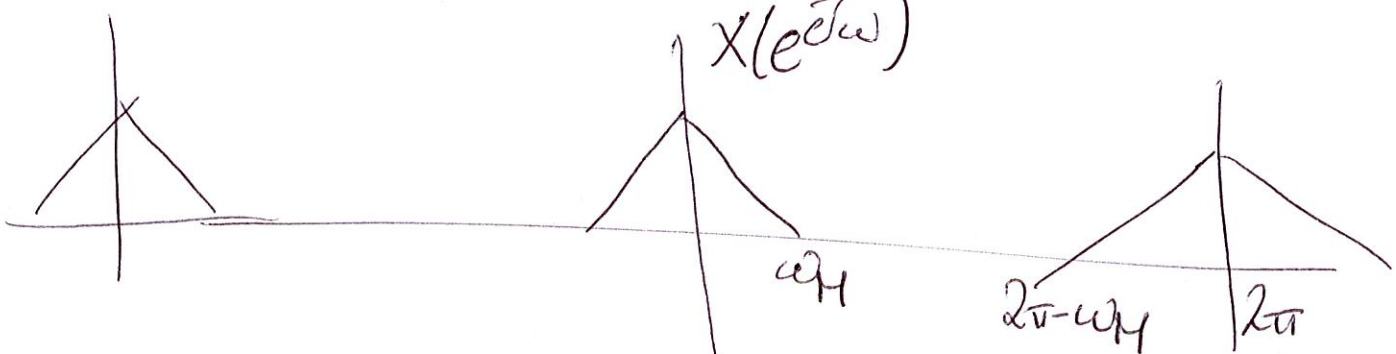
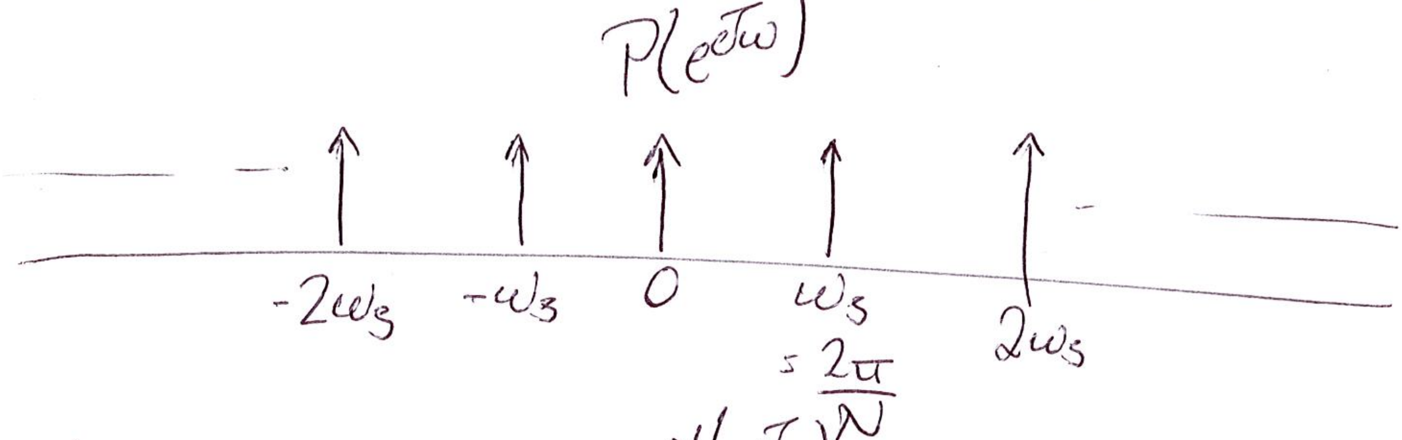
ECE 301 - Lecture # 35

Discrete-Time Sampling

$$x_p[n] = \begin{cases} x[n] & \text{if } n \text{ is a multiple of } N \\ 0 & \text{otherwise} \end{cases}$$





Example 7.4

$$X(e^{j\omega}) = 0 \qquad \text{if} \quad \frac{2\pi}{9} \le |\omega| \le \pi$$

How slow can we sample this signal and still be able to perfectly reconstruct it?

