ECE 301 - Lecture #6

$$x_k[n] = e^{jk\omega_0 n}$$

Assume that $x[n]$ is periodic with period N

$$\frac{\omega_0}{2\pi} = \frac{m}{N} \Rightarrow N\omega_0 = m\,2\pi$$

$$x_{k+N}[n] = e^{j(k+N)\omega_0 n}$$

$$= e^{jk\omega_0 n}\, e^{jN\omega_0 n}$$

$$= e^{jk\omega_0 n}\, \underbrace{e^{jm2\pi n}}_{\cos m2\pi n + \cancel{j\sin m2\pi n}}$$

$$1$$

$$= e^{jk\omega_0 n}$$

$$= x_k[n]$$

The set of harmonically related DT-complex exponentials $\{e^{jk\omega_0 n}\}$ and $\frac{\omega_0}{2\pi} = \frac{m}{N}$, has only N distinct elements

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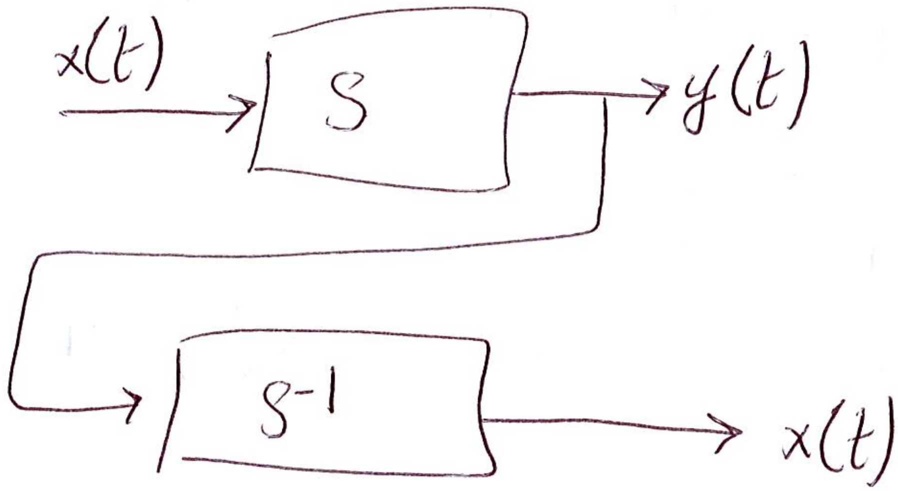
System Properties

* Memoryless
* Causal
* Stable

  Given any bounded input, the output is bounded
* Invertibility
* Linearity
* Time Invariance

