ECE 301: Signals and Systems
Homework Assignment #4

Due on October 28, 2015

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Problem 1

Let $x[n]$ be a real periodic signal with period N and Fourier coefficients a_k.

(a) Show that if N is even, at least two of the Fourier coefficients within one period of a_k are real.

(b) Show that if N is odd, at least one of the Fourier coefficients within one period of a_k is real.

Problem 2

Consider the function

$$a[k] = \sum_{n=0}^{N-1} e^{j(2\pi/N)kn}$$

(a) Show that $a[k] = N$ for $k = 0, \pm N, \pm 2N, \pm 3N$, ...

(b) Show that $a[k] = 0$ whenever k is not an integer multiple of N. (*Hint*: Use the finite sum formula.)

(c) Repeat parts (a) and (b) if

$$a[k] = \sum_{n=<N>} e^{j(2\pi/N)kn}$$

where $n =< N >$ means any consecutive N integer numbers.

Problem 3

Let $x[n]$ be a periodic signal with fundamental period N and Fourier series coefficients a_k. In this problem, we derive the time-scaling property

$$x_{(m)}[n] = \begin{cases} x[\frac{n}{m}], & n = 0, \pm m, \pm 2m, ... \\ 0, & \text{elsewhere} \end{cases}$$

in the textbook.

(a) Show that $x_{(m)}[n]$ has period of mN.

(b) Show that if

$$x[n] = v[n] + w[n]$$

then

$$x_{(m)}[n] = v_{(m)}[n] + w_{(m)}[n]$$

(c) Assuming that $x[n] = e^{j2\pi k_0 n/N}$ for some integer k_0, verify that

$$x_{(m)}[n] = \frac{1}{m} \sum_{l=0}^{m-1} e^{j2\pi(k_0+lN)n/mN}$$

Noe that here you may use the results from the Problem 2.

(d) Using the results of parts (a), (b), (c), show that if $x[n]$ has the Fourier coefficients a_k, then $x_{(m)}[n]$ must have the Fourier coefficients $\frac{1}{m}a_k$.

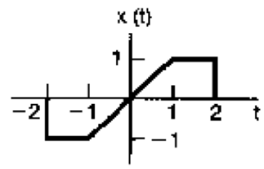
Problem 4

Compute the Fourier transform of each of the following signals

(a) $[e^{\alpha t} cos(w_0 t)]u(t)$, $a > 0$

(b) $e^{-3|t|} sin(2t)$

(c) $x(t)$ as show in Figure 1.



(a)

Figure 1: The graph of signal $x(t)$ in (c).

Problem 5

Consider the signal $x(t)$ in Figure 2.

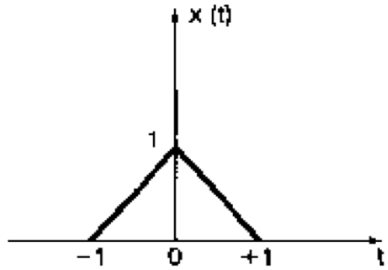


Figure 2: The graph of signal $x(t)$.

(a) Find the Fourier transform $X(jw)$ of $x(t)$.

(b) Sketch the signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

(c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\tilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

(d) Argue that, although $G(jw)$ is different from $X(jw)$, $G(j\frac{\pi k}{2}) = X(j\frac{\pi k}{2})$ for all integers k. You should not explicitly evaluate $G(jw)$ to answer the question.