$$\omega_M \le \frac{2\pi}{9}$$

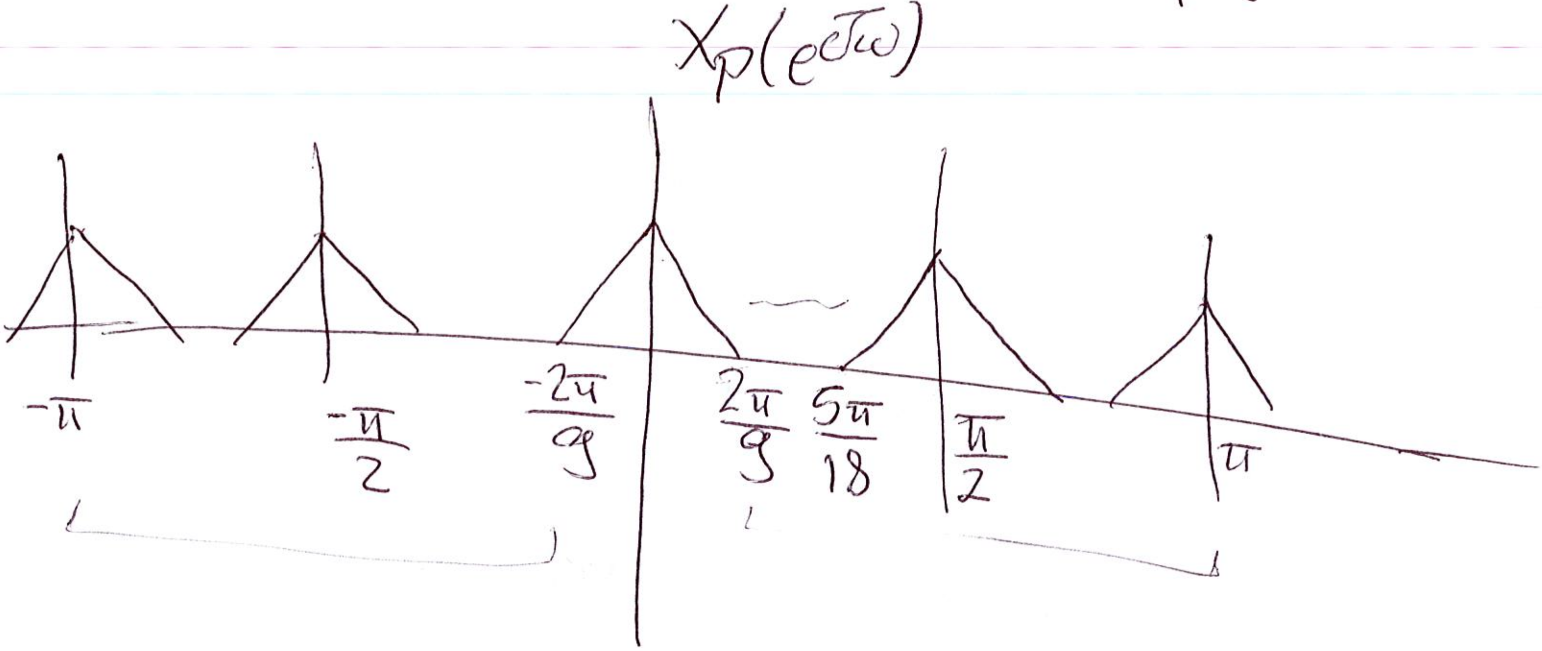
$$\omega_s \ge 2\omega_M \ge \frac{4\pi}{9}$$

