ECE 301: Signals and Systems
Homework Assignment #6

Due on November 30, 2015

Professor: ***Aly El Gamal***
TA: ***Xianglun Mao***

Note: Homework 6 will have only 4 problems since there will be a midterm exam and then Thanksgiving break, each problem is assigned 12.5 points.

Problem 1

Let $x_1[n]$ be the discrete-time signal whose Fourier transform $X_1(e^{jw})$ is depicted in Figure 1(a).

(a) Consider the signal $x_2[n]$ with Fourier transform $X_2(e^{jw})$, as illustrated in Figure 1(b). Express $x_2[n]$ in terms of $x_1[n]$. (*Hint*: First express $X_2(e^{jw})$ in terms of $X_1(e^{jw})$, and then use properties of the Fourier transform.)

(b) Repeat part (a) for $x_3[n]$ with Fourier transform $X_3(e^{jw})$, as shown in Figure 1(c).

(c) Let

$$\alpha = \frac{\sum_{n=-\infty}^{\infty} n x_1[n]}{\sum_{n=-\infty}^{\infty} x_1[n]}$$

This quantity, which is the center of gravity of the signal $x_1[n]$, is usually referred to as the *delay time* of $x_1[n]$. Find α. (You can do this without first determining $x_1[n]$ explicitly.)

(d) Consider the signal $x_4[n] = x_1[n] * h[n]$, where

$$h[n] = \frac{sin(\pi n/6)}{\pi n}$$

Sketch $X_4(e^{jw})$.

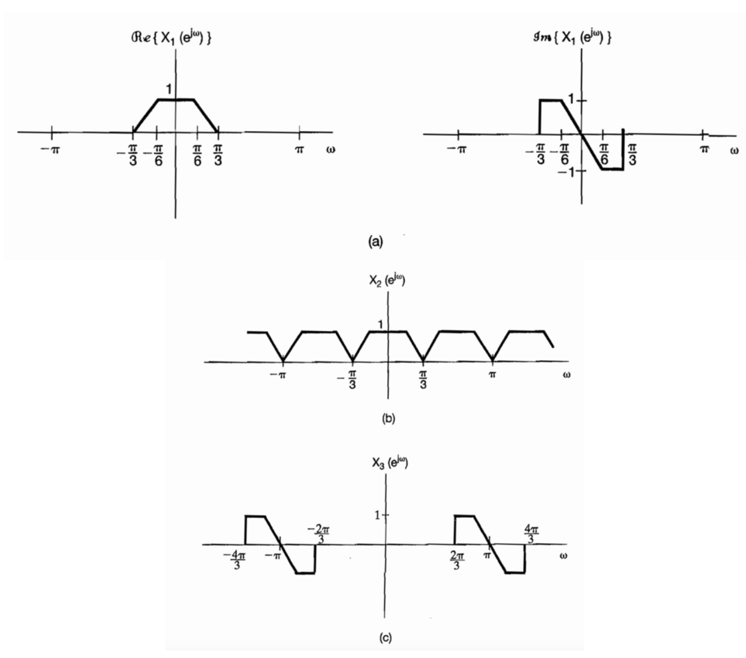

Figure 1: The graph of Fourier transforms of signals $X_1(e^{jw})$, $X_2(e^{jw})$, $X_3(e^{jw})$.

Problem 2

The signals $x[n]$ and $g[n]$ are known to have Fourier transforms $X(e^{jw})$ and $G(e^{jw})$, respectively. Furthermore, $X(e^{jw})$ and $G(e^{jw})$ are related as follows:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})G(e^{j(w-\theta)})d\theta = 1 + e^{-jw}$$

(a) If $x[n] = (-1)^n$, determine a sequence $g[n]$ such that its Fourier transform $G(e^{jw})$ satisfies the above equation. Are there other possible solutions for $g[n]$?

(b) Repeat the previous part for $x[n] = (\frac{1}{2})^n u[n]$.

Problem 3

In lecture, we indicated that the continuous-time LTI system with impulse response

$$h(t) = \frac{W}{\pi} sinc(\frac{Wt}{\pi}) = \frac{sin(Wt)}{\pi t}$$

plays a very important role in LTI system analysis. The same is true of the discrete-time LTI system with impulse response

$$h[n] = \frac{W}{\pi} sinc(\frac{Wn}{\pi}) = \frac{sin(Wn)}{\pi n}$$

(a) Determine and sketch the frequency response for the system with impulse response $h[n]$.

(b) Consider the signal

$$x[n] = sin(\frac{\pi n}{8}) - 2cos(\frac{\pi n}{4}).$$

Suppose that this signal is the input to LTI systems with the following impulse responses. Determine and sketch the frequency response of the output in each case.

(i) $h[n] = \frac{sin(\pi n/6)}{\pi n}$

(ii) $h[n] = \frac{sin(\pi n/6)}{\pi n} + \frac{sin(\pi n/2)}{\pi n}$

(iii) $h[n] = \frac{sin(\pi n/6) sin(\pi n/3)}{\pi^2 n^2}$

(iv) $h[n] = \frac{sin(\pi n/6) sin(\pi n/3)}{\pi n}$

(c) Consider an LTI system with unit sample response

$$h[n] = \frac{sin(\pi n/3)}{\pi n}.$$

Determine and sketch the frequency response of the output in each case.

(i) $x[n] =$ the square wave depicted in Figure 2.

(ii) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 8k]$

(iii) $x[n] = (-1)^n$ times the square wave depicted in Figure 2.

(iv) $x[n] = \delta[n + 1] + \delta[n - 1]$

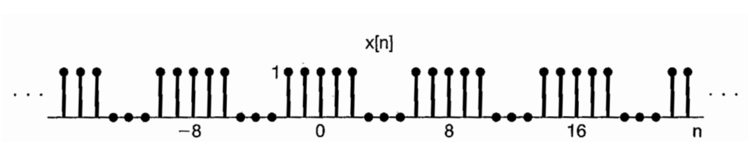

Figure 2: The square wave that construct $x[n]$.

Problem 4

An LTI system X with impulse response $h[n]$ and frequency response $H(e^{jw})$ is known to have the property that, when $-\pi \leq w_0 \leq \pi$,

$$cos(w_0 n) \rightarrow w_0 cos(w_0 n)$$

(a) Determine $H(e^{jw})$.

(b) Determine $h[n]$.