ECE 301: Signals and Systems
Homework Assignment #5

Due on November 11, 2015

Professor: ***Aly El Gamal***
TA: ***Xianglun Mao***

Problem 1

Compute the Fourier transform of each of the following signals

(a)

$$x(t) = \begin{cases} 1 + cos(\pi t), & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$$

(b)

$$\sum_{k=0}^{\infty} a^k \delta(t - kT), |a| < 1$$

(c)

$$[te^{-2t} sin(4t)]u(t)$$

(d)

$$x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$$

(e) $x(t)$ as show in Figure 1.

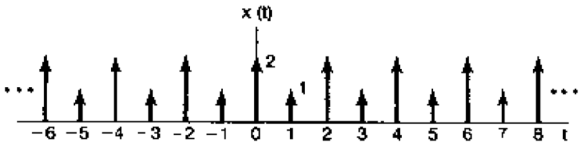

Figure 1: The graph of signal $x(t)$ in (e).

Problem 2

Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the Fourier transform of each of the signals shown in Figure 2. You should be able to do this by explicitly evaluating *only* the transform of $x_0(t)$ and then using properties of the Fourier transform.

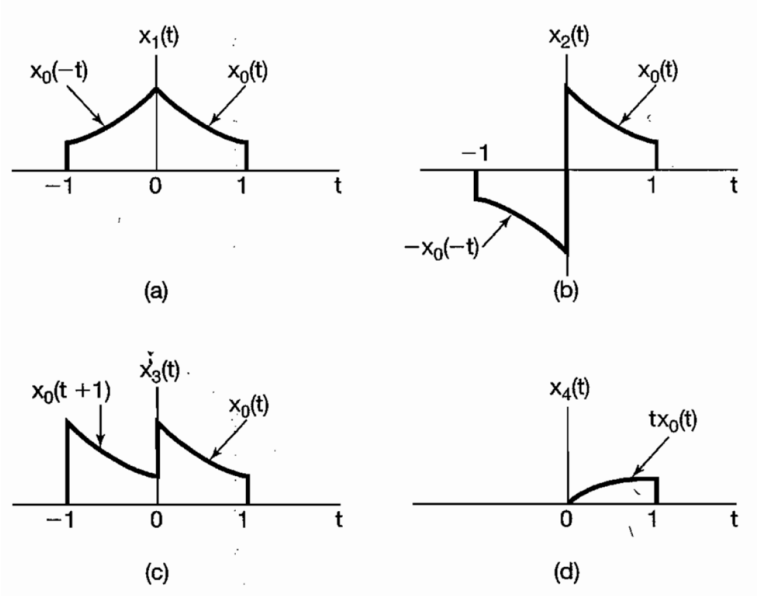

Figure 2: The graph of signals $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$.

Problem 3

Determine which, if any, of the real signals depicted in Figure 3 have Fourier tranforms that satisfy each of the following conditions:

(a) $\mathfrak{Re}\{X(jw)\} = 0$

(b) $\mathfrak{Im}\{X(jw)\} = 0$

(c) There exists a real α such that $e^{j\alpha w}X(jw)$ is real

(d) $\int_{-\infty}^{\infty} X(jw)dw = 0$

(e) $\int_{-\infty}^{\infty} wX(jw)dw = 0$

(f) $X(jw)$ is periodic

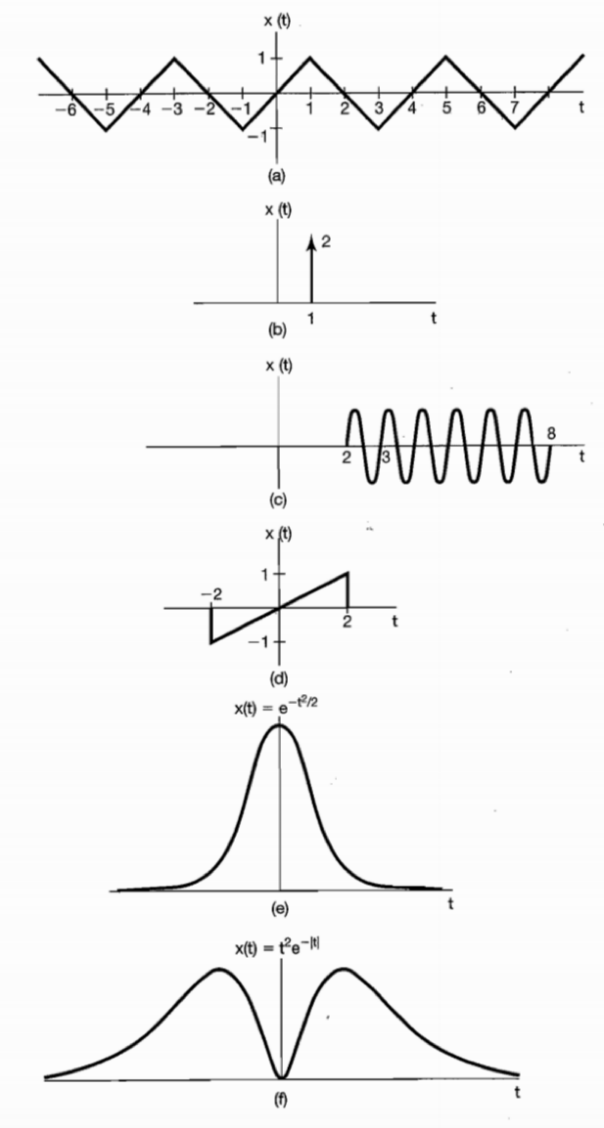

Figure 3: The graph of real signals (a), (b), (c), (d), (e), (f).

Problem 4

The input and the output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

(a) Find the impulse response of this system.

(b) What is the response of this system if $x(t) = te^{-2t}u(t)$?

(c) Repeat part (a) for the stable and causal LTI system described by the equation

$$\frac{d^2y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2x(t)}{dt^2} - 2x(t)$$

Problem 5

A causal and stable LTI system S has the frequency response

$$H(jw) = \frac{jw + 4}{6 - w^2 + 5jw}$$

(a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S.

(b) Determine the impulse response $h(t)$ of S.

(c) What is the output of S when the input is

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)?$$