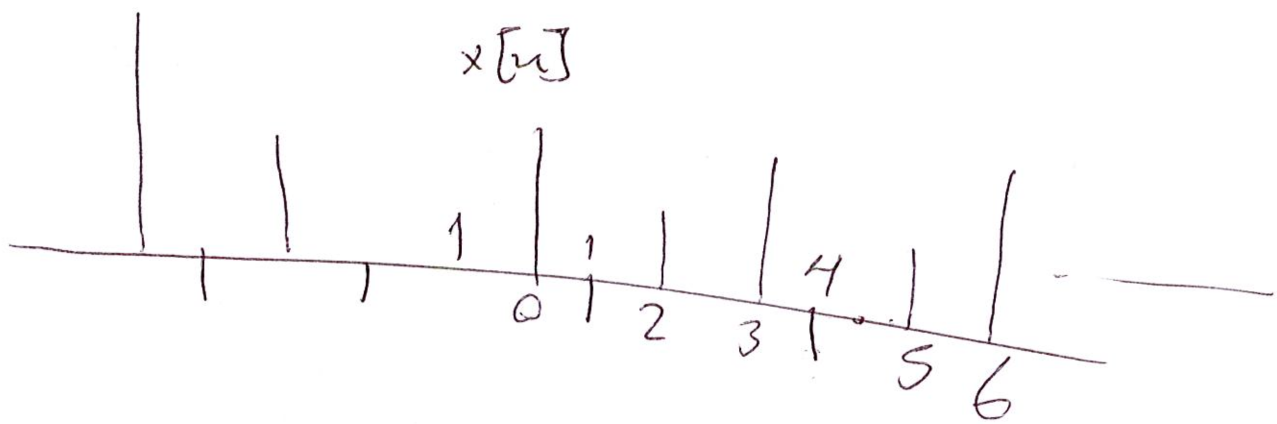
$$\cancel{\frac{2\pi}{N} \le \frac{2\pi \cdot 9}{4\pi} = \frac{9}{2}}$$

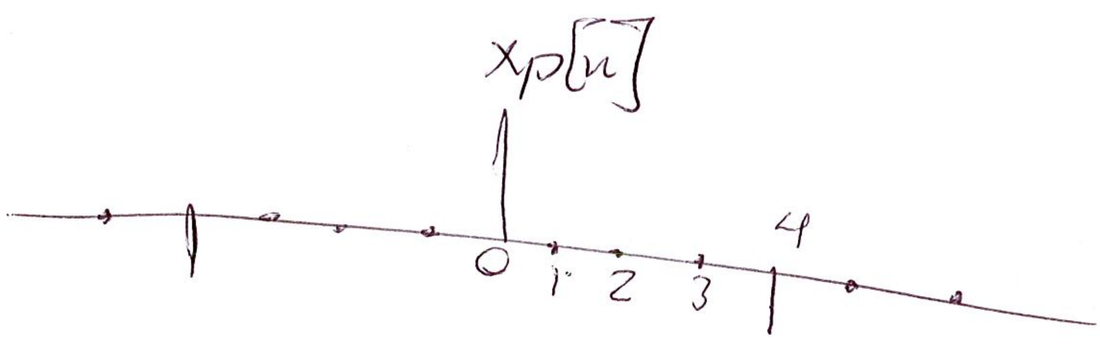
$$\frac{2\pi}{N} \ge \frac{4\pi}{9} \Rightarrow N \le \frac{2\pi \cdot 9}{4\pi} = \frac{9}{2}$$

$$N_{max} = 4 \qquad \frac{2\pi}{N_{max}} = \frac{\pi}{2}$$

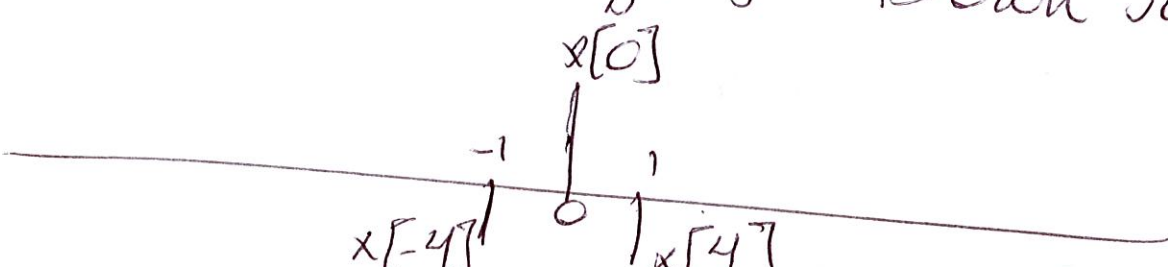
Is that the most efficient? (3)







$x_d[n]$ Down Sampling



Most Efficient Representation

$$x_b[n] = x_p[nN]$$
$$= x[nN]$$

"Throwing away zeros"

$$X_b(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x_b[k]\, e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} x_p[kN]\, e^{-j\omega k}$$

