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STRATEGIC DISCLOSURE OF INTERMEDIATE RESEARCH RESULTS

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Strategic Disclosure of Intermediate Research Results

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Abstract

We analyze the incentives to disclose intermediate research results. We find that despite the help that disclosure can give to a rival, the leading innovator sometimes chooses to disclose. Disclosure signals commitment to the research project, which may induce a rival to exit. With weak product market competition, the leader discloses intermediate results that are sufficiently promising, while secrecy may be employed for very good results. As spillovers from disclosure increase, the leader becomes more secretive. With strong product market competition, the leader may rely entirely on secrecy but perhaps surprisingly invests more often at the intermediate stage.

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1 Introduction

This paper analyzes the incentives to disclose intermediate research results in a two-stage research and development (R&D) competition. An innovator has reached an intermediate stage in the research first and has to decide whether to ditch the research project or invest in a second stage. The main novelty of the model is that the leader can also choose to disclose its intermediate results to the follower, who has yet to reach the intermediate stage. Disclosure has two effects. First, the disclosure may help the follower, depending on the level of knowledge spillovers and on whether the follower's intermediate results turn out to be more or less promising than the leader's - the *information transmission* effect. Second, disclosure signals to the follower that the leader is committed to the research project, which may induce the rival to exit the competition - the *signalling* effect. We analyze this trade-off, determining the conditions under which the leader will use disclosure to try to induce the rival to quit via *strategic deterrence*.

Considerable analytical complexity is introduced by the fact that we need to take into account the inferences the follower draws about the leader's likelihood of investment if the leader fails to disclose. The leader might have failed to disclose because it has thrown in the towel and discontinued the project or because it is investing but wants to rely on secrecy. We solve for Perfect Bayesian Equilibria of the game.

The results depend upon the three main parameters of the model: the strength of product market competition, the development cost at the second stage, and the level of knowledge spillovers if the leader discloses. An alternative interpretation of the strength of competition is the likelihood of being granted a patent if both firms succeed in innovating. There are four cases.

Case (a). Where development costs are low and product market competition is not too strong, the incentives to invest in the second stage of research are high. Disclosure can't put a rival off, but may give it succour, so the leader keeps its intermediate results secret. Both the leader and follower invest in the second stage, irrespective of their first stage results.

Case (b). Where competition is weak and development costs are in an intermediate range, the leader's disclosure strategy will exhibit non-monotonicity unless spillovers are low: if its intermediate results are poor, the leader will not want to continue with the project, so neither will it bother to disclose; where intermediate results are sufficiently promising but not too good, the leader will invest and also disclose its intermediate results in the hope that the disclosure induces its rival to exit the competition; if the results are very good the leader invests but relies on secrecy, unless spillovers are very low in which case the leader also discloses. As spillovers increase, eventually the leader will start to rely on secrecy for strong results, with the range of results for which it uses secrecy expanding in the level of the spillovers. Secrecy is used for very good results because disclosure gives too much help to the rival.

Case (c). Where competition is strong and costs are in an intermediate range, we find that in equilibrium the leader always invests but never uses disclosure for any level of spillovers. Despite the strong product market competition, the leader invests even if its intermediate results are poor. The reason is that strong competition makes the follower very cautious if it thinks that the leader is investing, so the leader always wants to invest (disclosing if that is required to make the follower believe that it is investing and disclosure is not too costly). Seeing no disclosure, the follower must therefore infer the leader is investing. Thus, in equilibrium disclosure cannot make the follower believe the leader is more likely to have invested, destroying any incentive to disclose in our model. The follower's inferences from no disclosure are sufficiently pessimistic that the leader always invests but does not need to disclose, as the disclosure makes no difference to the follower's beliefs but might instead help the rival to catch up. The leader is able to use its first-mover advantage to force many of the follower types out of the market.

Case (d). Where competition is not too weak and costs are high, the leader abandons the project unless intermediate results are good enough. There is an equilibrium in which it always discloses where it invests, irrespective of the level of spillovers. Given the high costs, disclosure always puts the rival off. If competition is sufficiently strong, there is a second equilibrium in which the leader invests as per the first one, but does not disclose. In this second equilibrium, the follower's inferences from no disclosure are very pessimistic (similarly to the equilibrium in case (c)), so the leader can put off the rival by not disclosing.

Section 4.5 provides a diagrammatic representation of the different cases, explaining how the leader tends to invest more often as product market competition rises.

