ECE 301: Signals and Systems
Class Participation Problems #1

Due on September 11, 2015

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Problem 1

(a) Show that if a system is *either* additive or homogeneous, it has the property that if the input is identically zero , then the output is also identically zero.

(b) Determine a system (either in continuous or discrete time) that is *neither* additive *nor* homogeneous but which has a zero output if the input is identically zero.

(c) From Part(a), can you conclude that if the input to a linear system is zero between times t_1 and t_2 in continuous time or between times n_1 and n_2 in discrete time, then its output must also be zero between these same times? Explain your answer.

Solution

(a) Assume that we are discussing a continuous-time system $y(t) = x(t) * h(t)$.

If this continuous-time system is additive, then,

$$0 = x(t) - x(t) \to y(t) - y(t) = 0$$

Also, if the continuous-time system is homogeneous, then,

$$0 = 0 \cdot x(t) \to 0 \cdot y(t) = 0$$

Note that this property can also be proved based on discrete-time systems, the proof is trivial and will not be included in this answer.

(b) One typical example will be $y(t) = x^2(t)$.

This system has a zero output if the input is identically zero, because

$$x(t) = 0 \to y(t) = x^2(t) = 0.$$

This system is not additive, since

$$x(t) = x_1(t) + x_2(t) \to y(t) = (x_1(t) + x_2(t))^2 = x_1^2(t) + 2x_1(t)x_2(t) + x_2^2(t) \neq x_1^2(t) + x_2^2(t).$$

This system is also not homogeneous, since

$$x(t) = 5x_1(t) \to y(t) = (5x_1(t))^2 = 25x_1^2(t) \neq 5x_1^2(t)$$

(c) No. For example, consider

$$y(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

with $x(t) = u(t) - u(t-1)$. Then $x(t) = 0$ for $t > 1$, but $y(t) = 1$ for $t > 1$.

Problem 2

Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:

(a) $x[n] = h[n] = \alpha^n u[n]$

(b) $x[n]$ and $h[n]$ as in Figure 1

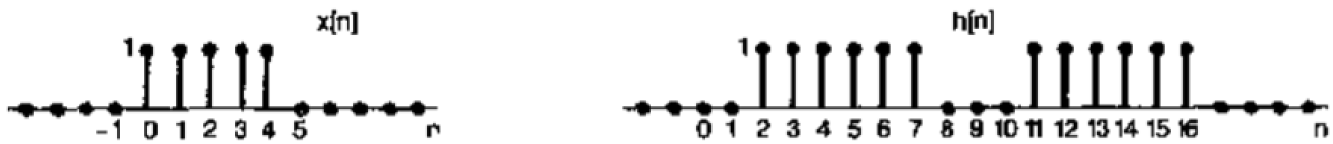

Figure 1: The discrete-time signal $x[n]$ and $h[n]$.

Solution

(a) $x[n] = h[n] = \alpha^n u[n]$. The desired convolution is

$$
\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= \alpha^n [\sum_{k=0}^{n} 1] u[n] \\
&= (n+1)\alpha^n u[n]
\end{aligned}
$$

(b) $x[n]$ and $h[n]$ as in Figure 1. The desired convolution is

$$
\begin{aligned}
y[n] &= x[n] * h[n] \\
&= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= x[0]h[n] + x[1]h[n-1] + x[2]h[n-2] + x[3]h[n-3] + x[4]h[n-4] \\
&= h[n] + h[n-1] + h[n-2] + h[n-3] + h[n-4]
\end{aligned}
$$

This is shown in Figure 2.

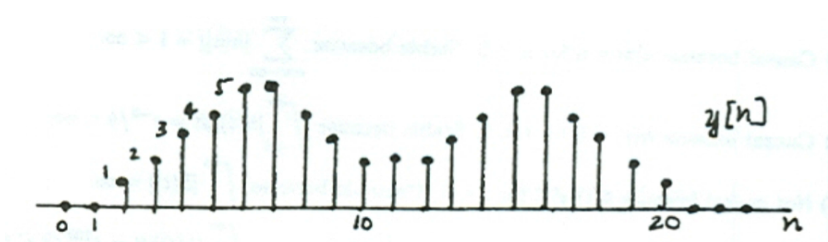

Figure 2: The resulting discrete-time signal $y[n]$.