ECE 301 **Name:** ____________________

Fall 2015

Midterm II **PUID:** ____________________

11/16/2015

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This exam contains 13 pages (including this cover page) and 6 questions.

Total of points is 110, (100 basic points + 10 nonspecific bonus points).
Coverage: Chapters 4-7 in the textbook.
Closed Book but **TWO** (both sides) handwritten A4/Letter size crib sheets are allowed.
Calculators **NOT** allowed.
This test contains **Six** problems.
Show your work in the space provided for each problem.
You must show all work for each problem to receive full credit.
Always simplify your answers as much as possible.

Grade Table

Question	Points	Score
1	20	
2	20	
3	15	
4	20	
5	20	
6	15	
Total:	110	

1. (20 points) Prove the following statements, you must show all the steps of your proof to receive the full credit.

   (a) (5 points) Show that if

   $$x(t) \stackrel{FS}{\longleftrightarrow} a_k,$$

   then

   $$x^*(t) \stackrel{FS}{\longleftrightarrow} a^*_{-k}.$$

   Here, FS stands for Fourier Series, whereas * stands for the conjugate operator.

   (b) (10 points) Show that if

   $$x[n] = \sum_{k=<N>} a_k e^{jkw_0 n},$$

   then

   $$a_k = \frac{1}{N} \sum_{k=<N>} a_k e^{-jkw_0 n}.$$

   Note that $n =< N >$ means any consecutive N integer numbers.

   (c) (5 points) Show that if $x(t)$ is real and odd then $X(jw)$ is purely imaginary and odd.

1.(cont.):

2. (20 points) Compute the Fourier transform $X(jw)$ of the signal $x(t)$ that is shown in Figure 1.

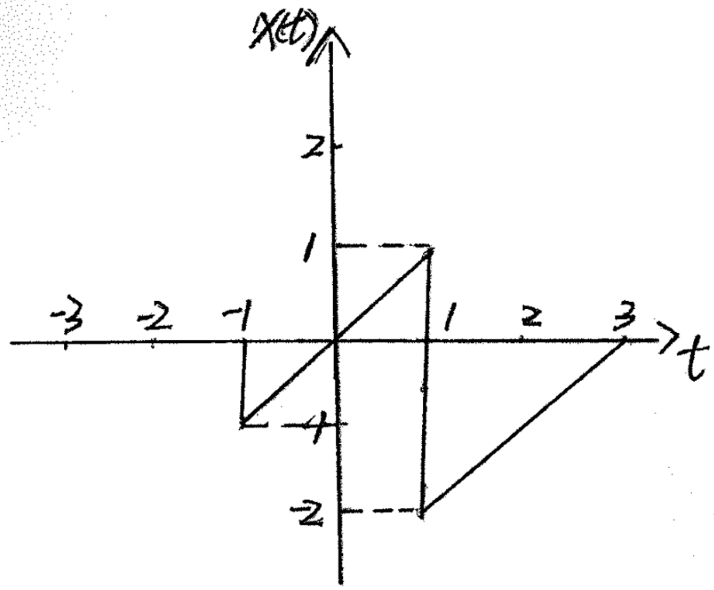

Figure 1: The continuous-time signal $x(t)$.

2.(cont.):

3. (15 points) Let $X(jw)$ be the Fourier transform of $x[n]$. Derive expressions in terms of $X(jw)$ for the Fourier transforms of the following signals. (Do not assume that $x[n]$ is real.)

   (a) (5 points) $\text{Re}\{x[n]\}$

   (b) (5 points) $x^*[-n]$

   (c) (5 points) $\text{Ev}\{x[n]\}$

3.(cont.):

4. (20 points) Sketch the Fourier transform $R(jw)$ where

$$r(t) = s(t) \cdot cos(w_0 t)$$

and $S(jw)$ is given in Figure 2.

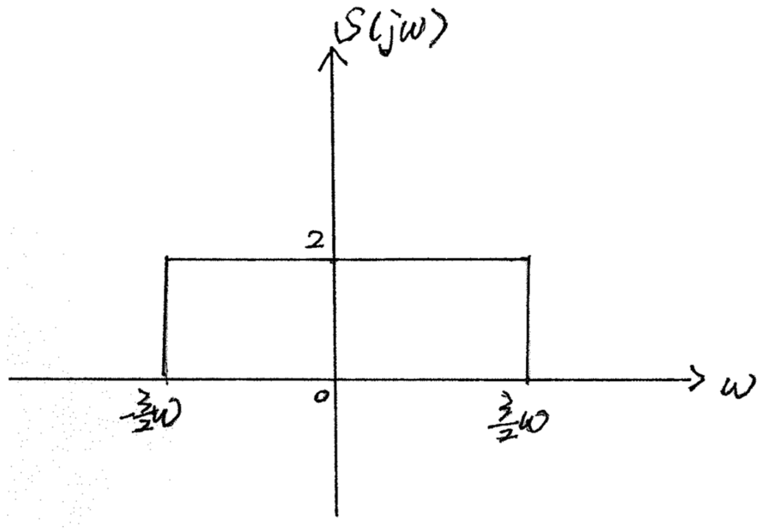

Figure 2: The Fourier transform of $S(jw)$.

4.(cont.):

5. (20 points) Use the Duality Property between the Discrete-Time Fourier transform and the Continuous-Time Fourier series to determine the Fourier transform of the following Discrete-Time signal $x[n]$.

$$x[n] = \frac{sin(\frac{7\pi n}{8})}{\pi n}.$$

5.(cont.):

6. (15 points) Use the sampling theory to solve the following problems.

   (a) (5 points) A real-valued signal $x(t)$ is known to be uniquely determined by its samples when the sampling frequency is $w_s = 10,000\pi$. For what values of w is $X(jw)$ guaranteed to be zero?

   (b) (10 points) A continuous-time signal $x(t)$ is obtained at the output of an ideal lowpass filter with cutoff frequency $w_c = 1,000\pi$. If impulse-train sampling is performed on $x(t)$, which of the following sampling periods would guarantee that $x(t)$ can be recovered from its sampled version using an appropriate lowpass filter?

      (i) $T = 0.5 \times 10^{-3}$

      (ii) $T = 2 \times 10^{-3}$

      (iii) $T = 10^{-4}$

6.(cont.):