ECE 301: Signals and Systems Class Participation Problems #2

Due on October 4, 2015

Professor: Aly El Gamal TA: Xianglun Mao

Problem 1

Problem 1

A continuous-time periodic signal x(t) is real valued and has a fundamental period T = 8. The nonzero Fourier series coefficients for x(t) are specified as

$$a_1 = a_{-1}^* = j, \ a_5 = a_{-5} = 2$$

Express x(t) in the form

$$x(t) = \sum_{k=0}^{\infty} A_k cos(w_k t + \phi_k)$$

Solution

A continuous-time periodic signal x(t) is real valued and has a fundamental period T = 8, therefore the fundamental frequency is $\omega_0 = \frac{\pi}{4}$. The nonzero Fourier series coefficients for x(t) are specified as

$$a_1 = a_{-1}^* = j, \ a_5 = a_{-5} = 2$$

Hence,

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t} \\ &= a_1 e^{j\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_5 e^{5j\omega_0 t} + a_{-5} e^{-5j\omega_0 t} \\ &= j e^{j(\pi/4)t} - j e^{-j(\pi/4)t} + 2 e^{(5\pi/4)t} + 2 e^{-(5\pi/4)t} \\ &= -2sin(\frac{\pi}{4}t) + 4cos(\frac{5\pi}{4}t) \\ &= -2cos(\frac{\pi}{4}t - \frac{\pi}{2}) + 4cos(\frac{5\pi}{4}t) \end{aligned}$$

Problem 2

Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1\\ 2-t, & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period T = 2 and Fourier coefficients a_k .

- (a) Determine the value of a_0 .
- (b) Determine the Fourier series coefficients of $\frac{dx(t)}{dt}$.

Solution

(a) We have

$$a_0 = \frac{1}{2} \int_0^1 t dt + \frac{1}{2} \int_1^2 (2-t) dt = \frac{1}{2}.$$

(b) The signal

$$g(t) = \frac{dx(t)}{dt}$$
$$= \begin{cases} 1, & 0 \le t \le 1\\ -1, & 1 \le t \le 2 \end{cases}$$

is as shown in Figure 1.



Figure 1: The signal $g(t) = \frac{dx(t)}{dt}$.

The FS coefficients b_k of g(t) may be found as follows:

$$b_0 = \frac{1}{2} \int_0^1 dt - \frac{1}{2} \int_1^2 dt = 0.$$

and

$$b_k = \frac{1}{2} \int_0^1 g(t) e^{-j(2\pi/T)kt} dt - \frac{1}{2} \int_1^2 g(t) e^{-j(2\pi/T)kt} dt$$
$$= \frac{1}{j\pi k} [1 - e^{-j\pi k}].$$