1.1 Strategic Disclosure in the Real World

Disclosure of intermediate research results is a common strategy, and can take a number of forms. Firms may preannounce their products, perhaps by describing in some detail the expected final features of the new product or demonstrating a prototype at a trade show. Alternatively, they can publish intermediate research findings in a commercial disclosure service, the best-known of which is Research Disclosure (RD)[2], or in an in-house research journal.[3] Finally, they may simply issue press releases or make other public statements.

Preannouncements with long lags to launch are common in the software industry. Bayus et al. (2001) report that of 123 important new software products announced in the period 1985-95, 60% had a lag of at least six months between announcement and launch, while 22% had a lag of at least twelve months. The marketing and management literature has recognized that such preannouncements may be designed to force rivals to exit innovation races. According to Lilly and Walters (1997, p.8): "Netscape's new product preannouncement of electronic publishing software was seen by experts as a preemptive move directed toward Microsoft, a firm with tremendous potential for entering the electronic publishing software market." Robertson et al. (1995, p.1) note that: "One dominant motivation for new product announcement signals is *preemption...* Microsoft's preannouncements of software could be considered as preemptive, and competitors have claimed that Microsoft's new product announcements signals are unfair, because it sometimes has not developed the software at the time the preannouncement occurs."

A recent example of strategic disclosure could be Boeing's preannouncement of its

[2] According to RD, over 90% of the world's leading companies have published disclosures in their pages (see www.researchdisclosure.com).

[3] According to Baker and Mezzetti (2003)'s calculations, one in six patents issued to IBM from 1996 to 2001 cited IBM's own previous disclosures from its journal the *IBM Technical Disclosure Bulletin*.

new 200-300 seater 7E7 Dreamliner aircraft, due to enter into service in 2008. Having announced the concept in December 2002, a January 2003 press release stated that "Boeing has given its new super-efficient, mid-sized airplane a development designation – the Boeing 7E7... The designation *signals the company's commitment* [emphasis added] to develop a new airplane with major breakthroughs in efficiency, economics, environmental performance, exceptional comfort and convenience, e-enabled systems, and more."[4] As development progressed, a number of further technical details were released. For example, a June 2003 press release announced that "Boeing has decided that the 7E7 will be the first commercial jet ever to have a majority of the primary structure – including the wing and fuselage – made of advanced composite materials. Following months of intensive study and analysis, the company has selected a graphite combined with a toughened epoxy resin as the main composite. The wings will also include TiGr composites – a combination of titanium and graphite."[5]

1.2 Relation to the Literature

A number of papers have studied the competitive dynamics within a multi-stage patent race,[6] but to our knowledge only Lichtman et al. (2000) and Jansen (2004b) begin to analyze disclosure of intermediate results to impact on a rival's innovatory behaviour in the presence of knowledge spillovers.[7] Lichtman et al. develop a highly stylized model in which, for certain parameter values, there is an equilibrium in which disclosure of intermediate knowledge can force a rival to exit. Jansen looks at the incentive to release information about investment costs, which can be either high or low. Release by a low cost firm signals that it will be a tough competitor, but may aid the rival. Jansen finds that for small enough knowledge spillovers, there is an equilibrium in which low cost firms release, thus reducing the rival's investment intensity.

In many respects our model is quite different. Our formulation is more general in

4 www.boeing.com/news/releases/2003/photorelease/q1/pr_030129h1.html

5 www.boeing.com/news/releases/2003/q2/nr_030612g.html

6 For example, Grossman and Shapiro (1987), Judd (1985, 2003) and Harris and Vickers (1987) were amongst the first to study how the intensity of innovatory investment depended on the relative positions of the competing firms in multi-stage races.

7 In Gordon (2004), firms can disclose intermediate results without any knowledge spilling over to rivals. The disincentive to release arises from the spur the disclosure may give to the investment intensity of a rival who has already reached the intermediate stage.

the sense that the first-stage information (intermediate results) which can be released is drawn from a continuous distribution, in contrast to Jansen's two-point high or low cost distribution and to Lichtman et al. where firms either achieve a fixed level of success or fail at the first stage. This allows a much richer analysis, where for example the leader's disclosure decision often turns out to be non-monotonic in how promising its intermediate results are. We also allow the value of the final innovatory output to vary, as opposed to Jansen and Lichtman et al. where the firm either succeeds and gets an innovation of fixed value or fails completely. Finally, we solve for all the equilibria for all parameter values, while Jansen and Lichtman et al. only show that for certain parameter values, disclosure is *an* equilibrium.

