ECE 301 **Name:** _____
Fall 2015
Midterm I **PUID:** _____
10/5/2015
Professor: Aly El Gamal **TA: Xianglun Mao**

This exam contains 13 pages (including this cover page) and 6 questions.

Total of points is 110, (100 basic points + 10 bonus points).
Coverage: Chapters 1-3 in the textbook.
Closed Book and Closed Notes.
Calculators NOT allowed.
This test contains **Six** problems.
Show your work in the space provided for each problem.
You must show all work for each problem to receive full credit.
Always simplify your answers as much as possible.

Grade Table

Question	Points	Score
1	15	
2	15	
3	20	
4	25	
5	20	
6	15	
Total:	110	

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1. (15 points) Determine the values of P_∞ and E_∞ for each of the following signals:
   (a) (5 points) $x_1(t) = e^t$
   (b) (5 points)
   $$x_2(t) = \begin{cases} e^t, & 0 \leq t \leq 5000 \\ 0, & \text{otherwise} \end{cases}$$
   (c) (5 points) $x_3(t) = e^0$

1.(cont.):

2. (15 points) A continuous-time signal $x(t)$ is shown in Figure 1, where $A = 1$, $T = 2$. Please answer the following questions. Note that you shall only give the resulting figures as answers. Please mark the corresponding values in your answers.

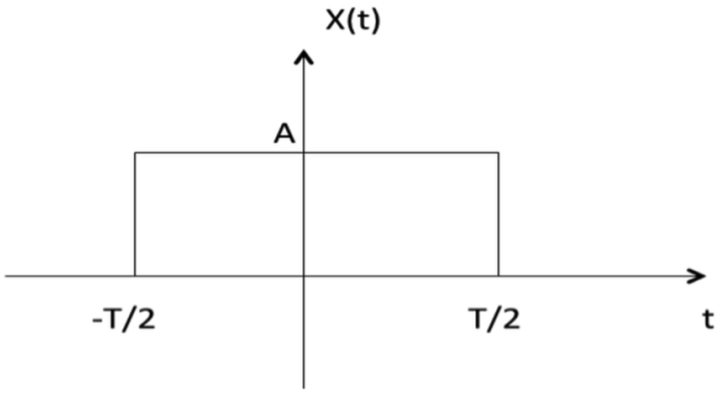

Figure 1: The continuous-time signal $x(t)$.

(a) (5 points) Calculate $x(t-1) + x(t+1)$.

(b) (5 points) Calculate $x(2t) - x(t/2)$.

(c) (5 points) Calculate $x(-t+1) - x(-t-1)$.

2.(cont.):

3. (20 points) Determine the fundamental period of the following continuous/discrete time signals.

   (a) (5 points) $x_1(t)$

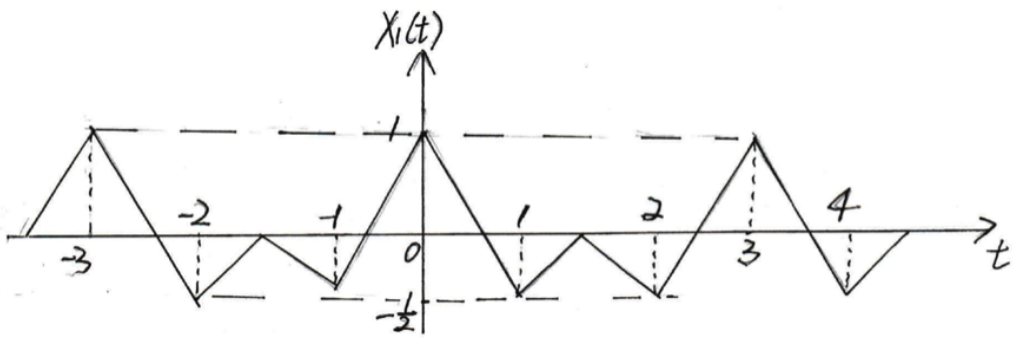

Figure 2: The continuous-time signal $x(t)$.

   (b) (5 points) $x_2(t) = cos(\frac{10\pi}{3}t) + sin(\frac{5\pi}{4}t)$

   (c) (5 points) $x_3[n] = e^{j(\frac{2\pi}{3})n} + e^{j(\frac{3\pi}{4})n}$

   (d) (5 points) $x_4(t) = 1$ for $t \in \mathbb{R}$.

3.(cont.):

4. (25 points) Let two continuous-time signal be $\delta_{\triangle}(t)$ and $r_{\triangle}(t)$, which are shown in Figure 3.

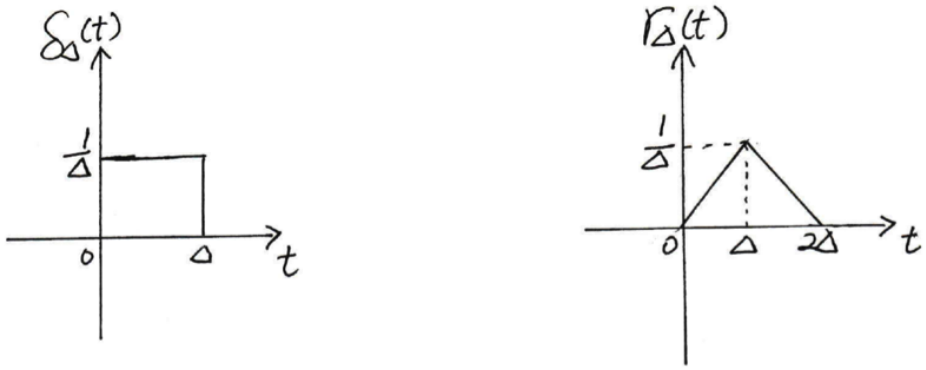

Figure 3: The continuous-time signal $\delta_{\triangle}(t)$ and $r_{\triangle}(t)$.

(a) (15 points) Calculate

$$\lim_{\triangle \to 0} S_{1,\triangle}(t)$$

, where $S_{1,\triangle}(t) = \delta_{\triangle}(t) * r_{\triangle}(t)$.

(b) (10 points) Calculate

$$\lim_{\triangle \to 0} S_{2,\triangle}(t)$$

, where $S_{2,\triangle}(t) = S_{1,\triangle}(t) * S_{1,\triangle}(t)$.

4.(cont.):

5. (20 points) Consider three continuous-time periodic signals whose Fourier series representations are as follows:

$$x_1(t) = \sum_{k=0}^{100} (\frac{1}{2})^k e^{jk\frac{2\pi}{50}t}$$

$$x_2(t) = \sum_{k=-100}^{100} cos(k\pi) e^{jk\frac{2\pi}{50}t}$$

$$x_3(t) = \sum_{k=-100}^{100} jsin(\frac{k\pi}{2}) e^{jk\frac{2\pi}{50}t}$$

Use Fourier series properties to help answer the following questions:

(a) (10 points) Which of the three signals is/are real valued? Explain the reasons.

(b) (10 points) Which of the three signals is/are even? Explain the reasons.

5.(cont.):

6. (15 points) Given a discrete-time LTI system $h[n]$, please answer the following questions.

   (a) (5 points) If this system is *causal*, then what is the condition that $h[n]$ should satisfy?

   (b) (10 points) (*Extra bonus question*) Please show that this condition is a necessary and sufficient condition of a causal LTI system.

   In other words, show that if this given discrete-time LTI system $h[n]$ is *causal*, then this condition should be satisfied. Meanwhile, show that if this condition is satisfied on this given LTI system $h[n]$, then this system is *causal*.

6.(cont.):