ECE 301: Signals and Systems
Class Participation Problems #5

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Problem 1

The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$$y(t) = x_1(t) * x_2(t)$$

where

$$X_1(jw) = 0 \text{ for } |w| > 1000\pi$$
$$X_2(jw) = 0 \text{ for } |w| > 2000\pi$$

Impulse-train sampling is performed on $y(t)$ to obtain

$$y_p(t) = \sum_{n=-\infty}^{\infty} y(nT)\delta(t - nT)$$

Specify the range of values for the sampling period T which ensures that $y(t)$ is recoverable from $y_p(t)$.

Solution

Using the properties of the Fourier transform, we obtain

$$Y(jw) = X_1(jw)X_2(jw)$$

Therefore, $Y(jw) = 0$ for $|w| > 1000\pi$. This implies that the Nyquist rate for $y(t)$ is $2 \times 1000 = 2000\pi$. Therefore, the sampling period T can at most be $\frac{2\pi}{2000\pi} = 10^{-3}$ sec. Therefore, we have to use $T < 10^{-3}$ sec in order to be able to recover $y(t)$ from $y_p(t)$.

Problem 2

A signal $x[n]$ with Fourier transform $X(e^{jw})$ has the property that

$$(x[n]\sum_{k=-\infty}^{\infty}\delta[n-3k]) * (\frac{sin(\frac{\pi}{3}n)}{\frac{\pi}{3}n}) = x[n].$$

For what values of w is it guaranteed that $X(e^{jw}) = 0$?

Solution

Let $y[n] = x[n]\sum_{k=-\infty}^{\infty}\delta[n-3k]$. Then

$$Y(e^{jw}) = \frac{1}{3}\sum_{k=0}^{3} X(e^{j(w-2\pi k/3)}).$$

Note that $\frac{sin(\frac{\pi n}{3})}{\frac{\pi n}{3}}$ is the impulse response of an ideal lowpass filter with cutoff frequency $\frac{\pi}{3}$ and passband gain of 3. Therefore, we now require that $y[n]$ when passed through this filter should yield $x[n]$. Therefore, the replicas of $X(e^{jw})$ contained in $Y(e^{jw})$ should not overlap with one another. This is possible only if $X(e^{jw}) = 0$ for $\frac{\pi}{3} \leq |w| \leq \pi$.