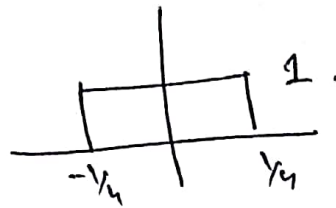


## Quiz 5.

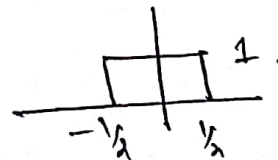
$$1. \quad x(t) = \frac{\sin(t/4) \cdot \sin(t/2)}{\pi t^2}$$

$$x(t) = \pi \cdot \frac{\sin(t/4)}{\pi t} \cdot \frac{\sin(t/2)}{\pi t}$$

$$\frac{\sin(t/4)}{\pi t}$$



$$\frac{\sin(t/2)}{\pi t}$$



$$\pi \cdot \sin(t/4) \cdot \sin(t/2) \longleftrightarrow \frac{1}{2\pi} \cdot \pi \cdot \left[ \text{rect}(t/4) * \text{rect}(t/2) \right]$$

$$= \frac{1}{2} \left[ \text{trapezoidal pulse from } -3/4 \text{ to } 3/4 \text{ with peak height } 1/2 \right]$$

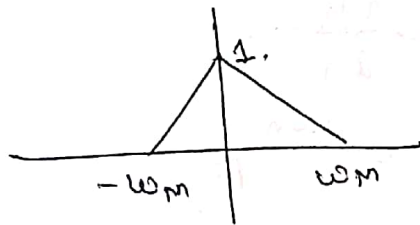
$$2. \quad \frac{dy(t)}{dt} + ay(t) = x(t) \quad \leftrightarrow \quad (j\omega) Y(j\omega) + aY(j\omega) = X(j\omega)$$

$$\Rightarrow \frac{Y(j\omega)}{X(j\omega)} = H(j\omega) = \frac{1}{a + j\omega}$$

$$\frac{1}{a+j\omega} \xleftrightarrow{\text{IFT}} e^{-at} u(t).$$

This system is an approximation for a low pass filter.

3.  $x(j\omega)$



$$\cos^2(\omega_c t) = \cos(\omega_c t) \cdot \cos(\omega_c t).$$

$$\cos(\omega_c t) = \pi \delta(\omega - \omega_c) + \pi \delta(\omega + \omega_c).$$

$$\begin{aligned} \Rightarrow \cos^2(\omega_c t) &= \frac{1}{2\pi} \cdot \left[ \pi \delta(\omega - \omega_c) + \pi \delta(\omega + \omega_c) \right] \\ &\quad * \left[ \pi \delta(\omega - \omega_c) + \pi \delta(\omega + \omega_c) \right] \\ &= \pi \left[ \delta(\omega - 2\omega_c) + 2\delta(\omega) + \delta(\omega + 2\omega_c) \right]. \end{aligned}$$

$$x(t) \cos^2(\omega_c t) \xleftrightarrow{\text{FT}} \frac{1}{2\pi} x(j\omega) * \left\{ \pi \left[ \delta(\omega - 2\omega_c) + 2\delta(\omega) + \delta(\omega + 2\omega_c) \right] \right\}$$

=

