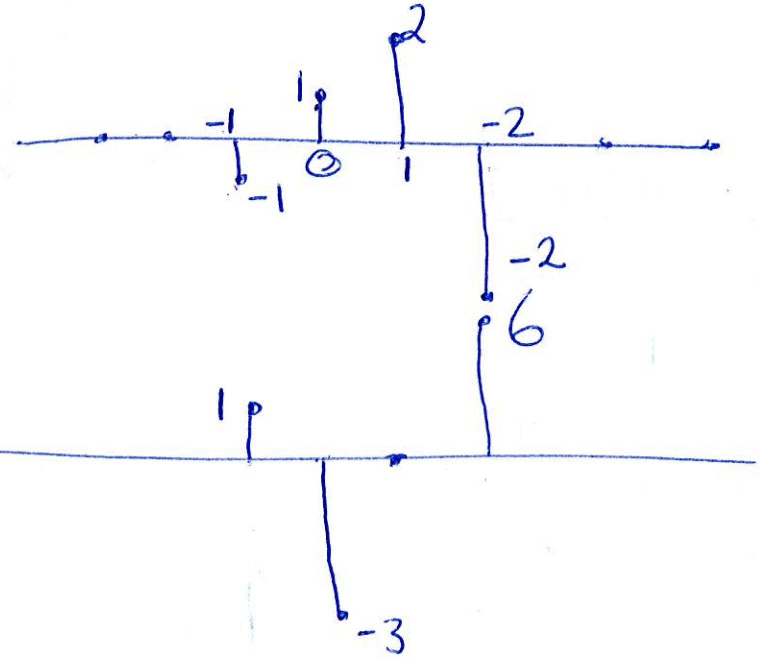
ECE 301 - Lecture #7

Time Invariance

Any shift in the input signal, results in the same shift in the output signal, and this is true for any input signal

Examples

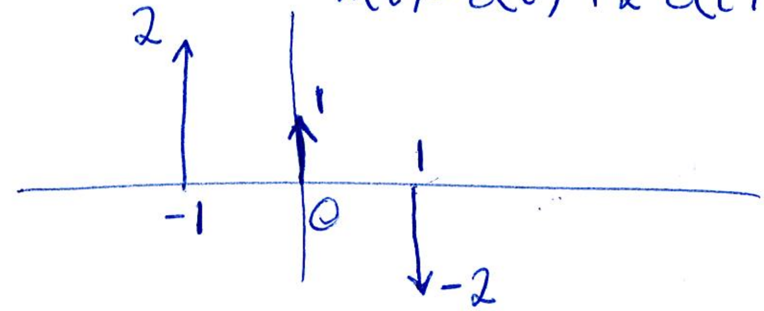
$y[n] = 2x[n-1] - x[n]$ Time Invariant



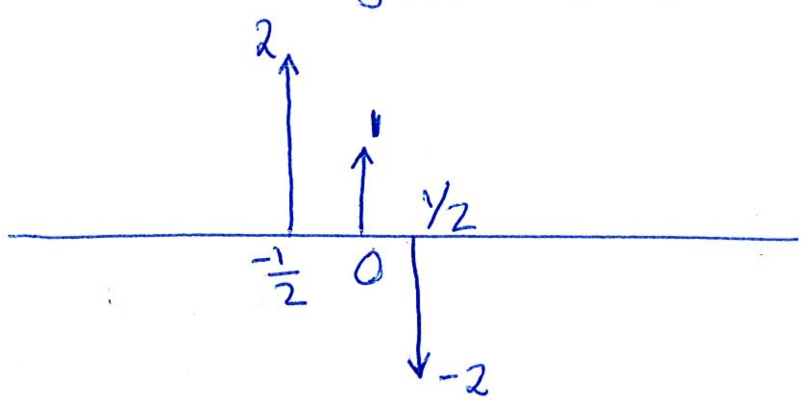
$y[n] = (n-1)\,x[n]$ Time Variant

$y(t) = x(2t)$ Time Invariant? $\boxed{\text{No}}$

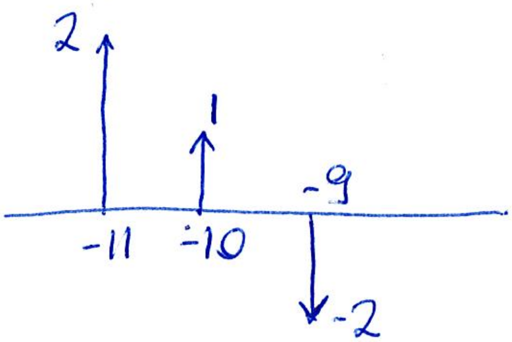
$x(t) = \delta(t) + 2\,\delta(t+1) - 2\,\delta(t-1)$