There are a number of other reasons why firms might disclose interim knowledge or preannounce products. In Bhattacharya and Ritter (1983), disclosure can be valuable as it acts as a credible signal to financial markets of the firm's innovation prospects. Where product market spillovers allow the loser in a patent race to still gain some benefit, De Fraja (1993) and Jansen (2004a) find that firms may disclose their knowledge to encourage rivals to invest more. In Anton and Yao (2002), the firm producing the intermediate research cannot innovate directly but must instead sell it on to a final innovator. Some knowledge is disclosed, acting as a signal of the value of non-disclosed knowledge. A recent literature (e.g., Parchomovsky (2000), Baker and Mezzetti (2003), Bar (2003)) analyzes the incentives of firms to publish results during the course of an R&D race to establish new "prior art", and hence make it more difficult for a rival to patent any final innovation given the novelty requirement under US patent law. Finally, the literature on *vaporware* looks at the preannouncement of a new product with the intention of strategically missing the announced launch date. For example, in Bayus et al. (2001) an early launch date can deter a rival innovator by signalling low development costs in the presence of antitrust penalties for missing announced launch dates.

Anton and Yao (2003, 2004) look at the related issue of how much final innovatory output to patent where a rival may be able to use the patent to better compete with the innovator in the product market, but the amount disclosed in the patent can signal strength. Similarly to our result, they find that for very large innovations the innovator may rely mainly on secrecy.

There is a growing literature on the strategic effects of product preannouncements via final market demand. An innovator may preannounce a new product in order to force consumers to wait rather than to buy from an incumbent in the presence of network externalities (Farrell and Saloner, 1986) or switching costs (Gerlach, 2004). Our paper can be seen as extending the understanding of the role of such preannouncements to include the effect on a rival innovator.

Finally, our paper should be distinguished from the literature on disclosing sequential innovations which have marketable value in and of themselves. In Scotchmer and Green (1990), patenting the first innovation allows a firm to earn initial profits, but helps rivals in the race to develop further innovations.

1.3 Overview of the Paper

The next section sets out the model. Section 3 analyzes the equilibrium of the game when the follower holds fixed or naive inferences having observed no disclosure from the leader. Section 4 analyzes the set of Perfect Bayesian Equilibria when the follower holds fully rational inferences, looking at the four different cases depending on the parameter values in turn. Section 5 concludes.

2 The Model

Two firms are competing to produce an innovation.[8] The innovation process is two-step, and both firms have already paid a sunk cost $q \in (0, 1)$ to start the race. The first step, the *research phase*, produces an independently and identically distributed intermediate output for each firm $x_i \sim U(0, 1)$, which is of no intrinsic value beyond its role in helping to produce a final innovation. The value x_i is private information. It is common knowledge that firm A, the leader, has produced its intermediate output x_a (the value of x_a, though, is not known to B), while firm B, the follower, has not yet

[8] Although we frame the model in terms of competing firms, the interpretation is broader and encompasses any agents or entities engaged in research races. For example, academics racing to produce papers analyzing a similar topic would fall under the model's scope, where disclosure consists of the publication of a working paper.

done so (perhaps because A started the race sooner). Having observed x_a, A decides whether or not to disclose its intermediate research results at a small cost $\varepsilon > 0$.[9] At the same time it makes an irreversible decision whether or not to invest to enter into the *development phase*.[10] Firm B observes $A's$ disclosure decision, but not whether A invests to continue, and then makes its own irreversible investment decision once it has produced x_b.[11] Both firms face a common development cost $c \in (0,1)$. If A discloses, then B will be able to make use of A's results as follows to increase its intermediate output to

$$\widehat{x}_b = \max\{x_b, x_b + s(x_a - x_b)\} \tag{1}$$

where $s \in (0,1)$ measures the level of knowledge spillovers from the leader's disclosure.[12] Thus the follower can make use of the leader's disclosure if and only if its intermediate output is below that of the leader, and can only partially catch up as $s < 1$. If a firm continues, it produces a final innovation $v_i \sim U(x_i, 1)$, which is independently distributed of its rival's final output if the rival also continues.[13] If i continues but j does not, $\pi_i = v_i - c - q$. If both firms continue, $\pi_i = \alpha v_i - c - q$, where $\alpha \in (0,1)$ is a measure of the degree of competition in the final product market (a lower α means the market is more competitive, so if both firms innovate, the ensuing product market competition reduces profits by a greater proportion).[14] If a firm does not continue, it gets $\pi_i = -q$. Throughout, we assume that where a firm is indifferent between investing or not, it invests.

