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ECE 368 Spring 2016

Homework 5

1) Trees as Graphs (15 Pts):

A tree is a graph that satisfies two properties

I. It is connected

II. There is a unique path between any pair of vertices.

a) What is the condition on the number of edges on a tree with N vertices?

➔ $|E| = N - 1$

b) Prove your answer to part a)

➔ We prove the result by using induction on n, the number of vertices. The result is obviously true for n = 1, 2 and 3. Let the result be true for all trees with fewer than n vertices. Let T be a tree with n vertices and let e be an edge with end vertices u and v. So the only path between u and v is e. Therefore deletion of e from T disconnects T. Now, T − e consists of exactly two components T1 and T2 say, and as there were no cycles to begin with, each component is a tree. Let n1 and n2 be the number of vertices in T1 and T2 respectively, so that n1 + n2 = n. Also, n1 < n and n2 < n. Thus, by induction hypothesis, number of edges in T1 and T2 are respectively n1 −1 and n2 −1. Hence the number of edges in T = n1 −1+n2 −1+1 = n1 +n2 −1 = n−1.

2) Graphs (15 Pts)

Discuss the most appropriate representations (with a justification) for the following types of graphs:

a) 1st type: Unweighted, directed and very dense (the degree of each vertex is O(n)).

➔ (Binary) Adjacency Matrix

b) 2nd type: Weighted, undirected and the degree of each vertex is O(1).

➔ Adjacency list

c) 3rd type: Weighted, undirected, and sparse with a large number of vertices (the total number of edges is O(n)).

➔ Binary relation list (Adjacency list, Lower-triangular adjacency matrix)

3) Graphs: Finding Connected Components (35 Pts):

Do not submit any file, submit the provided file without any changes → 5 pts

Otherwise → 20-35 pts

4) Graphs: Shortest Path (Dijkstra's Algorithm) (35 Pts):

Do not submit any file, submit the provided file without any changes → 5 pts

Otherwise → 20-35 pts