ECE 301: Signals and Systems
Homework Assignment #7

Due on December 11, 2015

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Note: Homework 7 will have only 4 problems since this is the last homework. The first 2 problems are assigned 10 pts, and the last 2 problems are assigned 15 pts. Enjoy the last homework!

Problem 1

A signal $x(t)$ with Fourier transform $X(jw)$ undergoes impulse-train sampling to generate

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $T = 10^{-4}$. For each of the following sets of constraints on $x(t)$ and/or $X(jw)$, does the sampling theorem guarantee that $x(t)$ can be recovered exactly from $x_p(t)$?

(a) $X(jw) = 0$ for $|w| > 5000\pi$

(b) $X(jw) = 0$ for $|w| > 15000\pi$

(c) $\text{Re}\{X(jw)\} = 0$ for $|w| > 5000\pi$

(d) $x(t)$ real and $X(jw) = 0$ for $w > 5000\pi$

(e) $x(t)$ real and $X(jw) = 0$ for $w < -15000\pi$

(f) $X(jw) * X(jw) = 0$ for $|w| > 15000\pi$

(g) $|X(jw)| = 0$ for $w > 5000\pi$

Problem 2

A signal $x[n]$ has a Fourier transform $X(e^{jw})$ that is zero for $\frac{\pi}{4} \leq |w| \leq \pi$. Another signal

$$g[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-1-4k]$$

is generated. Specify the frequency response $H(e^{jw})$ of a lowpass filter that produces $x[n]$ as output when $g[n]$ is the input.

Problem 3

Let $x(t)$ be a band-limited signal such that $X(jw) = 0$ for $|w| \geq \frac{\pi}{T}$.

(a) If $x(t)$ is sampled using a sampling period T, determine an interpolating function $g(t)$ such that

$$\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} x(nT)g(t - nT).$$

(b) Is the function $g(t)$ unique?

Problem 4

In this problem we develop the dual to the time-domain sampling theorem, whereby a time-limited signal can be reconstructed from *frequency-domain* samples. To develop this result, consider the frequency-domain sampling operation in Figure 1.

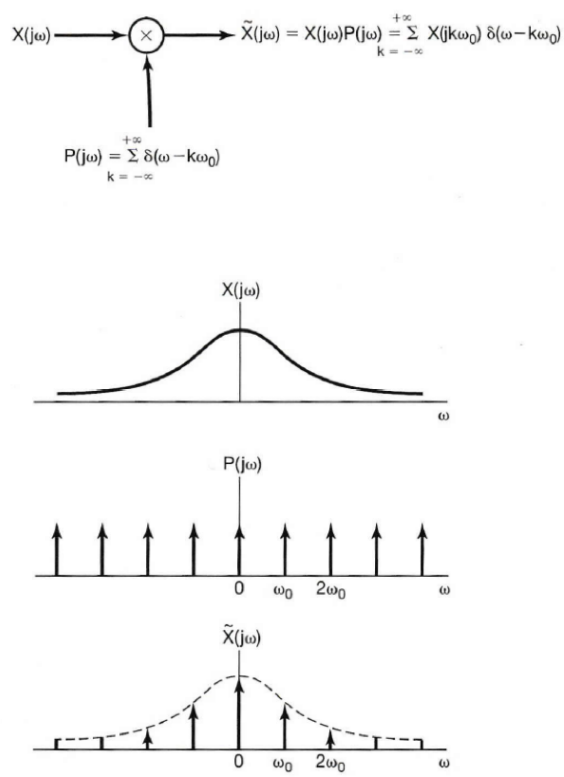

Figure 1: The frequency-domain sampling operation.

(a) Show that

$$\tilde{x}(t) = x(t) * p(t)$$

where $\tilde{x}(t)$, $x(t)$, and $p(t)$ are the inverse Fourier transforms of $\tilde{X}(jw)$, $X(jw)$, and $P(jw)$, respectively.

(b) Assuming that $x(t)$ is time-limited so that $x(t) = 0$ for $|t| \geq \frac{\pi}{w_0}$, show that $x(t)$ can be obtained from $\tilde{x}(t)$ through a "low-time windowing" operation. That is,

$$x(t) = \tilde{x}(t)w(t)$$

where

$$w(t) = \begin{cases} w_0, & |t| \leq \frac{\pi}{w_0} \\ 0, & |t| > -\frac{\pi}{w_0} \end{cases}$$

(c) Show that $x(t)$ is not recoverable from $\tilde{x}(t)$ if $x(t)$ is not constrained to be zero for $|t| \geq \frac{\pi}{w_0}$.