Let $n = kN \Rightarrow k = \frac{n}{N}$

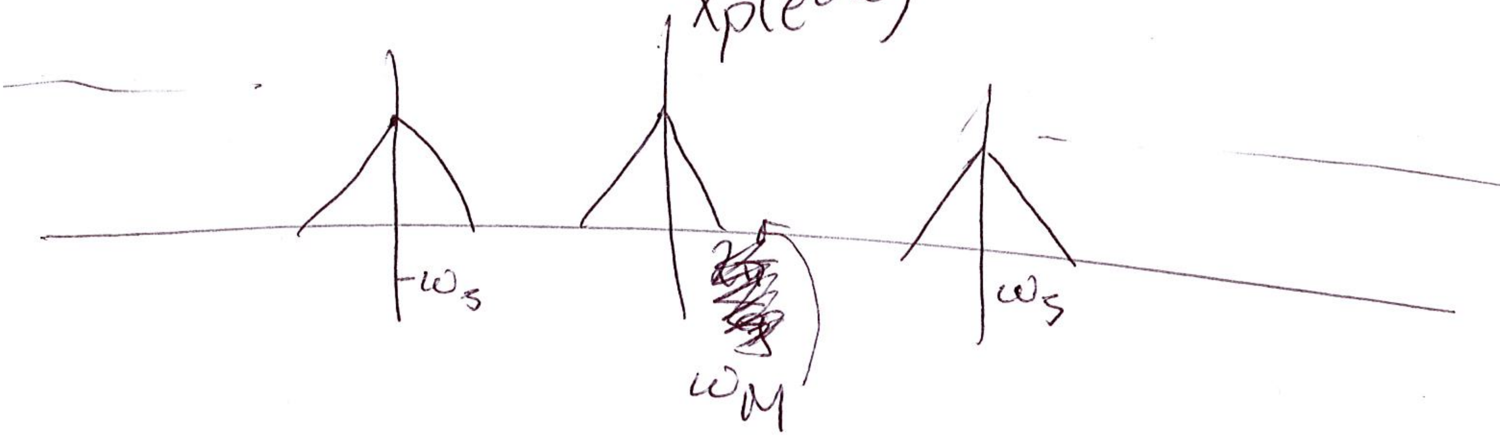
$$= \sum_{\substack{n:\ \text{integer} \\ \text{multiple of } N}} x_p[n]\, e^{-j\omega \frac{n}{N}}$$

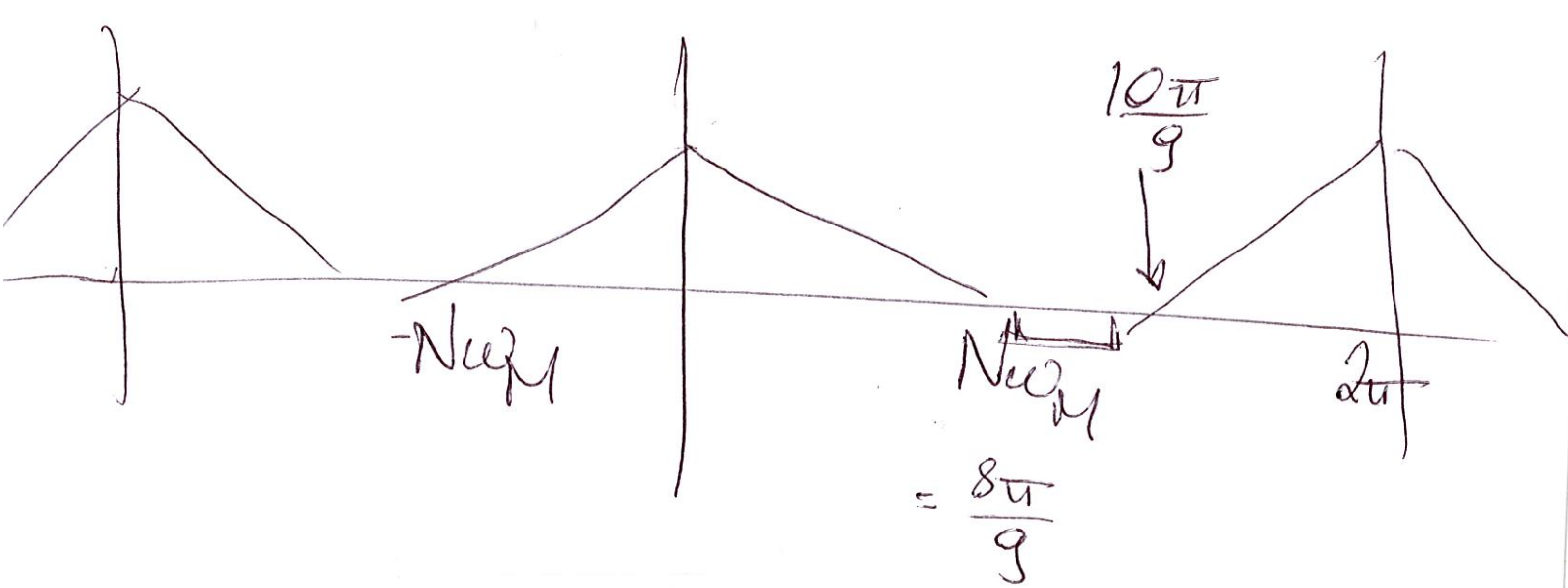
(5)

