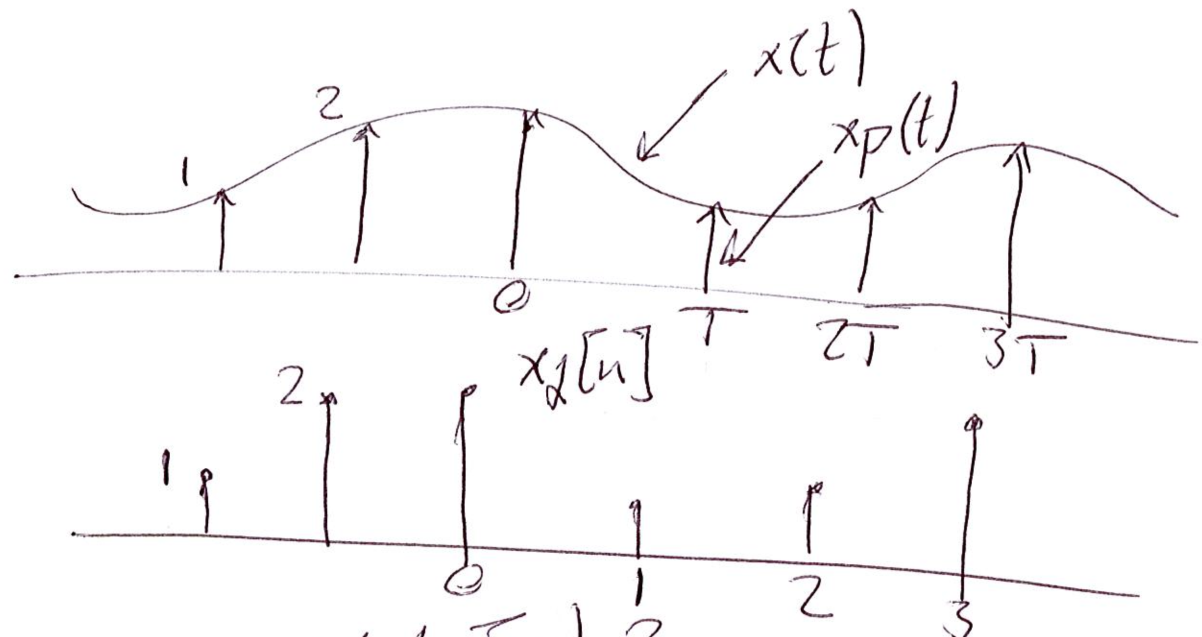




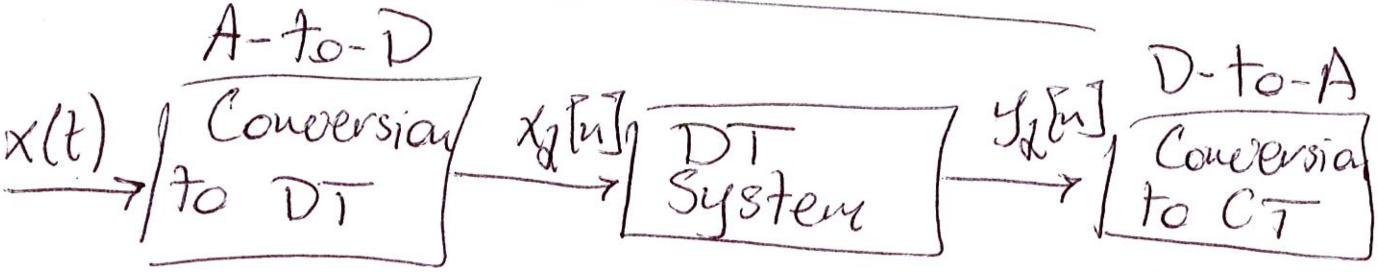
Freq. Domain

$$H(j\omega) = \frac{1}{T}\left[\frac{\sin(\omega T/2)}{\omega/2}\right]^2$$





$X_d(e^{j\omega})$?