The above assumes that the innovatory tracks of the two firms are sufficiently different that each is able to market its final output without infringing any patent its rival

[9] For example, publishing in RD (see Section 1) costs $120 per page of A4.

[10] The assumption that firms make an irreversible decision at the intermediate stage as to whether or not to pay a fixed cost to continue the project follows Choi (1991).

[11] Because B cannot directly observe A's investment decision, the decisions to invest or not are effectively simultaneous. This formulation of the game implies that the leader's decision whether or not to invest to continue the project is not verifiable. Nonetheless, as we will see below, if the leader releases its intermediate results, this will act as a credible signal that the leader intends to invest.

[12] Note that (1) implies that $\widehat{x}_b \in (0,1)$.

[13] This formulation implies that the variance at the development phase is decreasing in the quality of the intermediate output. We believe this to be a natural assumption.

[14] It would perhaps be more realistic to assume that the profit loss from competition is a function of how close together the two innovations are (although the introduction of even a low quality competitor might be very costly if it reduces trust in the market as a whole or imposes significant congestion costs). The analysis in that case, however, quickly becomes intractable.

may have taken out, while disclosure by the leader does not preclude the follower from using its own intermediate results. In effect, we assume that the final outputs are not perfect substitutes, so $\alpha > 0$. We even permit weak substitutability where $\alpha > \frac{1}{2}$, so that with $v_i = v_j$ joint product market profits are higher when both firms innovate.[15] An equally valid interpretation of α is that if both firms succeed in innovating they enter into a legal battle for a single monopolizing patent, with α measuring each firm's symmetric probability of winning. Then $\alpha \in (0, \frac{1}{2}]$, with $\alpha < \frac{1}{2}$ corresponding to the situation where there is a chance that no patent is granted given the rival claims (the firms then facing perfect competition from entrants copying the technology). Throughout we will take the first interpretation, but all the results for $\alpha \in (0, \frac{1}{2}]$ are equally valid under the second interpretation.

Since the seminal work of authors such as Loury (1979), Dasgupta and Stiglitz (1980), Lee and Wilde (1980) and Reinganum (1981, 1982), papers on R&D rivalry have tended to model the competition as a *race* in which the first to reach a defined finishing line, the *winner*, gains a fixed prize. In our model, there is no concept of a winner or a first firm to finish. By investing to develop, a firm can guarantee an innovation. However, the firm cares about how good its innovation is, which depends stochastically on how good that firm's intermediate results are, and also values being the *only* innovator because of post-innovation product market competition.

The level of knowledge spillovers s will be determined by a number of factors. First, the closeness of the innovatory tracks will affect s via the *adaptability* of the research to the rival. Second, s will depend on the extent to which the leader can rely on patent or copyright protection on its disclosure.[16] Third, s will depend on how much detail

[15] As reported by Eswaran and Gallini (1996, p.722): "In the pharmaceutical industry, several imperfectly substitutable drugs are available in the product categories of corticosteroids, tranquilizers, antidiabetic drugs, and antibiotics."

[16] Grossman and Shapiro (1987, p.382) note that: "While current U.S. policy stipulates that patents are to be granted only for 'useful' innovations, there may be considerable scope for discretion on the part of the patent-granting authority concerning what discoveries satisfy this criterion. A patent may well be awarded at a stage where substantial additional development expenses will be necessary before the product can be made available to consumers."

Even if the intermediate results are not directly patented or protected under copyright law, the disclosed information may still incorporate some degree of protection. For example, under the US Patent Code an innovator is not entitled to patent an innovation previously published in printed form unless he himself was the publisher and the publication was within a year of the patent application. The Code also incorporates a general 'first-to-invent' principle. (See US Code Title 35, Section 102 at www.uspto.gov/web/offices/pac/mpep/consolidated_laws.pdf).

about the intermediate results needs to be released for disclosures to be *certifiable*, and hence believed by the rival. The leader will of course want to disclose as little detail as possible. The leader's disclosure of the level of its intermediate output is assumed to be the only certifiable disclosure it can make, so it either discloses nothing or its precise type.