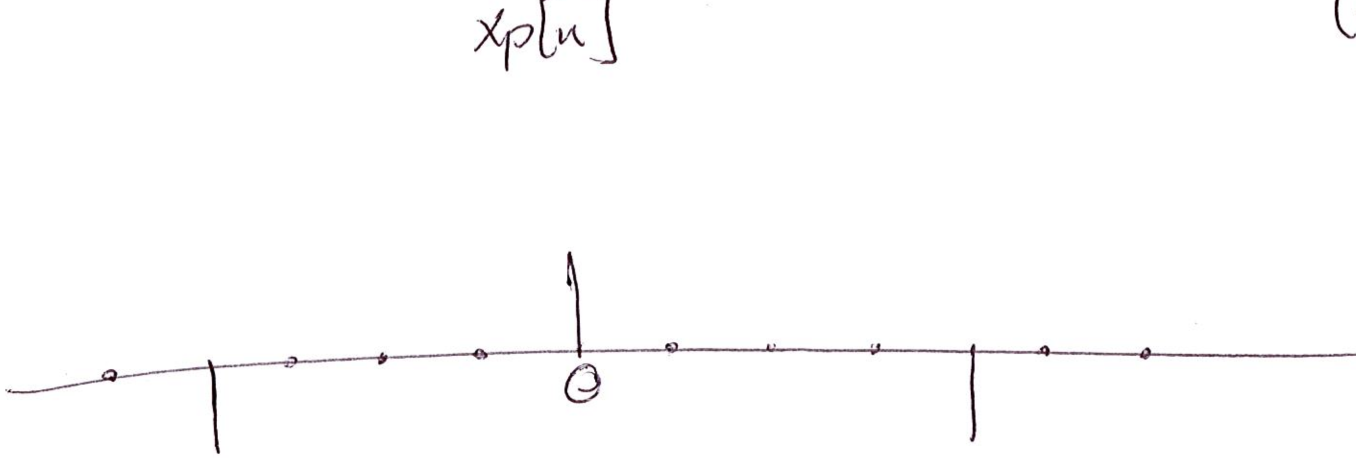
$$X_b(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_p[n]\, e^{-j\omega \frac{n}{N}}$$

$$X_p(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_p[n]\, e^{-j\omega n}$$

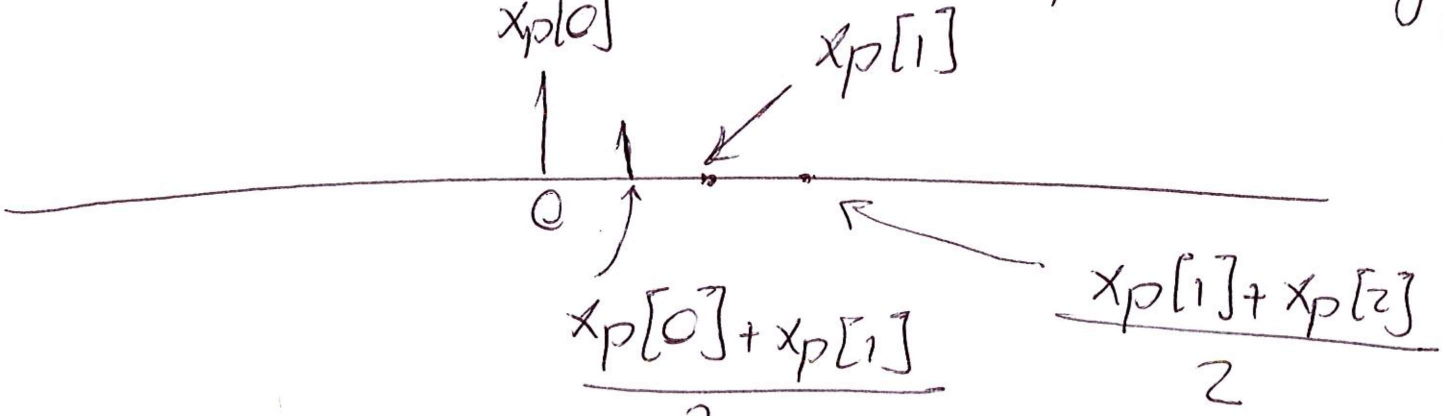
$$X_b(e^{j\omega}) = X_p(e^{j\omega/N})$$