Sampling & Reconstruction

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

$$\mathcal{F}\{\delta(t-nT)\} = e^{-j\omega nT}$$

$$X_p(j\omega) = \sum_{n=-\infty}^{\infty} x(nT)\,e^{-j\omega nT}$$

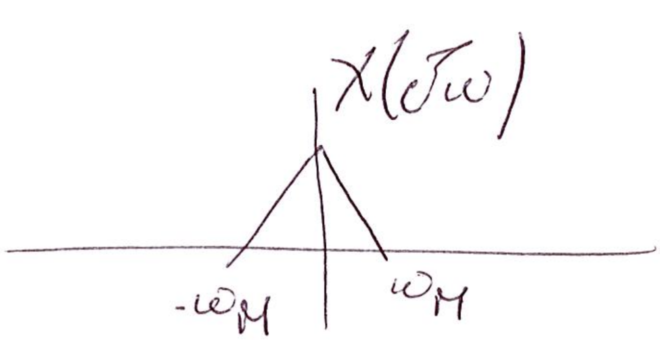
$$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_d[n]\,e^{-j\Omega n}$$

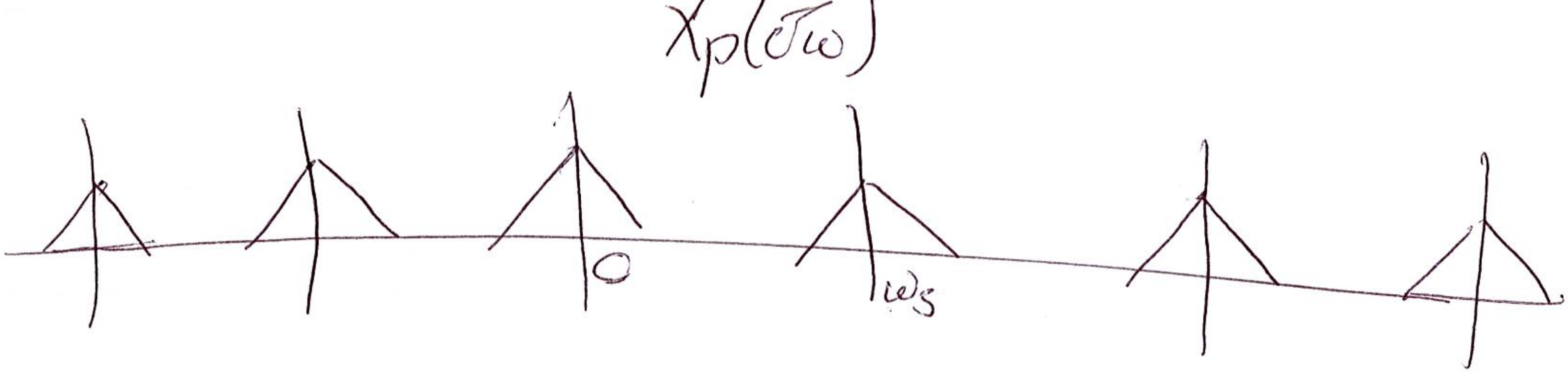
$$= \sum_{n=-\infty}^{\infty} x(nT)\,e^{-j\Omega n}$$

