ECE 301 - Lecture #36

Laplace Transform

$$\mathcal{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt = \int_{-\infty}^{\infty} \underbrace{x(t) e^{-\alpha t}}\, e^{-j\omega t}\, dt$$

$$s = \alpha + j\omega$$

$$\cancel{\mathcal{F}\{x(t)\} = \mathcal{L}}$$

$$\mathcal{L}\{x(t)\} = \mathcal{F}\{x(t) e^{-\alpha t}\}$$

Example 9-1

$$x(t) = e^{-at} u(t), \quad a > 0$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \frac{1}{a + j\omega}$$

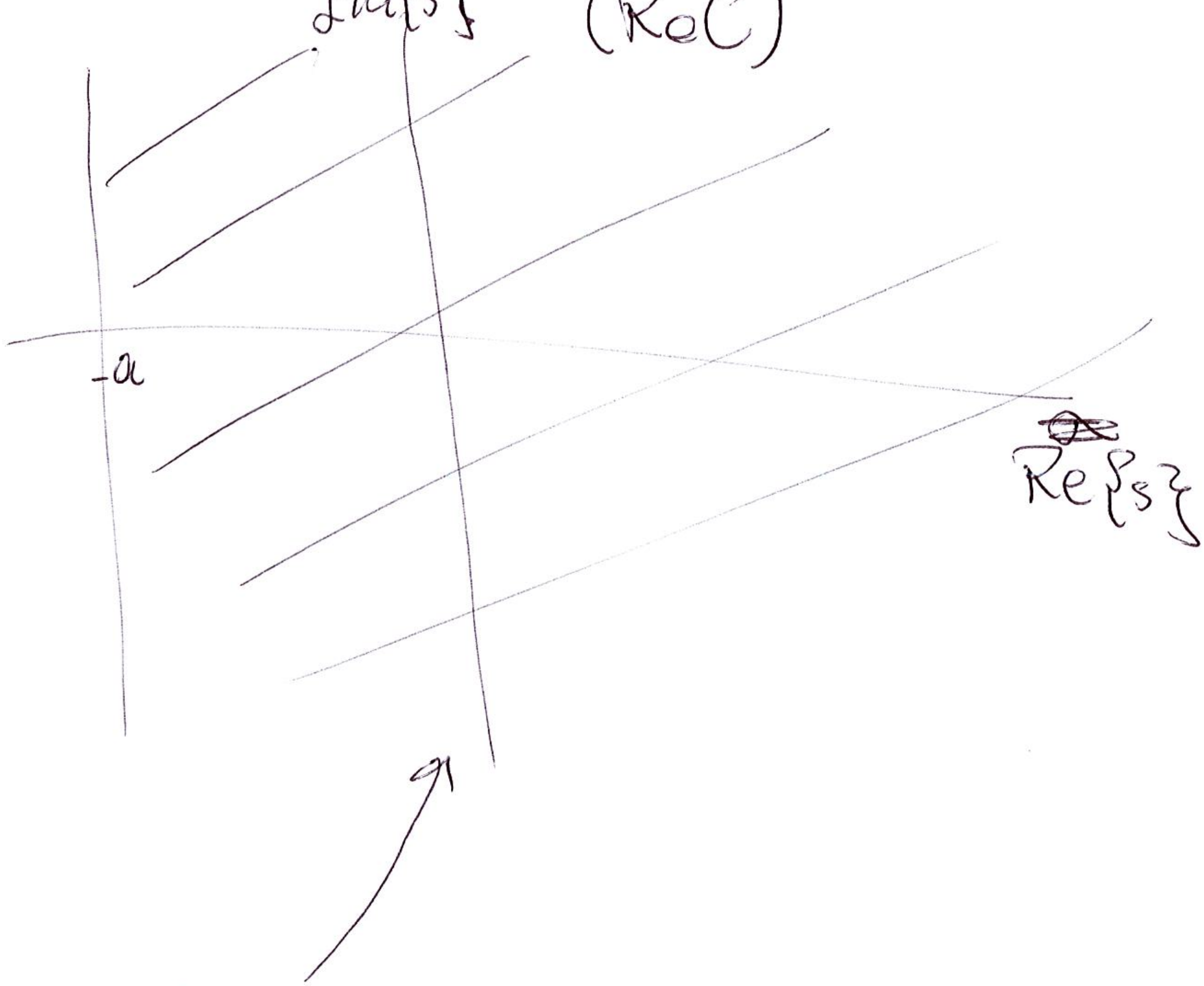
$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st}\, dt = \int_{0}^{\infty} e^{-at} e^{-st}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+s)t}\, dt$$

$$X(s) = \frac{1}{s+a}, \quad \underbrace{\sigma + a > 0}$$

$$\underbrace{\mathrm{Re}\{s\} > -a}$$

Region of Convergence (RoC)



