ECE 301: Signals and Systems
Class Participation Problems #4

Due on November 13, 2015

Professor: ***Aly El Gamal***
TA: ***Xianglun Mao***

Problem 1

Consider a system consisting of the cascade of two LTI systems with frequency responses

$$H_1(e^{jw}) = \frac{2 - e^{-jw}}{1 + \frac{1}{2}e^{-jw}}$$

and

$$H_2(e^{jw}) = \frac{1}{1 - \frac{1}{2}e^{-jw} + \frac{1}{4}e^{-j2w}}$$

(a) Find the difference equation describing the overall system.

(b) Determine the impulse response of the overall system.

Solution

(a) Since the two systems are cascaded, the frequency response of the overall system is

$$\begin{aligned} H(e^{jw}) &= H_1(e^{jw})H_2(e^{jw}) \\ &= \frac{2 - e^{-jw}}{1 + \frac{1}{8}e^{-j3w}} \end{aligned}$$

Therefore, the Fourier transforms of the input and output of the overall system are related by

$$\frac{Y(e^{jw})}{X(e^{jw})} = \frac{2 - e^{-jw}}{1 + \frac{1}{8}e^{-j3w}}.$$

Cross-multiplying and taking the inverse Fourier transform, we get

$$y[n] + \frac{1}{8}y[n-3] = 2x[n] - x[n-1].$$

(b) We may rewrite the overall frequency response as

$$H(e^{jw}) = \frac{3/4}{1 + \frac{1}{2}e^{jw}} + \frac{(1 + j\sqrt{3})/3}{1 - \frac{1}{2}e^{j120}e^{-jw}} + \frac{(1 - j\sqrt{3})/3}{1 - \frac{1}{2}e^{-j120}e^{-jw}}$$

Taking the inverse Fourier transform we get

$$h[n] = \frac{4}{3}(-\frac{1}{2})^n u[n] + \frac{1 + j\sqrt{3}}{3}(\frac{1}{2}e^{j120})^n u[n] + \frac{1 - j\sqrt{3}}{3}(\frac{1}{2}e^{-j120})^n u[n]$$

Problem 2

Consider the signal depicted in Figure 1. Let the Fourier transform of this signal be written in rectangular form as

$$X(e^{jw}) = A(w) + jB(w).$$

Sketch the function of time corresponding to the following Fourier transform (i.e., sketch the time domain signal)

$$Y(e^{jw}) = [B(w) + A(w)e^{jw}].$$

Hint: You probably need to divide $x[n]$ into two parts, *even* part and *odd* part.

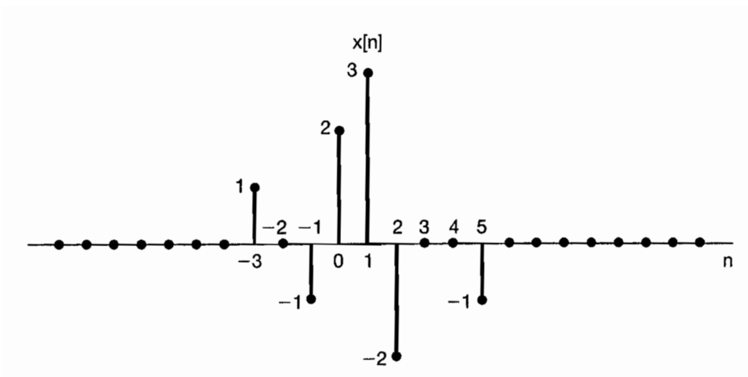

Figure 1: The signal of $x[n]$.

Solution

If the inverse Fourier transform of $X(e^{jw})$ is $x[n]$, then

$$x_e[n] = \mathrm{Ev}\{x[n]\} = \frac{x[n] + x[-n]}{2} \overset{FT}{\longleftrightarrow} A(w)$$

and

$$x_o[n] = \mathrm{Od}\{x[n]\} = \frac{x[n] + x[-n]}{2} \overset{FT}{\longleftrightarrow} jB(w)$$

Therefore, the inverse Fourier transform of $B(w)$ is $-jx_o[n]$. Also, the inverse Fourier transform $A(w)e^{jw}$ is $x_e[n+1]$. therefore, the time function corresponding to the inverse Fourier transform of $B(w) + A(w)e^{jw}$ will be $x_e[n+1] - jx_o[n]$. This is as shown in the Figure 2.

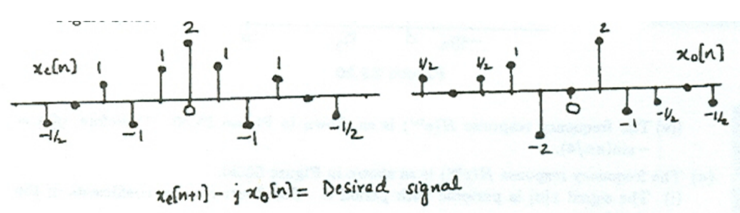

Figure 2: The signal of $x[n]$.