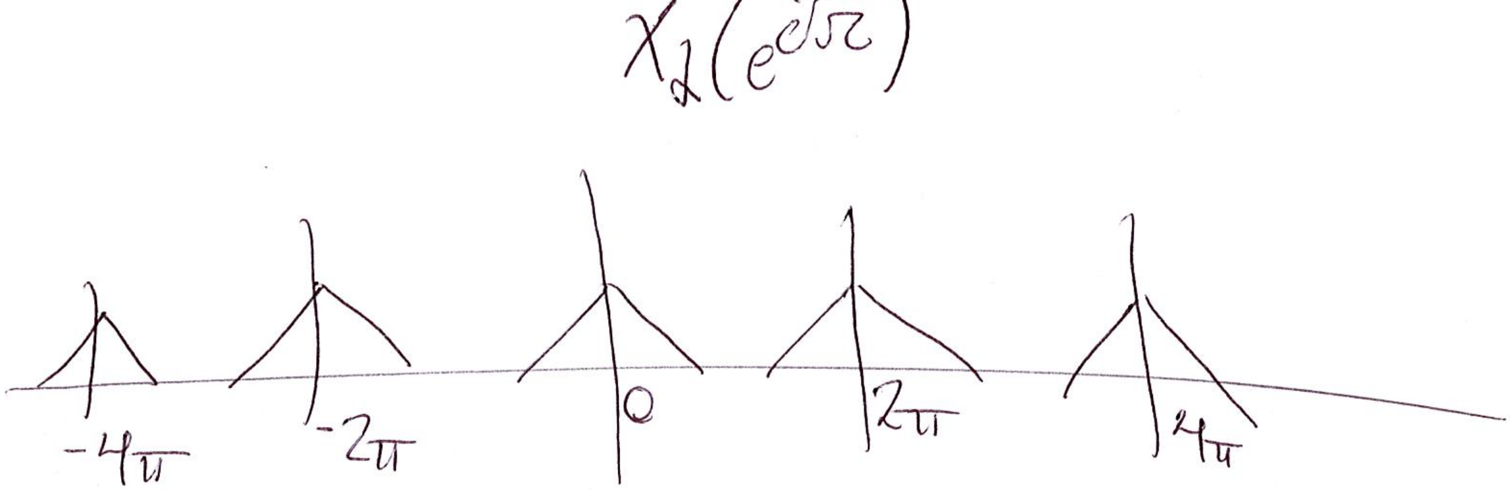
$$X_d(e^{j\Omega}) = X_p(j\Omega/T)$$

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega-k\omega_s))$$

$$X_d(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j(\Omega-k2\pi)/T\right)$$







$\Omega = \omega T$

Sampling of DT Signals

Impulse-Train Sampling

$$x_p[n] = \begin{cases} x[n] & \text{if } n \text{ multiple of } N \\ 0 & \text{otherwise} \end{cases}$$

Sampling Period

$$x_p[n] = x[n]\,p[n]$$

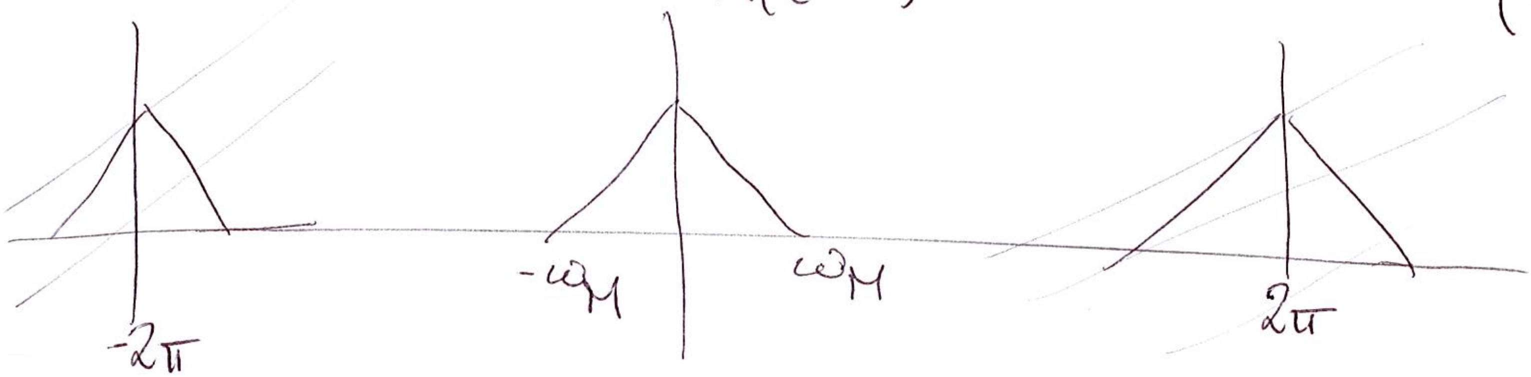
$$x_p[n] = \sum_{k=-\infty}^{\infty} x[kN]\,\delta[n-kN]$$

