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ECE 368 Spring 2016.

Homework 1

1) Explain the difference between set, list (array or linked list), and map. Then fill the boxes with the container to use for each case.

List order matters, is accessed through indexing and allow duplicates.

Set must be hashable, order doesn't matter and doesn't allow duplicates.

Map must be hashable, must store a key-value pair and doesn't allow duplicates.

Desired Task:	Container Type
Making a dictionary	map
Creating a spellchecker	set
Defining the possible outcomes of two dice roll	set
Queuing printing jobs from different computers.	list

2) Use induction to prove $10n < 2^n \ \forall \mathrm{n} : \mathrm{n} \geq n_0 \ where\ n_0 > 0\ is\ a\ constant.$

$Base\ case\ n_0 = 6.$

$60 < 64$

$assume\ 10k < 2^k$

$10(k+1) < 2^{k+1}$

$10k + 10 < 2 * 2^k$

$10k + 10 < 2^k + 2^k$ (another approach is to say $10\mathrm{k} + 10 < 2^k + 10$)

$10k < 2^k\ (from\ previous\ assumption)$

$10 < 2^k\ for\ k > 4\ (valid\ since\ n_0 = 6).$

$Therefore\ 10(k+1) < 2^{k+1}$

3) For any two functions f(n) and g(n), we say that f(n) ~ g(n) if and only if

$$\lim_{n\to\infty}\frac{f(n)}{g(n)} = c(n) \text{ and } \lim_{n\to\infty}\frac{c(n)}{n} = 0$$

a. Which of these functions are related? (specify all the related pairs)

$$n\,, \sqrt{n}, logn, 5n, n^2, e^n$$

We can group the functions in the following groups:

Group 1: log n, Group 2: $\sqrt{n}$, Group 3: n, 5n, Group 4: n^2, Group 5: e^n

Each function in a group is related to all functions belonging to groups with higher numbers. For example, log n ~ n

Further, all pairs of functions in Groups 1,2 and 3 are related

b. Is ~ an equivalence relation and why?

No, because symmetry is not always satisfied. For example, log n ~ n^2, but it is not true that n^2 ~ log n

4) Solve the following sub-problems:

a. Given a set {a, b, c, d, e, f, g, h}, how many ways can you choose 4 items

$$\binom{8}{4} = \frac{8!}{4!\,4!} = 70$$

b. $\lim_{n\to\infty}\frac{1+2+\cdots+n+(n+1)}{n(n+1)} = \lim_{n\to\infty}\frac{\sum_{i=1}^{i=n+1} i}{n(n+1)} = \lim_{n\to\infty}\frac{\sum_{i=1}^{i=n} i+n+1}{n(n+1)} = \lim_{n\to\infty}\frac{\frac{n(n+1)}{2}+n+1}{n(n+1)} = \frac{1}{2}$

$$c.\ \sum_{k=0}^{n^2}\left(\frac{1}{3}\right)^k = \frac{1-\left(\frac{1}{3}\right)^{n^2+1}}{1-\frac{1}{3}} (From \sum_{0}^{n} a^k \ formula)$$

5) For each function f(n) and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes f(n) microse conds.

f(n)	1 second	1 hour	1 day	1 year (365 days)
$\log_2 n$	2^{10^6}	$2^{3.6*10^9}$	$2^{8.64*10^{10}}$	$2^{3.11*10^{13}}$
$\sqrt{n}$	10^{12}	$1.29*10^{19}$	$7.46*10^{21}$	$9.67*10^{26}$
n	10^6	$3.6*10^9$	$8.64*10^{10}$	$3.11*10^{13}$
$\mathrm{n}\log_2 n$	$6.27*10^4$	$1.33*10^8$	$2.75*10^9$	$7.87*10^{11}$
n^2	$1.00*10^3$	$6*10^4$	$2.93*10^5$	$5.57*10^6$
2^n	19	31	36	44
$n!$	9	12	13	16

6) Rank the following functions in ascending order of growth. (1 = slowest, 9 = fastest)
   a. n^5
   b. $n!$
   c. $nlog_2(n)$
   d. $log_2(n!)$
   e. $(5log_2(n))^2$
   f. $\sqrt{n}$
   g. $2^{log_2 n}$
   h. $n^{1/log_2 n}$
   i. $n2^n$

1	2	3	4	5	6	7	8	9
h	e	f	g	d	c	a	i	b

1) $n^{\frac{1}{\lg n}} = 2$
2) $(5\log(n))^2 \rightarrow$ *Compare it with* $\sqrt{n}$ *using large numbers.*
3) $\sqrt{n}$
4) $2^{\log n} = n$
5) $\log n! \rightarrow \ \log n + \log(n-1) + \log(\mathrm{n}-2) \ldots + \log(1) < \mathrm{n} * \log n$
6) $nlogn$
7) n^5
8) $n\,2^n$
9) $n!$