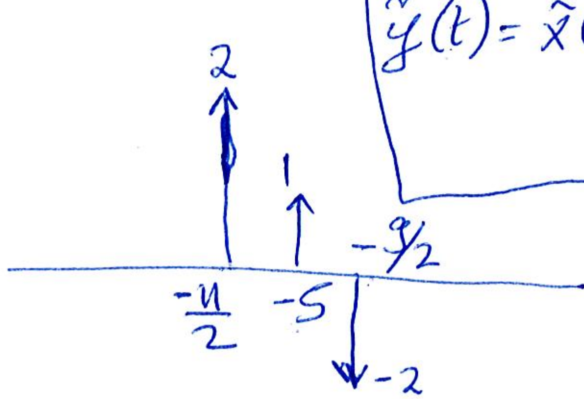
$y(t) = x(2t)$



$\tilde{x}(t) = x(t+10)$



$$\boxed{\tilde{y}(t) = \tilde{x}(2t) = y(t+5) \neq y(t+10)}$$



Linear and Time Invariant (LTI) Systems

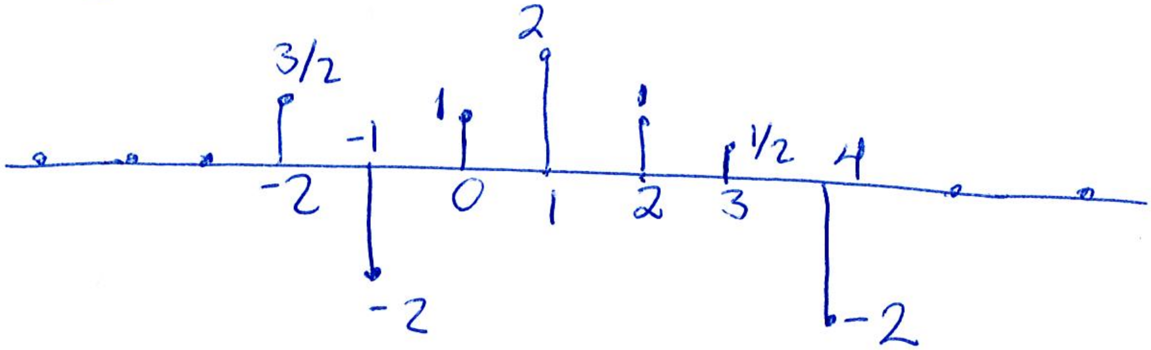
Consider any DT signal $x[n]$

$$x[n]\underbrace{\delta[n]}_{\begin{cases}1 & n=0\\ 0 & \text{otherwise}\end{cases}} = \begin{cases} x[0] & \text{if } n=0 \\ 0 & \text{otherwise}\end{cases}$$

$$x[n]\underbrace{\delta[n-1]}_{\begin{cases}1 & n=1\\ 0 & \text{otherwise}\end{cases}} = \begin{cases} x[1] & \text{if } n=1 \\ 0 & \text{otherwise}\end{cases}$$

$$x[n]\,\delta[n+1] = \begin{cases} x[-1] & \text{if } n=-1 \\ 0 & \text{otherwise}\end{cases}$$