Invertibility

We say that a system is invertible if it has an inverse system



$y(t) = 2\,x(t)$ Invertible

$\tilde{y}(t) = \frac{1}{2}\tilde{x}(t)$ Inverse system

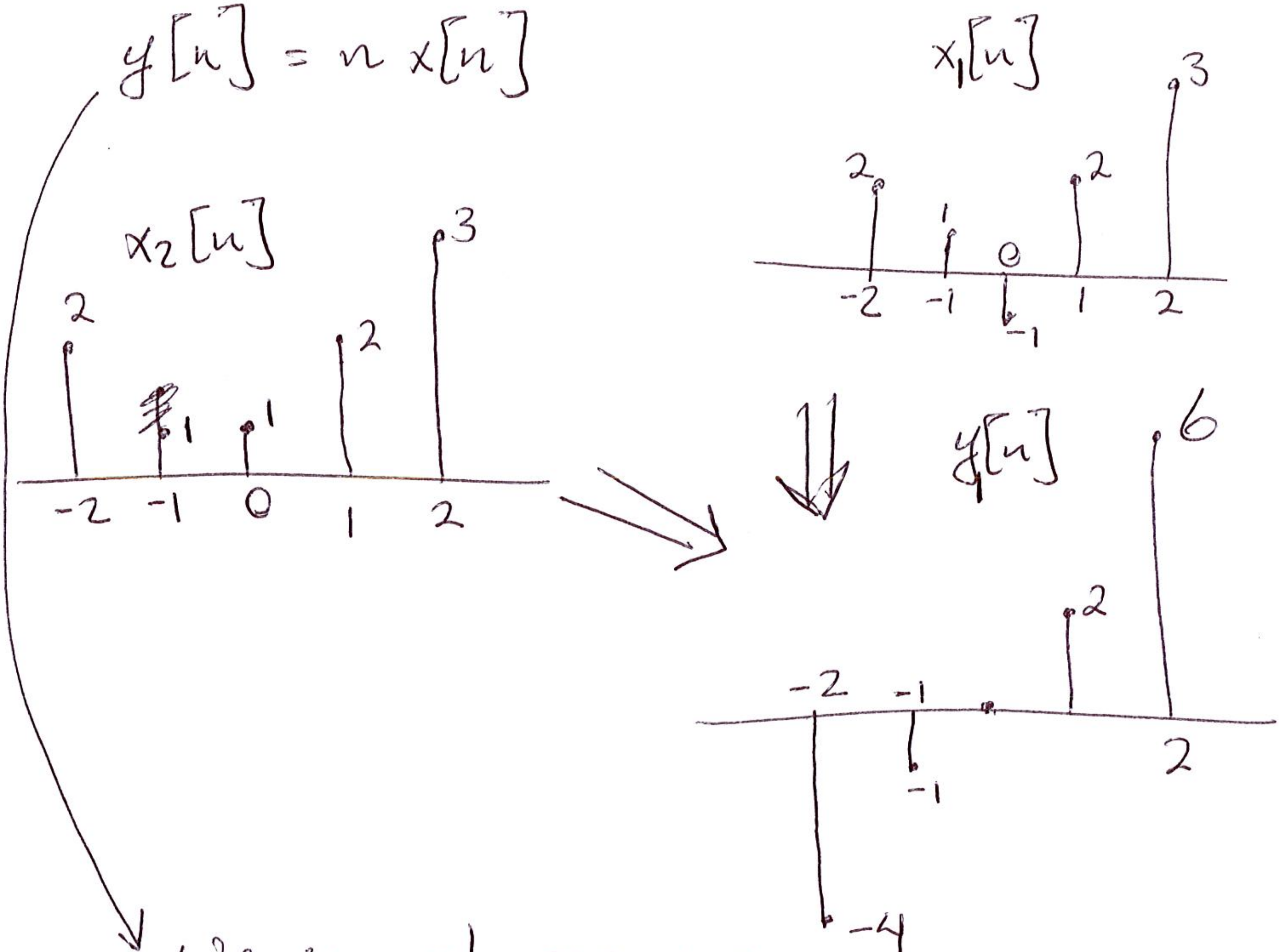
$\tilde{x}(t) = y(t) \Longrightarrow \tilde{y}(t) = x(t)$

$y(t) = x(t-1)$ Invertible

$\tilde{y}(t) = \tilde{x}(t+1)$ Inverse System

$y(t) = 2$ Not invertible

$y[n] = n\,x[n]$

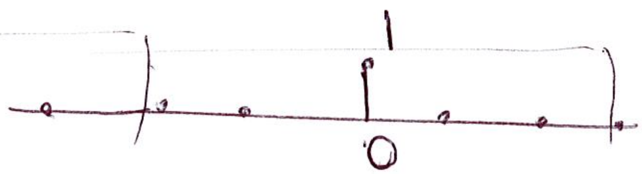


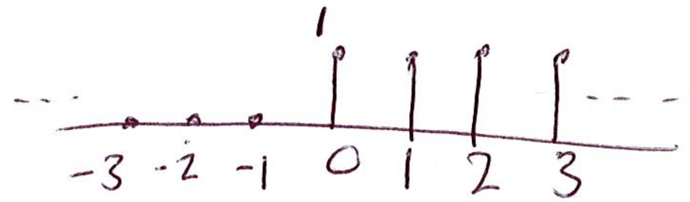
We cannot recover the value of $x[0]$ from the output

$\Rightarrow$ Not invertible

$$y[n] = \sum_{k=-\infty}^{n} x[k] \quad \text{Accumulator}$$

If $x[n] = \delta[n]$ then $y[n] = u[n]$





Is the accumulator invertible?