3 Equilibrium with Fixed or Naive Inferences

We start by deriving two useful remarks. The first remark gives i's optimal behaviour given j is definitely investing. As costs or the level of post-innovation competition rise, the cut-off level $\overline{\overline{x}}$ above which i invests goes up, so i becomes less eager to invest.

Remark 1 *If j is investing, i should invest if and only if $x_i \geq \overline{\overline{x}}$. Where $2c > \alpha$, $\overline{\overline{x}} = \frac{2c}{\alpha} - 1 > 0$. Where $2c \leq \alpha$, $\overline{\overline{x}} = 0$, so i always invests. As $c \to \alpha$, $\overline{\overline{x}} \to 1$. Where $c \geq \alpha$, $\overline{\overline{x}} \geq 1$, so i never invests.*

Proof. See Appendix. ■

Next, we derive i's cut-off level $\overline{\overline{\overline{x}}}$ given j is not investing. As costs rise, the cut-off level above which i invests goes up, so i becomes less eager to invest.

Remark 2 *If j is not investing, i should invest if and only if $x_i \geq \overline{\overline{\overline{x}}}$. Where $c > \frac{1}{2}$, $\overline{\overline{\overline{x}}} = 2c - 1 \in (0, 1)$. Where $c \leq \frac{1}{2}$, $\overline{\overline{\overline{x}}} = 0$, so i always invests. As $c \to 1$, $\overline{\overline{\overline{x}}} \to 1$.*

Proof. See Appendix. ■

As a first step to solving for the full equilibrium of the game, in this section we consider the case where the follower B has fixed inferences if the leader A does not disclose its intermediate results. Suppose B infers a probability ρ that A will invest if A fails to release. The following remark tells us that B is less likely to invest for higher c and more likely to invest for higher α, as is natural. It also tells us that cut-off value $\overline{x}_b$ must always lie between $\overline{\overline{\overline{x}}}$, the cut-off value for a firm that is sure its rival will not invest, and $\overline{\overline{x}}$, the cut-off value for a firm that is sure its rival will invest, which again is natural.

Remark 3 *Given A does not release and given B's inference ρ about the probability that A invests, B should invest if and only if $x_b \geq \overline{x}_b$. Where $2c > 1 - \rho(1-\alpha)$, $\overline{x}_b = \frac{2c}{\{1-\rho(1-\alpha)\}} - 1 > 0$. Where $2c = 1 - \rho(1-\alpha)$, $\overline{x}_b = \frac{2c}{\{1-\rho(1-\alpha)\}} - 1 = 0$. Where $2c < 1 - \rho(1-\alpha)$, $\overline{x}_b = 0$. For $\rho = 0$, $\overline{x}_b = \overline{\overline{\overline{x}}}$, for $\rho = 1$, $\overline{x}_b = \overline{\overline{x}}$ and as ρ ranges from 0 to 1, $\overline{x}_b$ increases from $\overline{\overline{\overline{x}}}$ to $\overline{\overline{x}}$.*

Proof. See Appendix. ■

If the leader discloses its results, then we can immediately determine the follower's inferences, which we assume to be fully rational in that case. Given the cost of release ε, the leader would only ever release if it intended to invest, as otherwise the cost of release would be wasted.[17] Thus, following release the follower infers that the leader will in fact continue the project. The release acts as a credible signal that the leader will in fact invest, raising the follower's threshold for investing from $\overline{x}_b$ to $\overline{\overline{x}}$. Thus we get the following remark.

Remark 4 *Following release by A, B will invest iff $\widehat{x}_b \geq \overline{\overline{x}}$.*

Given $\overline{x}_b$, we can solve for A's cut-off value if it does not release. The following remark tells us that A is less likely to invest for higher c and more likely to invest for higher α, as is natural. It also tells us that the cut-off value must always lie between $\overline{\overline{\overline{x}}}$, the cut-off value for a firm that is sure its rival will not invest, and $\overline{\overline{x}}$, the cut-off value for a firm that is sure its rival will invest (which again is natural), and is decreasing in the value of $\overline{x}_b$, i.e., increasing in B's probability of investing.

Remark 5 *If A does not release, it should invest if and only if $x_a \geq \overline{x}_a$.*

Suppose that (i) $2c \leq \alpha$. Then $\overline{x}_a = \overline{\overline{x}} = \overline{\overline{\overline{x}}} = 0$.

