# ECE 301: Signals and Systems
# Homework Assignment #2

Due on September 30, 2015

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# Problem 1

Show that causality for a continuous-time linear system is equivalent to the following statement:

For any time $t_0$ and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

# Problem 2

The initial rest assupmtion corresponds to a zero-valued auxiliary condition being imposed at a time determined in accordance with the input signal. In this problem we show that if the auxiliary condition used is nonzero or if it is always applied at a fixed time (regardless of the input signal) the corresponding system cannot be LTI. Consider a system whose input $x(t)$ and output $y(t)$ satisfy the first-order differential equation:

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \tag{1}$$

(a) Given the auxiliary condition $y(1) = 1$, use a counterexample to show that the system is not linear.

(b) Given the auxiliary condition $y(1) = 1$, use a counterexample to show that the system is not time invariant.

(c) Given the auxiliary condition $y(1) = 1$, show that the system is incrementally linear.

(d) Given the auxiliary condition $y(1) = 0$, show that the system is linear but not time invariant.

(e) Given the auxiliary condition $y(0) + y(4) = 0$, show that the system is linear but not time invariant.

# Problem 3

Let

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{elsewhere} \end{cases} \quad \text{and } h[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{elsewhere} \end{cases}$$

where $N \leq 9$ is an integer. Determine the value of $N$, given that $y[n] = x[n] * h[n]$ and

$$y[4] = 5, \ y[14] = 0$$

# Problem 4

One of the important properties of convolution, in both continuous and discrete time, is the associativity property. In this problem, we will check and illustrate this property.

(a) Prove the equality

$$[x(t) * h(t)] * g(t) = x(t) * [h(t) * g(t)] \tag{2}$$

by showing that both sides of (2) equal

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)h(\delta)g(t-\delta-\tau)d\tau d\delta$$

(b) Consider two LTI systems with the unit sample responses $h_1[n]$ and $h_2[n]$ shown in Figure 1(a). These two systems are cascaded as shown in Figure 1(b). Let $x[n] = u[n]$.

(i) Compute $y[n]$ by first computing $w[n] = x[n] * h_1[n]$ and then computing $y[n] = w[n] * h_2[n]$; that is, $y[n] = [x[n] * h_1[n]] * h_2[n]$

(ii) Now find $y[n]$ by first convolving $h_1[n]$ and $h_2[n]$ to obtain $g[n] = h_1[n]*h_2[n]$ and then convolving $x[n]$ with $g[n]$ to obtain $y[n] = x[n] * (h_1[n] * h_2[n])$.

The answers to (i) and (ii) should be identical, illustrating the associativity property of discrete-time convolution.

(c) Consider the cascade of two LTI system as in Figure 1(b), where in this case

$$h_1[n] = sin(8n)$$

and

$$h_2[n] = a^n u[n], |a| < 1$$

and where the input is

$$x[n] = \delta[n] - a\delta[n-1]$$

Determine the output $y[n]$. (*Hint*: The use of the associative and communicative properties of convolution should greatly facilitate the solution.)

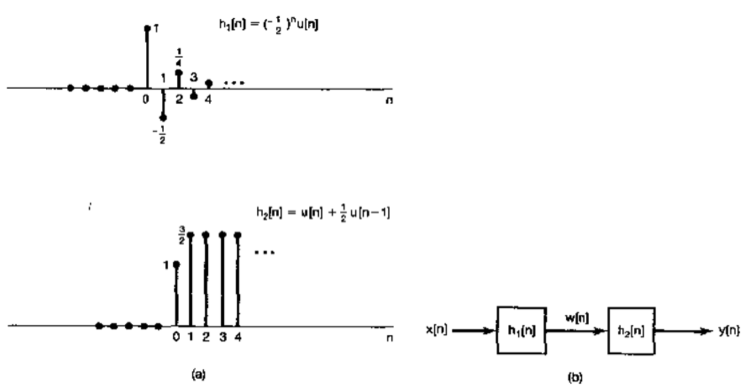


Figure 1: The discrete-time signal $h_1(t)$, $h_2(t)$ (a) and the cascaded system (b).

# Problem 5

In the text, we showed that if $h[n]$ is absolutely summable, i.e., if

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

then the LTI system with impulse response $h[n]$ is stable. This means that absolute summability is a *sufficient* condition for stability. In this problem, we shall show that it is also a *necessary* condition. Consider an LTI system with impulse response $h[n]$ that is not absolutely summable; that is,

$$\sum_{k=-\infty}^{\infty} |h[k]| = \infty$$

(a) Suppose that the input to this system is

$$x[n] = \begin{cases} 0, & \text{if } h[-n] = 9 \\ \frac{h[-n]}{|h[-n]|}, & \text{if } h[-n] \neq 9 \end{cases}$$

Does this input signal represent a bounded input? If so, what is the smallest number $B$ such that

$$|x[n]| \leq B \text{ for all } n?$$

(b) Calculate the output at $n = 0$ for this particular choice of input. Does the result prove the statement that absolute summability is a necessary condition for stability?

(c) In a similar fashion, show that a continuous-time LTI system is stable if any only if its impulse response is absolutely integrable.