ECE 301: Signals and Systems
Class Participation Problems #3

Due on October 23, 2015

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Problem 1

(a) Compute the convolution of the following pair of signals $x(t)$ and $h(t)$ by calculating $X(jw)$ and $H(jw)$, using the convolution property, and inverse transforming.

$$x(t) = e^{-t}u(t), h(t) = e^{t}u(-t)$$

(b) Given the Fourier transform pair

$$e^{-|t|} \overset{FT}{\longleftrightarrow} \frac{2}{1+w^2}$$

Use the differentiation property and the duality property to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

Hint: You need to firstly determine the Fourier transform of $te^{-|t|}$.

Solution

(a) We have

$$\begin{aligned} Y(jw) &= X(jw)H(jw) \\ &= [\frac{1}{1+jw}][\frac{1}{1-jw}] \\ &= \frac{1/2}{1+jw} + \frac{1/2}{1+jw} \end{aligned}$$

Taking the inverse Fourier transform, we obtain

$$y(t) = \frac{1}{2}e^{-|t|}$$

(b) We know that

$$e^{-|t|} \overset{FT}{\longleftrightarrow} \frac{2}{1+w^2}$$

Using the differentiation in frequency property, we have

$$te^{-|t|} \overset{FT}{\longleftrightarrow} j\frac{d}{dw}\{\frac{2}{1+w^2}\} = -\frac{4jw}{(1+w^2)^2}$$

The duality property states that if

$$g(t) \overset{FT}{\longleftrightarrow} G(jw)$$

then

$$G(t) \overset{FT}{\longleftrightarrow} 2\pi g(jw)$$

Now, since

$$te^{-|t|} \overset{FT}{\longleftrightarrow} -\frac{4jw}{(1+w^2)^2}$$

we may then sue the duality to write

$$-\frac{4jt}{(1+t^2)^2} \overset{FT}{\longleftrightarrow} 2\pi we^{-|w|}$$

Multiplying both sides by j, we obtain

$$\frac{4t}{(1+t^2)^2} \overset{FT}{\longleftrightarrow} 2j\pi we^{-|w|}$$

Problem 2

Show that the three LTI systems with impulse responses

$$h_1(t) = u(t),$$
$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t),$$
$$h_3(t) = 2te^{-t}u(t)$$

all have the same response to $x(t) = cos(t)$.

Solution

We have

$$x(t) = cos(t) \stackrel{FT}{\longleftrightarrow} X(jw) = \pi[\delta(w+1) + \delta(w-1)]$$

Then for each LTI system,

(i) we have

$$h_1(t) = u(t) \stackrel{FT}{\longleftrightarrow} H_1(jw) = \frac{1}{jw} + \pi\delta(w)$$

Therefore,

$$Y(jw) = X(jw)H_1(jw) = \frac{\pi}{j}[\delta(w+1) + \delta(w-1)]$$

Taking the inverse Fourier transform, we obtain

$$y(t) = sin(t)$$

(ii) We have

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t) \stackrel{FT}{\longleftrightarrow} H_3(jw) = -2 + \frac{5}{2+jw}$$

Therefore,

$$Y(jw) = X(jw)H_2(jw) = \frac{\pi}{j}[\delta(w+1) + \delta(w-1)]$$

Taking the inverse Fourier transform, we obtain

$$y(t) = sin(t)$$

(iii) We have

$$h_3(t) = 2te^{-t}u(t) \stackrel{FT}{\longleftrightarrow} H_3(jw) = \frac{2}{(1+jw)^2}$$

Therefore,

$$Y(jw) = X(jw)H_3(jw) = \frac{\pi}{j}[\delta(w+1) + \delta(w-1)]$$

Taking the inverse Fourier transform, we obtain

$$y(t) = sin(t)$$