# ECE 301: Signals and Systems Class Participation Problems \#2 

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## Problem 1

A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T=8$. The nonzero Fourier series coefficients for $x(t)$ are specified as

$$
a_{1}=a_{-1}^{*}=j, a_{5}=a_{-5}=2
$$

Express $x(t)$ in the form

$$
x(t)=\sum_{k=0}^{\infty} A_{k} \cos \left(w_{k} t+\phi_{k}\right)
$$

## Solution

A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T=8$, therefore the fundamental frequency is $\omega_{0}=\frac{\pi}{4}$. The nonzero Fourier series coefficients for $x(t)$ are specified as

$$
a_{1}=a_{-1}^{*}=j, a_{5}=a_{-5}=2
$$

Hence,

$$
\begin{aligned}
x(t) & =\sum_{k=-\infty}^{\infty} a_{k} e^{j \omega_{0} t} \\
& =a_{1} e^{j \omega_{0} t}+a_{-1} e^{-j \omega_{0} t}+a_{5} e^{5 j \omega_{0} t}+a_{-5} e^{-5 j \omega_{0} t} \\
& =j e^{j(\pi / 4) t}-j e^{-j(\pi / 4) t}+2 e^{(5 \pi / 4) t}+2 e^{-(5 \pi / 4) t} \\
& =-2 \sin \left(\frac{\pi}{4} t\right)+4 \cos \left(\frac{5 \pi}{4} t\right) \\
& =-2 \cos \left(\frac{\pi}{4} t-\frac{\pi}{2}\right)+4 \cos \left(\frac{5 \pi}{4} t\right)
\end{aligned}
$$

## Problem 2

Let

$$
x(t)= \begin{cases}t, & 0 \leq t \leq 1 \\ 2-t, & 1 \leq t \leq 2\end{cases}
$$

be a periodic signal with fundamental period $T=2$ and Fourier coefficients $a_{k}$.
(a) Determine the value of $a_{0}$.
(b) Determine the Fourier series coefficients of $\frac{d x(t)}{d t}$.

## Solution

(a) We have

$$
a_{0}=\frac{1}{2} \int_{0}^{1} t d t+\frac{1}{2} \int_{1}^{2}(2-t) d t=\frac{1}{2}
$$

(b) The signal

$$
\begin{aligned}
g(t) & =\frac{d x(t)}{d t} \\
& = \begin{cases}1, & 0 \leq t \leq 1 \\
-1, & 1 \leq t \leq 2\end{cases}
\end{aligned}
$$

is as shown in Figure 1.


Figure 1: The signal $g(t)=\frac{d x(t)}{d t}$.
The FS coefficients $b_{k}$ of $g(t)$ may be found as follows:

$$
b_{0}=\frac{1}{2} \int_{0}^{1} d t-\frac{1}{2} \int_{1}^{2} d t=0
$$

and

$$
\begin{aligned}
b_{k} & =\frac{1}{2} \int_{0}^{1} g(t) e^{-j(2 \pi / T) k t} d t-\frac{1}{2} \int_{1}^{2} g(t) e^{-j(2 \pi / T) k t} d t \\
& =\frac{1}{j \pi k}\left[1-e^{-j \pi k}\right]
\end{aligned}
$$

