Quiz 3.

$\omega_0^* = \omega_0.$

a) $e^{j\omega_0 t}$. $\quad a_1 = 1. \quad a_k = 0 \quad k \neq 1.$

b). $\sin(\omega_0 t) = \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$\omega_0^* = \omega_0$

$a_1 = 1/2j \qquad a_{-1} = -1/2j$

c) $\cos(2\omega_0 t)$. $\quad \omega_0^* = 2\omega_0$

$\cos(2\omega_0 t) = \dfrac{e^{j(\cancel{2}\omega_0)t} + e^{-j(\cancel{2}\omega_0)t}}{2}.$

$a_1 = \frac{1}{2} \qquad a_{-1} = 1/2 \qquad a_k = 0 \quad k \neq +1, -1.$

d) $\sin(\omega_0 t) + \cos(2\omega_0 t). \qquad \omega_0^* = \omega_0.$

$= \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + \dfrac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2}.$

$a_1 = 1/2j \qquad a_{-1} = -1/2j \qquad a_2 = 1/2 \qquad a_{-2} = 1/2 \qquad a_k = 0$

$k \notin \{1, -1, 2, -2\}.$

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e)

$a_k = \frac{1}{T}\int_T x(t)\, e^{jk\omega_0 t}$. $T = 4,\ \omega_0 = \frac{2\pi}{4} = \pi/2$.

$\Rightarrow a_k = \frac{1}{4}\int_1^3 x(t)\, e^{jk\frac{\pi}{2}t} = \frac{1}{4}\int_{-1}^{1} 1 \cdot e^{jk\frac{\pi}{2}t} = \frac{1}{4}\left[\frac{e^{jk\frac{\pi}{2}t}}{jk\frac{\pi}{2}}\right]_{-1}^{1}$

$= \frac{1}{4}\,\frac{e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}}{2j\,[k\pi]} = \frac{\sin k\frac{\pi}{2}}{4k\pi}$

$\cancel{f)}$

$a_0 = \frac{1}{4}\cdot 2 = 1/2$.

f) Similar to (e).

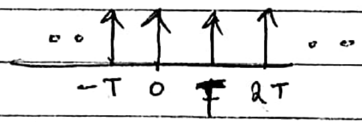
$a_k = \frac{1}{4}\left[\frac{e^{jk\frac{\pi}{2}t}}{jk\frac{\pi}{2}}\right]_{-1/2}^{1/2} = \frac{1}{4}\,\frac{e^{j\frac{\pi}{4}k} - e^{-jk\frac{\pi}{4}}}{2j\,(k\pi)} = \frac{\sin(\pi/4)}{4k\pi}$.

$\cancel{g}$

$a_0 = \frac{1}{4}\cdot 1 = 1/4$.

g. $x(t) = 1$ $a_0 = 1$ $a_k = 0 \quad k \neq 0$.

h. $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$.



$a_k = \frac{1}{T}\int_T x(t)\, e^{jk\omega_0 t}$ $\omega_0 = \frac{2\pi}{T}$

$= \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{jk\frac{2\pi}{T}t} = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, e^{jk\omega_0 t}\, dt$

$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, dt = 1/T$.

(i), (j) → Check HW3 solutions.

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