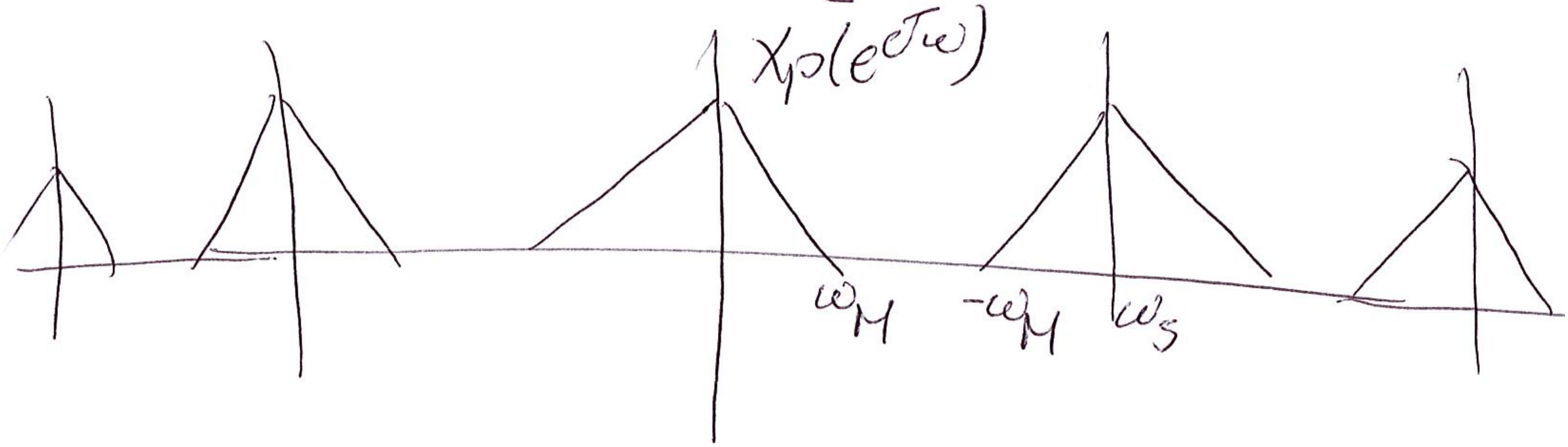




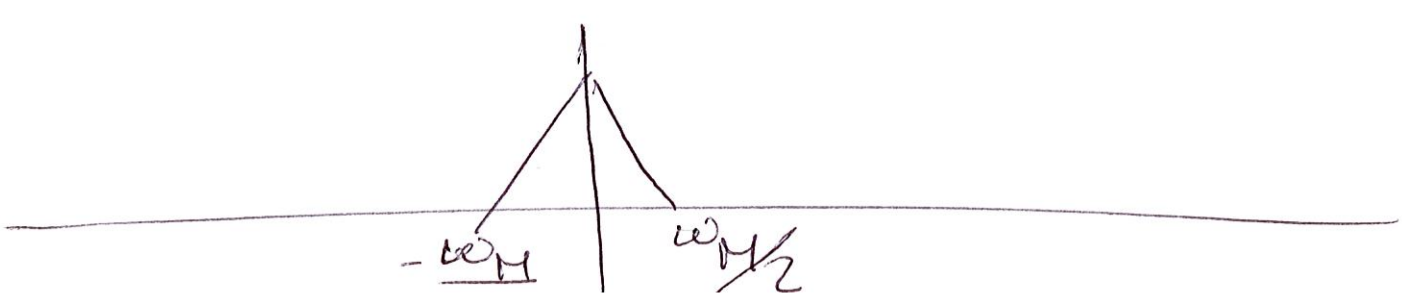


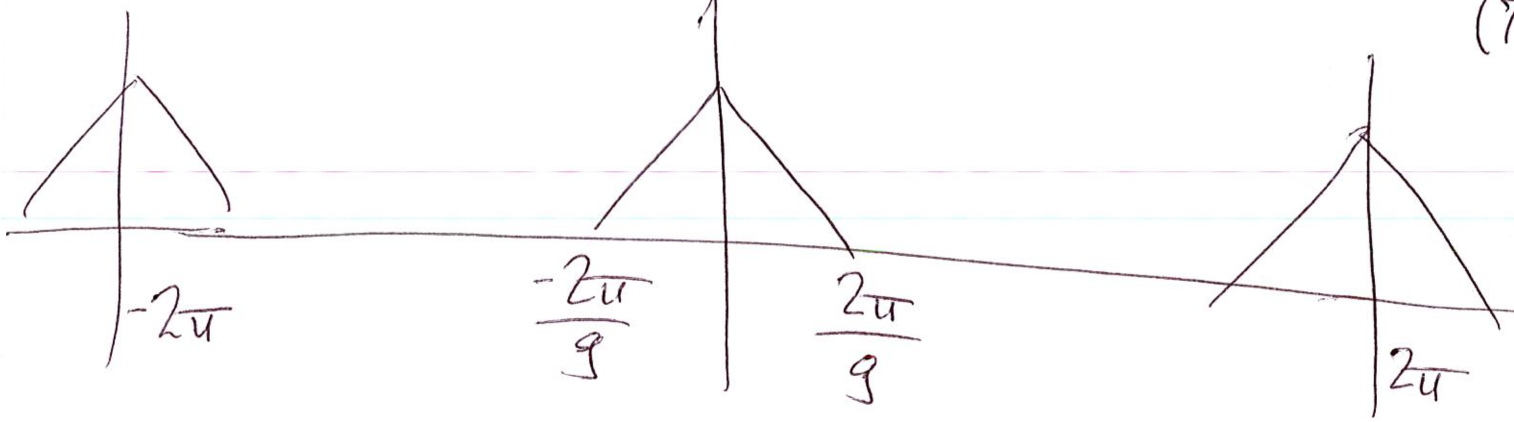
Up Sampling



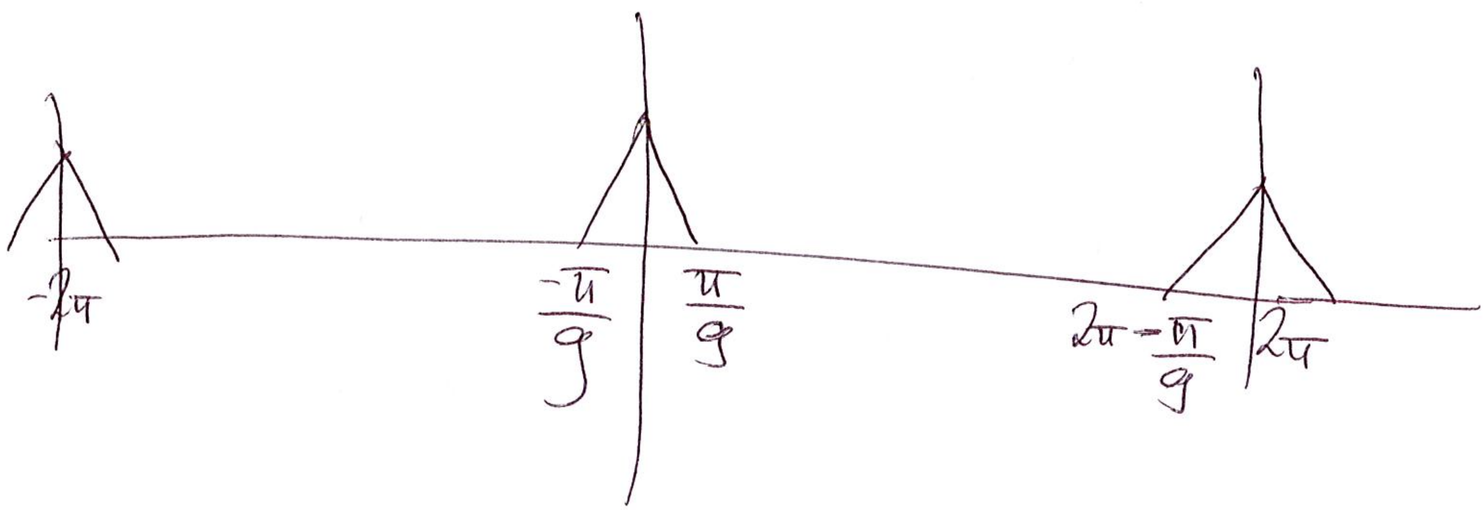


⇓ Up Sampling by 2



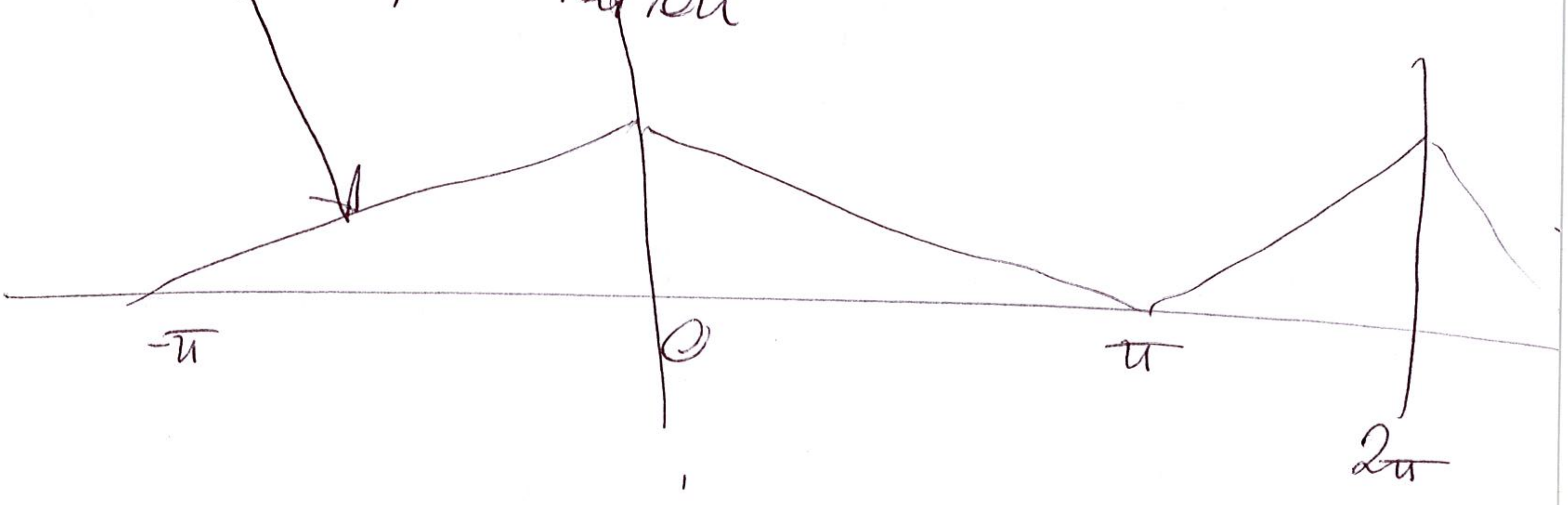


$\Downarrow$ UpSampling by 2



$\Downarrow$ DownSampling by 9

Most Efficient Representation



Introduction to Laplace Transform

$$\int_{\tau=-\infty}^{\infty} |h(\tau)| < \infty$$

Condition for stability of an LTI System whose impulse response is $h(t)$

Also <u>first Dirichlet Condition</u> needed for <u>convergence</u> of
existence
Fourier Transform

$\Rightarrow$ FT is only useful for analyzing stable LTI Systems

$\Rightarrow$ Generalization of FT for Continuous-Time Systems : Laplace Transform
DT Systems : Z-Transform