Ex $x[n]$



$=$

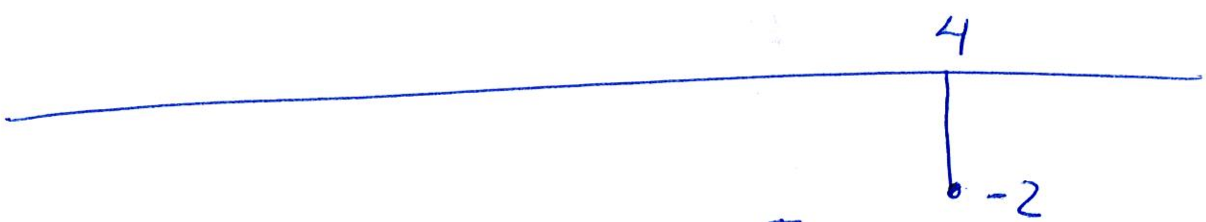
$x[n]\delta[n] = \delta[n]$

$+$ $x[+1]\delta[n-1]$

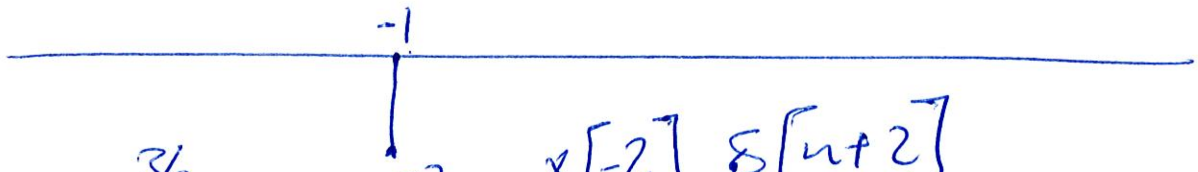
$+$ $x[2]\delta[n-2]$

$+$ $x[3]\delta[n-3]$

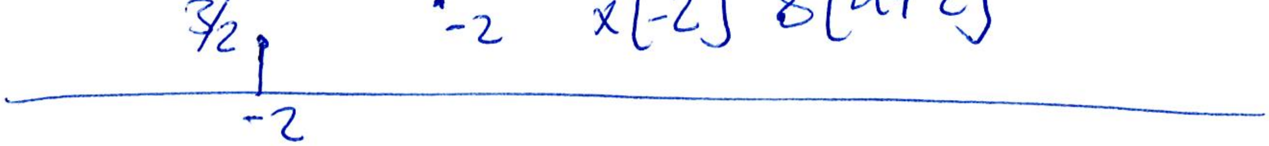
$+$ $x[4]\delta[n-4]$



$+$ $x[-1]\delta[n+1]$



$+$ $x[-2]\ \delta[n+2]$



$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$\Downarrow$ Input to an LTI System

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,\underbrace{h_k[n]}_{\text{Response to } \delta[n-k]}$$

$\boxed{\text{By Linearity}}$

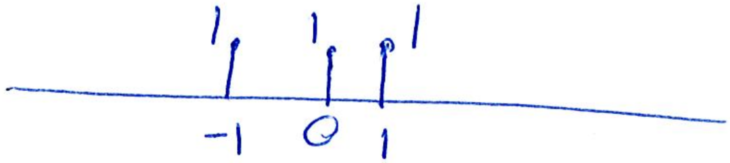
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \triangleq x[n] * h[n]$$

where $\underbrace{h[n]}$ is the output when the input is $\delta[n]$

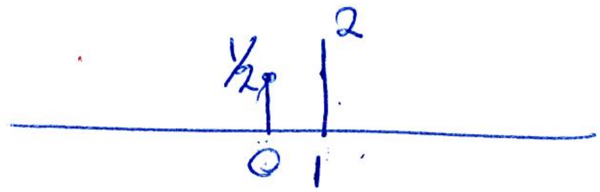
$\boxed{\text{Impulse Response}}$

Example 2.1

$h[n]$

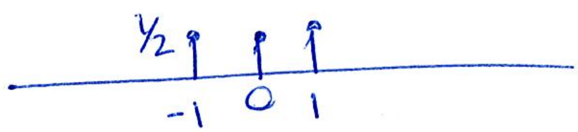


$x[n]$

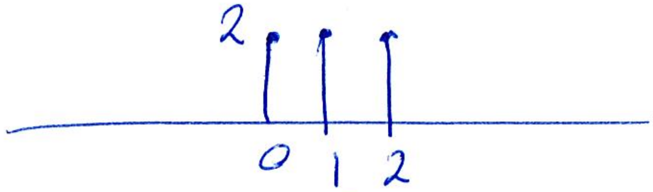


$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k=0}^{1} x[k]\, h[n-k] = \frac{1}{2} h[n] + 2\, h[n-1]$$