Suppose that (ii) $2c \in (\alpha, 1]$, where $\frac{2c-\alpha}{1-\alpha} \in (0,1]$. Then where $\overline{x}_b \in [0, \frac{2c-\alpha}{1-\alpha})$, $\overline{x}_a = \frac{2c}{\{\alpha + \overline{x}_b(1-\alpha))\}} - 1 > 0$ and where $\overline{x}_b \geq \frac{2c-\alpha}{1-\alpha}, \overline{x}_a = \overline{\overline{\overline{x}}} = 0$.

Suppose that (iii) $c > \frac{1}{2}$. Then where $\overline{x}_b \in [0,1)$, $\overline{x}_a = \frac{2c}{\{\alpha + \overline{x}_b(1-\alpha))\}} - 1 > 0$ and where $\overline{x}_b \geq 1, \overline{x}_a = 2c - 1 = \overline{\overline{\overline{x}}} > 0$.

[17] Neither disclosing nor investing strictly dominates disclosing but not investing.

In all cases, for $\overline{x}_b \geq 1$, $\overline{x}_a = \overline{\overline{\overline{x}}}$, *for* $\overline{x}_b = 0$, $\overline{x}_a = \overline{\overline{x}}$, *and as* $\overline{x}_b$ *ranges from* 1 *to* 0, $\overline{x}_a$ *increases from* $\overline{\overline{\overline{x}}}$ *to* $\overline{\overline{x}}$.

Proof. See Appendix. ■

Next we look at the effect of disclosure on the follower's investment decision where $x_a < \overline{\overline{x}}$. We find that in that case, release can never encourage more investment from the follower but may deter the follower in some cases. Because $x_a < \overline{\overline{x}}$, the release can never raise $\widehat{x}_b$ above $\overline{\overline{x}}$ where $x_b < \overline{\overline{x}}$, so extra investment can never be encouraged. However, where $x_b \in [\overline{x}_b, \overline{\overline{x}}) \cap (0, 1)$ the release deters the follower's investment due to the credible signalling effect which raises the follower's investment threshold from $\overline{x}_b$ to $\overline{\overline{x}}$. This range is of strictly positive length if and only if $\overline{x}_b < \min(\overline{\overline{x}}, 1)$.

Remark 6 *Where* $x_a < \overline{\overline{x}}$, *if (i)* $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ *disclosure deters the follower's investment with strictly positive probability but never encourages it where it would not otherwise have occurred, and if (ii)* $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, *i.e.,* $\overline{x}_b = \overline{\overline{x}} \leq 1$ *or* $\overline{x}_b \in \left[1, \overline{\overline{x}}\right]$,[18] *disclosure makes no difference to the follower's investment decision.*

Proof. See Appendix. ■

We are now in a position to analyze the leader's release and investment decisions, looking at all the possible ranges for x_a in turn.

Where $x_a < \overline{\overline{\overline{x}}}$, the leader would not want to invest even if it could be sure that the release would deter its rival from investing. Thus, the leader will not release to avoid the ε cost of release and will not invest either, giving the following claim.

Claim 1 *Where* $x_a < \overline{\overline{\overline{x}}}$, *then the leader neither releases its intermediate results nor invests.*

The next claim looks at the case where $x_a \in [\overline{\overline{\overline{x}}}, \overline{x}_a)$, in which the leader would not invest if it did not choose to release as $x_a < \overline{x}_a$.

For $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, we found in Remark 6 that the signalling effect of release, which raises the follower's threshold for investing from $\overline{x}_b$ to $\overline{\overline{x}}$, deters the follower's investment

[18] As $\overline{x}_b \leq \overline{\overline{x}}$ always.

in the range $x_b \in [\overline{x}_b, \overline{\overline{x}})$. Because investment is not optimal absent release, for release to be worthwhile the range within which the follower's investment is deterred must be sufficiently large. We derive a threshold $\widetilde{x}$ such that for $x_a > \widetilde{x}$, the deterrence effect is big enough to make post-release investment strictly optimal, so the leader releases. $\widetilde{x}$ is decreasing in $\overline{\overline{x}}$: a higher $\overline{\overline{x}}$ means the release is more likely to deter the follower, so disclosure is worthwhile in a bigger range of x_a. The direct effects of c and α on $\widetilde{x}$ are as expected, but note that there are also indirect effects via $\overline{\overline{x}}$. We find that $\widetilde{x} < \overline{x}_a$, so there is a non-empty range where the leader releases and then invests, where it would not have invested absent release. As $\overline{\overline{x}} \to 1$, the release almost certainly deters the follower's investment, so release tends towards being worthwhile $\forall x_a > \overline{\overline{\overline{x}}}$, i.e., for all x_a where the investment is optimal if the rival does not invest.