$j\omega$-axis is in RoC

$\Longleftrightarrow \mathcal{F}\{x(t)\} \underbrace{\text{exists}}_{\text{converges}}$

Example 9.3

$$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$$

$$X(s) = \int_{-\infty}^{\infty} \left[3e^{-2t} - 2e^{-t}\right] u(t)\, e^{-st}\, dt$$

$$= \int_{0}^{\infty} \left[3e^{-2t} - 2e^{-t}\right] e^{-st}\, dt$$

$$3e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} \frac{3}{s+2}, \quad Re\{s\} > -2$$

$$2e^{-t}u(t) \xleftrightarrow{\mathcal{L}} \frac{2}{s+1}, \quad Re\{s\} > -1$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}, \quad Re\{s\} > -1$$

$$\boxed{\text{FT exists}}$$

Example 9.4

$$x(t) = e^{-2t} u(t) + e^{-t} (\cos 3t)\, u(t)$$

$$= e^{-2t} u(t) + \frac{1}{2} e^{-(1-3j)t} u(t) + \frac{1}{2} e^{-(1+3j)t} u(t)$$

$$e^{-2t} u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+2}, \quad Re\{s\} > -2$$

$$\frac{1}{2} e^{-(1-3j)t} u(t) \xleftrightarrow{\mathcal{L}} \frac{1/2}{s+(1-3j)}, \quad Re\{s\} > -1$$

$$\frac{1}{2} e^{-(1+3j)t} u(t) \xleftrightarrow{\mathcal{L}} \frac{1/2}{s+(1+3j)}, \quad Re\{s\} > -1$$

$$X(s) = \frac{1}{s+2} + \frac{1/2}{s+(1-3j)} + \frac{1/2}{s+(1+3j)}, \quad Re\{s\} > -1$$

$\mathcal{F}\{x(t)\}$ exists

Example 9.5

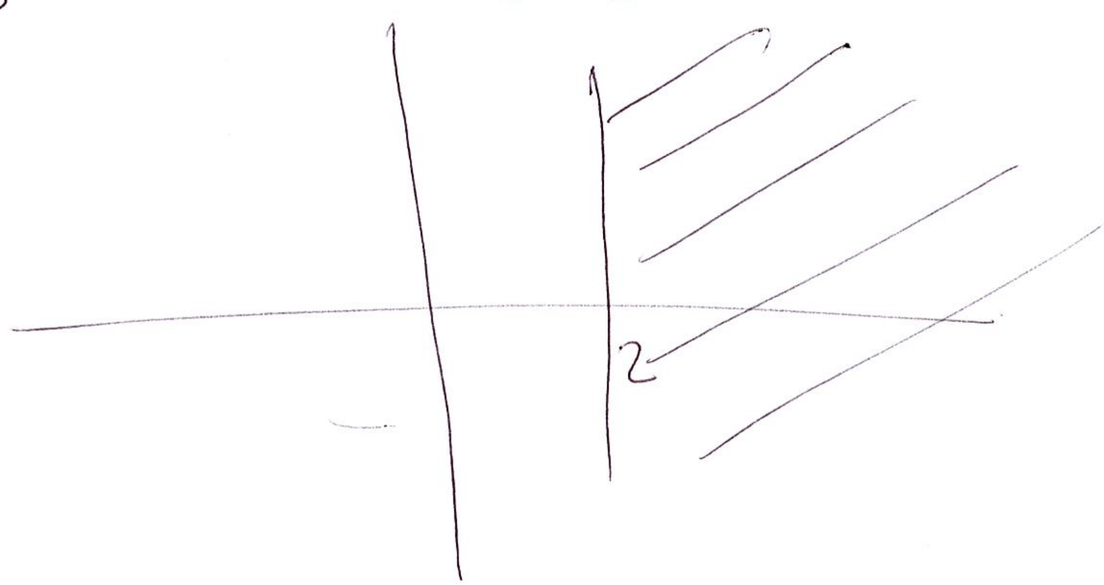
$$x(t) = \delta(t) - \frac{4}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$

$$\mathcal{L}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)\, e^{-st}\, dt = 1$$

RoC is the entire s-plane

$$\mathcal{L}\left\{\frac{4}{3}e^{-t}u(t)\right\} = \frac{4/3}{s+1}, \quad Re\{s\} > -1$$

$$\mathcal{L}\left\{\frac{1}{3}e^{2t}u(t)\right\} = \frac{1/3}{s-2}, \quad Re\{s\} > 2$$



Fourier Transform does not exist