Let's say we only have $y[n]$ for all n and we want to recover $x[4]$

$$y[4] = \sum_{k=-\infty}^{4} x[k] \; - \; y[3] = \sum_{k=-\infty}^{3} x[k]$$

$$= x[4]$$

$$y[n] = \sum_{k=-\infty}^{n} x[k] \; - \; y[n-1] = \sum_{k=-\infty}^{n-1} x[k]$$

$$= x[n]$$

$$y[n] = \sum_{k=-\infty}^{n} x[n] \quad \text{Accumulator}$$

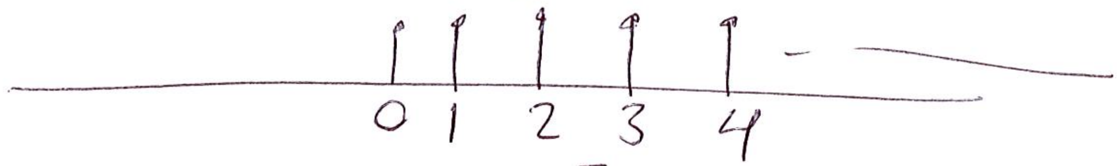
$$\tilde{y}[n] = \tilde{x}[n] - \tilde{x}[n-1] \quad \text{Differentiator}$$

$\boxed{\text{Inverse System}}$

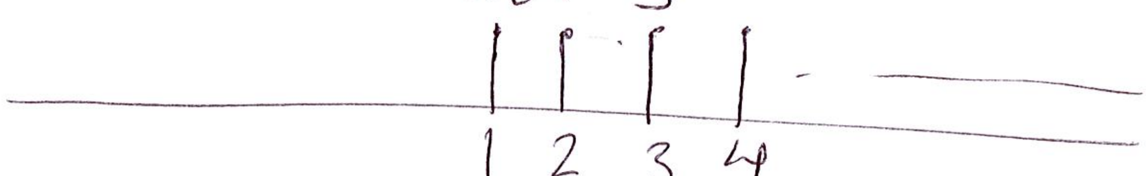
If $x[n] = \delta[n]$, then $y[n] = u[n]$

If $\tilde{x}[n] = u[n]$, then $\tilde{y}[n] = \delta[n]$

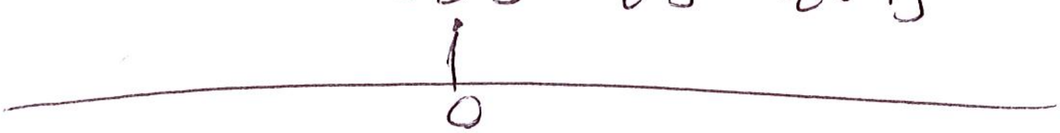
$u[n]$



$u[n-1]$



$\delta[n] = u[n] - u[n-1]$



CT

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{Accumulator}$$

$$\tilde{y}(t) = \frac{d\, \tilde{x}(t)}{dt} \qquad \text{Differentiator}$$

$\boxed{\text{Inverse System}}$

Linearity

* Addition

$$x_1[n] \longrightarrow \boxed{S} \longrightarrow y_1[n]$$

$$x_2[n] \longrightarrow \boxed{S} \longrightarrow y_2[n]$$

$$x_1[n] + x_2[n] \longrightarrow \boxed{S} \longrightarrow y_1[n] + y_2[n]$$

* Scaling

$$x[n] \longrightarrow \boxed{S} \longrightarrow y[n]$$

$$\alpha\, x[n] \longrightarrow \boxed{S} \longrightarrow a\, y[n]$$

$y[n] = x[n-1]$

Additive? ✓

Scaling? ✓

Linear

$y[n] = 2$

Additive? x

Not Linear

$y[n] = x^2[n]$

Additive? x

Not Linear

$y_1[n] = x_1^2[n]$ $y_2[n] = x_2^2[n]$

$$y[n] = \left(x_1[n] + x_2[n]\right)^2 = x_1^2[n] + x_2^2[n] + \underbrace{2x_1[n]\,x_2[n]}_{\text{Not always zero}}$$

