Quiz 4

(Q1)

1. a) $\sum_{n=60}^{74} x[n]$

$a_k = \sum_{\langle N \rangle} x[n] e^{jk\omega_0 n} \Rightarrow a_0 = N \sum_{\langle N \rangle} x[n] = 15 \sum_{n=60}^{74} x[n] = 2.15$

$= 30$

b). Average power of the signal.

$\frac{1}{N} \sum_{\langle N \rangle} |x[n]|^2 = \sum_{\langle N \rangle} |a_k|^2 = 48$

c) $x[90]$.

$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$

$x[90] = x[0] = \sum_{\langle N \rangle} a_k = 10$

d). $x[91] = x[1] = \sum_{k=\langle N \rangle} a_k e^{jk\frac{2\pi}{15}}$

$= 2 + \sum_{k=1}^{6} (-1) e^{j\frac{2\pi}{15}k} + \sum_{k=7}^{12} e^{j\frac{2\pi}{15}k} + 4 \sum_{k=13}^{14} e^{j\frac{2\pi}{15}k}.$

e) $x[n+1] \Rightarrow b_k = a_k e^{j\frac{2\pi}{15}}$

2. $h[n] = \left(\frac{1}{4}\right)^n u[n]$. $x[n] = \cos\left(\frac{3\pi n}{10}\right) \rightsquigarrow \omega_0 = \frac{3\pi}{10}$ $a_1 = \frac{1}{2}, a_{-1} = \frac{1}{2}$

$y[n] = \sum_{k=-\infty}^{\infty} a_k H(e^{jk\omega_0}) e^{jk\omega_0 n}.$

$H(e^{j\omega}) = \frac{1}{1 - \frac{1}{4} e^{-j\omega}}.$

For $y[n]$

$\therefore \omega_0 = \frac{3\pi}{10}$

$b_1 = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{4} e^{-j\frac{3\pi}{10}}}$

$b_{-1} = \frac{1}{2} \cdot \frac{1}{1 - \frac{1}{4} e^{+j\frac{3\pi}{10}}}$