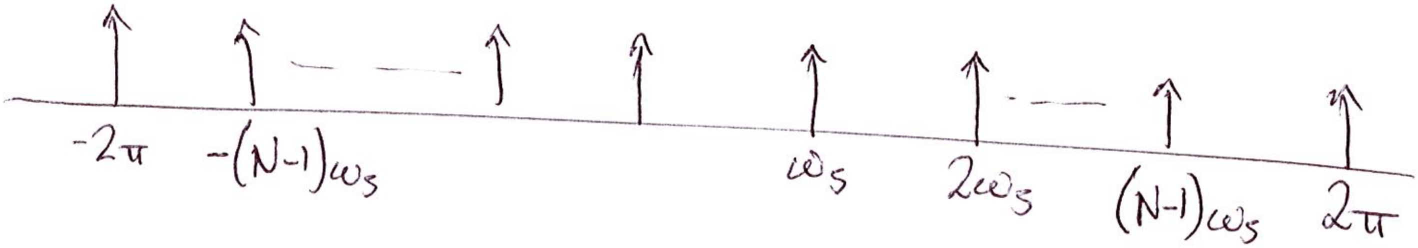
$$X_p(e^{j\omega}) = \frac{1}{2\pi}\int_{2\pi} P(e^{j\theta})\,X(e^{j(\omega-\theta)})\,d\theta$$

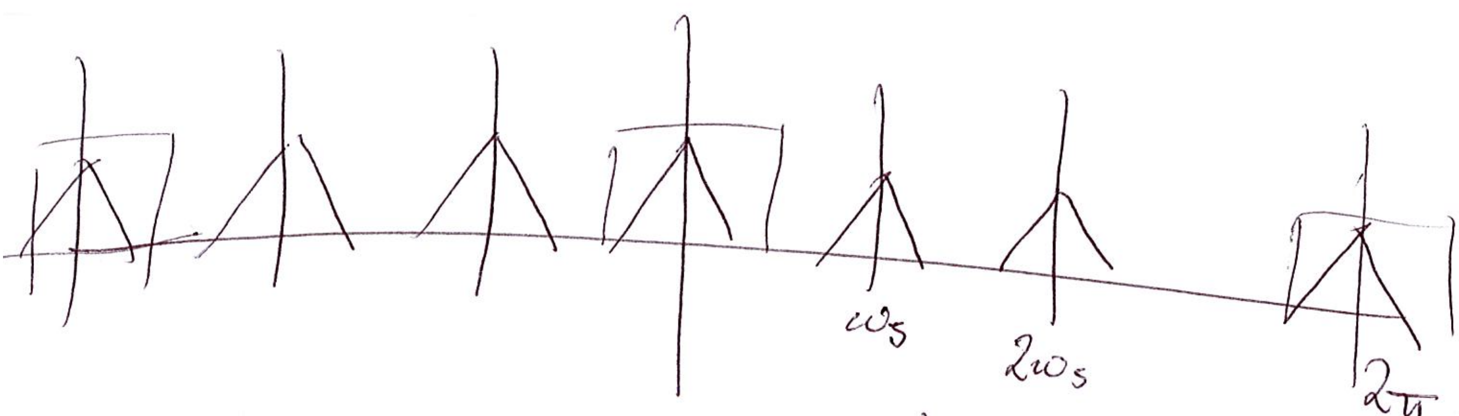
$$P(e^{j\omega}) = \frac{2\pi}{N}\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

$\frac{2\pi}{N}$

$X(e^{j\omega})$







$$X_p(e^{j\omega}) = \frac{1}{N} \sum_{k=0}^{N-1} X(e^{j(\omega - k\omega_s)})$$