ECE 301: Signals and Systems
Homework Assignment #3

Due on October 14, 2015

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Problem 1

Consider a causal LTI system S whose input $x[n]$ and output $y[n]$ are related by the difference equation

$$2y[n] - y[n-1] + y[n-3] = x[n] - 5x[n-4].$$

(a) Verify that S may be considered a cascade connection of two causal LTI systems S_1 and S_2 with the following input-output relationship:

$$S_1 : 2y_1[n] = x_1[n] - 5x_1[n-4],$$
$$S_2 : y_2[n] = \frac{1}{2}y_2[n-1] - \frac{1}{2}y_2[n-3] + x_2[n]$$

(b) Draw a block diagram representation of S_1.

(c) Draw a block diagram representation of S_2.

(d) Draw a block diagram representation of S as a cascade connection of the block diagram representation of S_1 followed by the block diagram representation of S_2.

(e) Draw a block diagram representation of S as a cascade connection of the block diagram representation of S_2 followed by the block diagram representation of S_1.

(f) Show that the four delay elements in the block diagram representation of S obtained in part (e) may be collapsed to three. The resulting block diagram is referred to as a *Direct Form II* realization of S, while the block diagrams obtained in parts (d) and (e) are referred to as *Direct Form I* realizations of S.

Problem 2

Determine the Fourier series representations for the following signals. You shall not only give the Fourier series coefficients, but also give the Fourier series expression of the signals.

(a) Each $x[n]$ or $x(t)$ illustrated in Figure 1.

(b) $x(t)$ periodic with period 2 and

$$x(t) = e^{-t} \text{ for } -1 \leq t \leq 1.$$

(c) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} sin(\pi t), & 0 \leq t \leq 2, \\ 0, & 2 < t \leq 4. \end{cases}$$

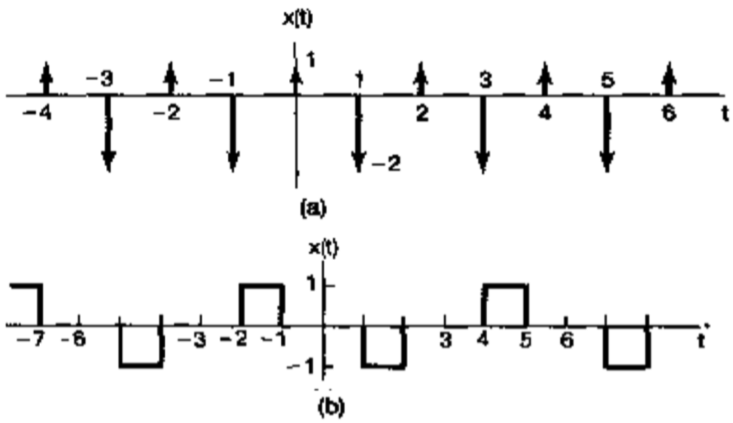

Figure 1: The signal of $x[n]$ or $x(t)$ in sub-question (a).

Problem 3

Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$x(t) = cos(4\pi t),$$
$$y(t) = sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

(a) Determine the Fourier series coefficients of $x(t)$.

(b) Determine the Fourier series coefficients of $y(t)$.

(c) Use the results of parts (a) and (b), along with the *multiplication* property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.

(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part(c).

Note that you shall only give the nonzero Fourier series coefficients as answers.

Problem 4

Let $x(t)$ be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0 \\ j(\frac{1}{2})^{|k|}, & \text{otherwise} \end{cases}$$

Known that the period of signal $x(t)$ is T, use the Fourier series properties to answer the following questions:

(a) Is $x(t)$ real?

(b) Is $x(t)$ even?

(c) Is $\frac{dx(t)}{dt}$ even?

Problem 5

Let

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 7 \\ 0, & 8 \leq n \leq 9 \end{cases}$$

be a periodic signal with fundamental period $N = 10$ and Fourier series coefficients a_k. Also, let

$$g[n] = x[n] - x[n-1].$$

(a) Draw the graph of signal $g[n]$, and determine the fundamental period of $g[n]$.

(b) Determine the Fourier series coefficients of $g[n]$.

(c) Using the Fourier series coefficients of $g[n]$ and the First-Difference property, determine a_k for $k \neq 0$.