Not always Zero

$$y[n] = 2x[n] + 3$$

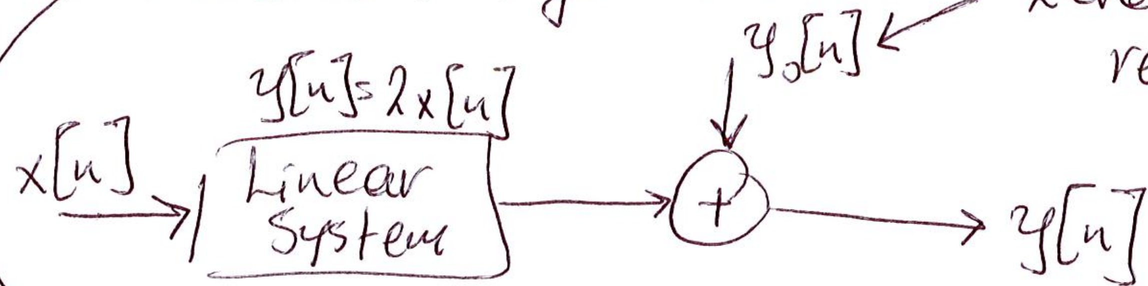
Additive? ✗ Not Linear

$$y_1[n] = 2x_1[n] + 3 \qquad y_2[n] = 2x_2[n] + 3$$

$$y[n] = 2\left(x_1[n] + x_2[n]\right) + 3 \neq y_1[n] + y_2[n]$$

Incrementally Linear

Zero-Input response



$$x_1[n] \rightarrow \boxed{S} \rightarrow y_1[n] = 2x_1[n] + 3$$

$$x_2[n] \rightarrow \boxed{S} \rightarrow y_2[n] = 2x_2[n] + 3$$

$$\left(y_1[n] - y_2[n]\right) = f\left(x_1[n] - x_2[n]\right)$$

$$= 2\left(x_1[n] - x_2[n]\right)$$

Linear

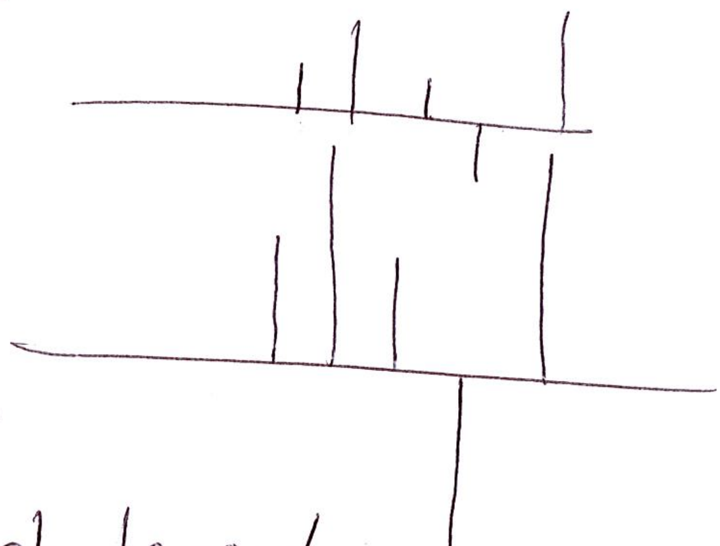
Time Invariance

$$x[n] \longrightarrow \boxed{S} \longrightarrow y[n]$$

$$x[n-n_0] \longrightarrow \boxed{S} \longrightarrow y[n-n_0]$$

constant

$$y[n] = 2\,x[n]$$



Another Definition

Dependence between Input-Output does not depend on the value of time