$+$



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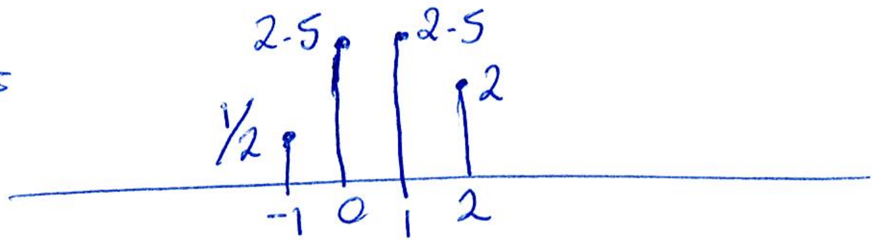
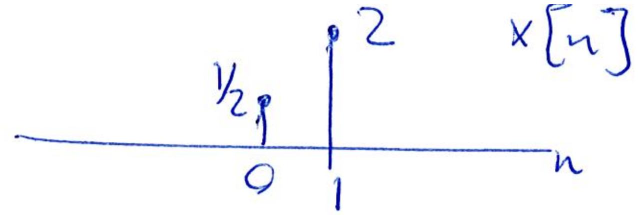
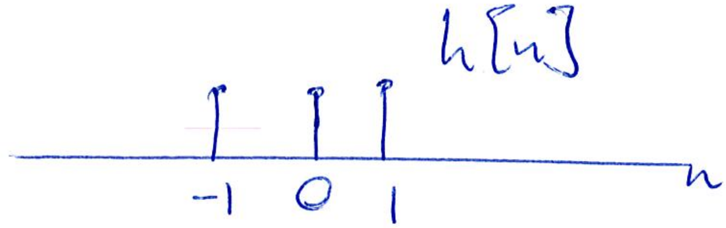
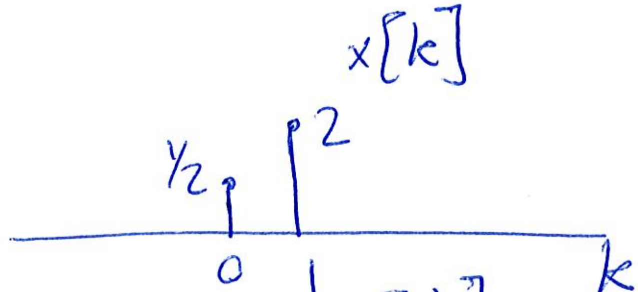
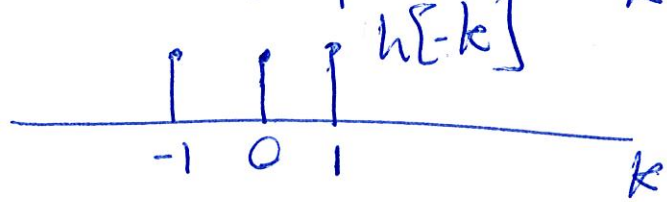


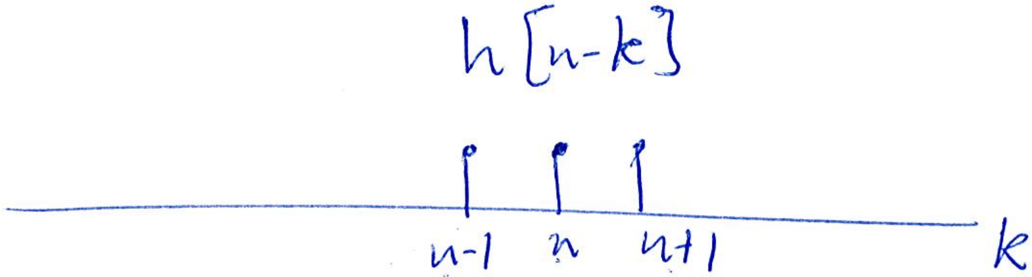




$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$







If $n > 2$, $y[n] = 0$

If $n < -1$, $y[n] = 0$