ECE 301 - Lecture #5

$e^{j\omega_0 n}$ periodic if and only if

$\frac{\omega_0}{2\pi}$ is a rational number.

and in this case,

$\frac{\omega_0}{2\pi} = \frac{m}{N}$, we reduce the rational number, and then the denominator N would be the fundamental period

Example 1-6

$$x[n] = e^{j(2\pi/3)n} + e^{j\left(\frac{3\pi}{4}\right)n}$$

$$= x_1[n] + x_2[n]$$

$$x_1[n] = e^{j\left(\frac{2\pi}{3}\right)n}$$

$$\omega_{0,1} = \frac{2\pi}{3} \qquad \frac{\omega_{0,1}}{2\pi} = \frac{1}{3}$$

$$N_1 = 3$$

$$x_2[n] = e^{j\left(\frac{3\pi}{4}\right)n}$$

$$\omega_{0,2} = \frac{3\pi}{4} \qquad \frac{\omega_{0,2}}{2\pi} = \frac{3}{8}$$

$$N_2 = 8$$

$$x[n] = x_1[n] + x_2[n]$$

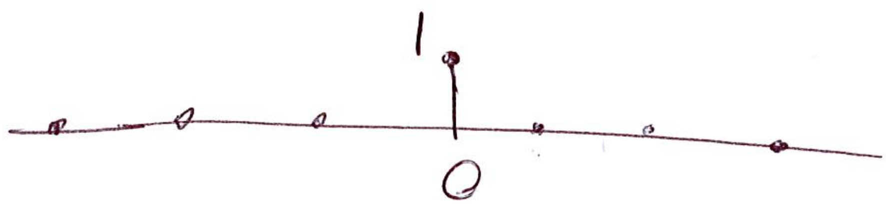
is periodic with fundamental period $N = \mathrm{Lcm}(N_1, N_2) = 24$

Unit step and the unit impulse

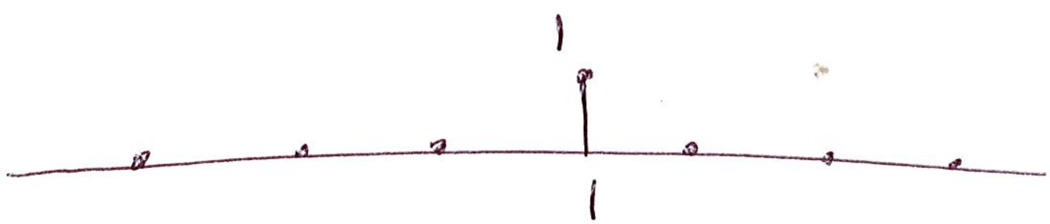
Discrete-time

$$\delta[n] = \begin{cases} 1 & \text{if } n=0 \\ 0 & \text{otherwise} \end{cases}$$

