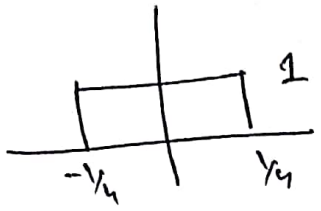
Quiz 5.

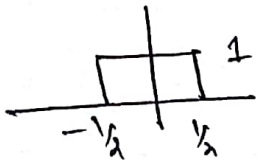
1. $x(t) = \dfrac{\sin(t/4).\ \sin(t/2)}{\pi t^2}$.

$$x(t) = \pi \cdot \frac{\sin t/4}{\pi t} \cdot \frac{\sin(t/2)}{\pi t}.$$

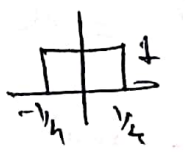
$$\frac{\sin(t/4)}{\pi t} \longleftrightarrow$$



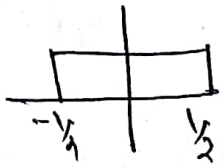
$$\frac{\sin(t/2)}{\pi t} \longleftrightarrow$$



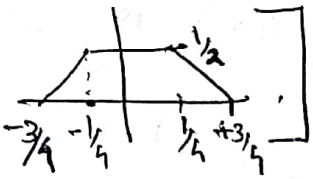
$$\pi \cdot \sin(t/4).\ \sin(t/2) \longleftrightarrow \frac{1}{2\pi} \cdot \pi \cdot$$



$*$



$$= \frac{1}{2}$$



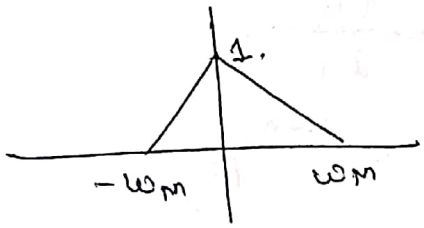
2. $\dfrac{dy(t)}{dt} + a\,y(t) = x(t). \longleftrightarrow (j\omega)\,Y(j\omega) + a\,Y(j\omega) = X(j\omega)$

$$\Rightarrow \frac{Y(j\omega)}{X(j\omega)} = H(j\omega) = \frac{1}{a + j\omega}.$$

$$\frac{1}{a+j\omega} \xleftrightarrow{IFT} e^{-at}u(t).$$

This system is an approximation for a low pass filter.

3. $x(j\omega)$



$$\cos^2(\omega_c t) = \cos(\omega_c t) \cdot \cos(\omega_c t).$$

$$\cos(\omega_c t) = \pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c).$$

$$\Rightarrow \cos^2(\omega_c t) = \frac{1}{2\pi} \cdot \left[\pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c)\right] * \left[\pi(\delta(\omega-\omega_c) + \pi(\delta(\omega+\omega_c)\right]$$

$$= \pi\left[\delta(\omega-2\omega_c) + 2\delta(\omega) + \delta(\omega+2\omega_c)\right].$$

$$x(t)\cos^2(\omega_c t) \xleftrightarrow{FT} \frac{1}{2\pi} X(j\omega) * \left\{\pi\left[\delta(\omega-2\omega_c) + 2\delta(\omega) + \delta(\omega+2\omega_c)\right]\right\}$$

$=$