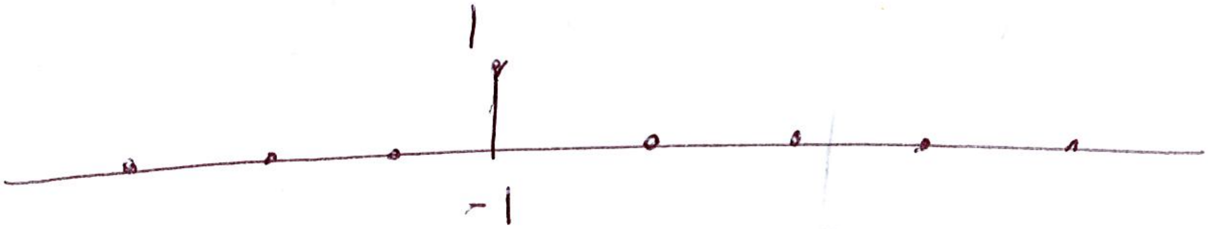
Unit impulse



$\delta[n-1]$

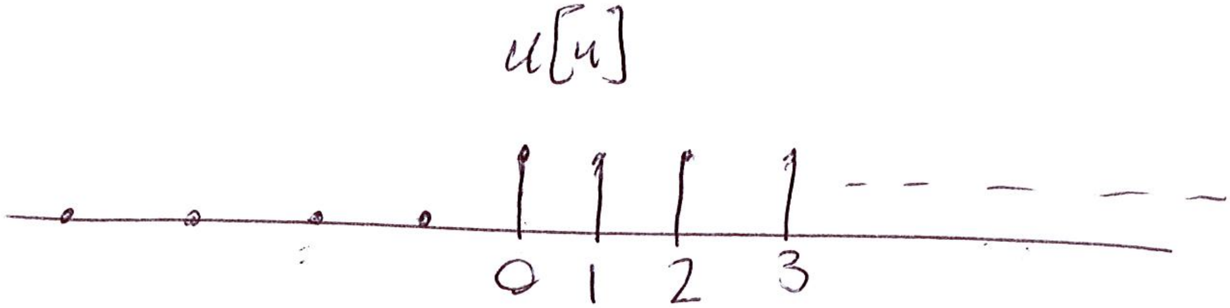


$\delta[n+1]$



$$u[n] = \begin{cases} 1 & , \text{ if } n \geq 0 \\ 0 & , \text{ otherwise} \end{cases}$$

Unit step

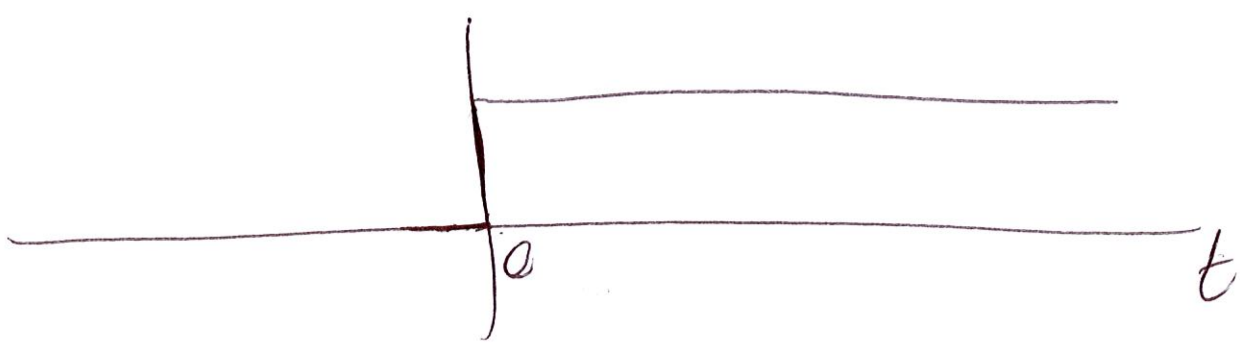


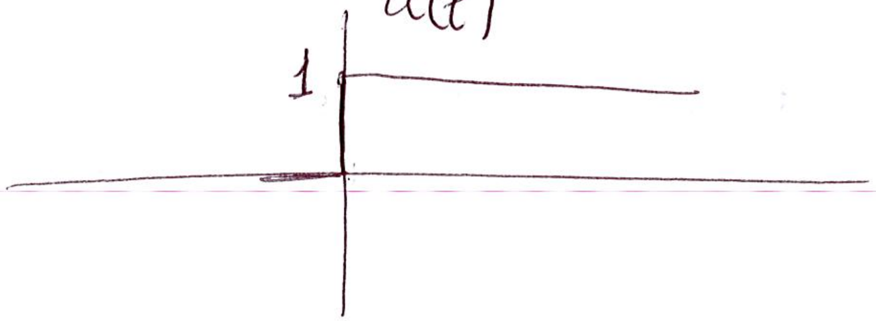
$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

$$\delta[n] = u[n] - u[n-1]$$

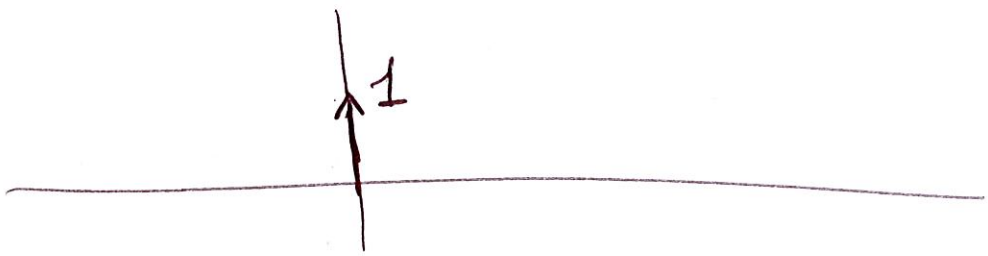
Continuous-time

$$u(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



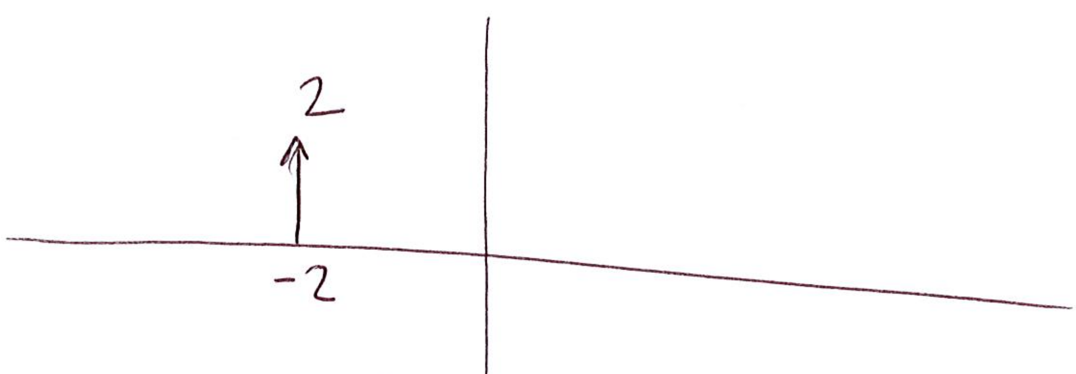


$$\delta(t) = \frac{du(t)}{dt}$$

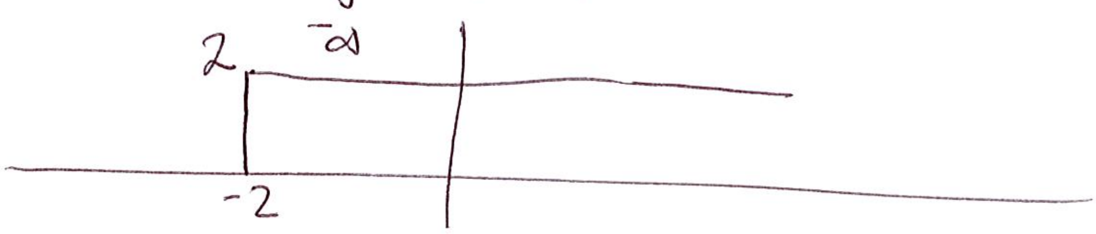


$$\int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

$$2\delta(t+2)$$



$$\int_{-\infty}^{t} 2\delta(\tau+2)$$



Properties of Systems

* Memoryless

we say that a system is memoryless if the output at any point in time depends on the input only through its value at the same point in time

$y[n] = 5\,x[n]$ Memoryless

$y[n] = x[n-1]$ Not memoryless

$y[n] = x[n+1]$ Not memoryless

$y[n] = (n-1)\,x[n]$ Memoryless

* Causality

~~Th~~ We say that a system is causal if the output at any point in time depends on the input only its current and past values

$y[n] = x[n-1]$ Causal

$y[n] = x[n]$ Causal

$y[n] = x[n+1]$ Not Causal

$y[n] = (n+1)\, x[n-1]$ Causal

Any system that is ~~the~~ memoryless, is also causal

* Stability

Bounded Signal

we say that a signal is bounded.
if there is ~~an~~ a positive real number B
such that $|x(t)| \leq B$ for every
value of t

A system is stable if whenever the input is bounded, the output is also bounded

$y(t) = x(t)$ stable

$y(t) = 2\,x(t)$ stable

$y(t) = -1000\,x(t)$ stable

$$y(t) = \frac{x(t-1)}{100}$$ Stable

$$y(t) = e^{x(t)}$$ Stable

$$y(t) = t\,x(t)$$ Unstable

$$y(t) = t$$ Unstable