For $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, we found in Remark 6 that release is not a deterrent to the follower, so the leader should not release, and nor should it invest as $x_a < \overline{x}_a$.

Claim 2 *Suppose* $x_a \in [\overline{\overline{\overline{x}}}, \overline{x}_a)$:

Where (i) $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ *the leader releases and invests if* $x_a > \widetilde{x}$, *and otherwise neither releases nor invests. For* $c \leq \alpha$ *(i.e.,* $\overline{\overline{x}} \leq 1$ *),* $\widetilde{x} = \frac{2c}{\alpha + \overline{\overline{x}}(1-\alpha)} - 1$ *and for* $c > \alpha$ *(i.e.,* $\overline{\overline{x}} > 1$*),* $\widetilde{x} = 2c - 1$.

Where (ii) $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, *which here is equivalent to* $\overline{x}_b = \overline{\overline{x}} < 1$,[19] *the leader neither releases nor invests.*

In case (i):

$\widetilde{x} < \overline{x}_a$ *and* $\widetilde{x} < 1$, *so the leader releases and invests in the non-empty range* $x_a \in (\widetilde{x}, \overline{x}_a) \cap (0, 1)$ *(where it would not have invested had it chosen not to release).*

$\frac{\partial \widetilde{x}}{\partial \overline{\overline{x}}} \leq 0$.

If $\max(c, \alpha) \leq \frac{1}{2}$, $\widetilde{x} \leq \overline{\overline{\overline{x}}} = 0$, *in which case the leader releases and invests in the whole range* $x_a \in (0, \overline{x}_a)$.

If $\max(c, \alpha) > \frac{1}{2}$, $\widetilde{x} > \overline{\overline{\overline{x}}}$ *for* $\overline{\overline{x}} < 1$ *and* $\widetilde{x} = \overline{\overline{\overline{x}}}$ *for* $\overline{\overline{x}} \geq 1$. *As* $\overline{\overline{x}} \to 1$ *from below,* $\widetilde{x} \to \overline{\overline{\overline{x}}}$ *from above, so the leader tends towards investing and releasing in the range* $x_a \in (\overline{\overline{\overline{x}}}, \overline{x}_a)$, *the release and investment range for* $\overline{\overline{x}} \geq 1$.

[19] Note that from Remark 5 $\overline{x}_b \geq 1 \Rightarrow \overline{x}_a = \overline{\overline{\overline{x}}}$ which cannot occur here. Also, $\overline{x}_b \leq \overline{\overline{x}}$. So $\overline{x}_b \geq \min(\overline{\overline{x}}, 1) \Rightarrow \overline{x}_b = \overline{\overline{x}} < 1$.

Proof. See Appendix. ■

Where $x_a \in [\overline{x}_a, \overline{\overline{x}})$, the leader wants to invest whether or not release is worthwhile as $x_a \geq \overline{x}_a$. The leader's release decision depends upon whether the release deters the rival's investment with positive probability or not, which question was analyzed in Remark 6.

Claim 3 *If* $x_a \in [\overline{x}_a, \overline{\overline{x}})$,[20] *where (i)* $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ *the leader releases and invests and where (ii)* $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$ *the leader does not release but still invests.*

Proof. See Appendix. ■

Where $x_a \in [\overline{\overline{x}}, 1)$ and $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, disclosure may encourage or deter investment by the follower, depending on the value of x_b. Releasing information is valuable because it sends a signal that you are serious about innovating, thus raising the follower's threshold for investing from $\overline{x}_b$ to $\overline{\overline{x}}$. However, the release is potentially harmful: because $x_a \geq \overline{\overline{x}}$, the information transmission effect may raise the value of the follower's intermediate output from below $\overline{x}_b$ to above $\overline{\overline{x}}$, thus encouraging extra investment. Clearly, the higher is x_a, the higher is this risk. In the following claim, we derive a threshold $\tilde{\tilde{x}}$ below which the leader will want to release, where the signalling effect outweighs the information transmission effect. As s goes down, the informational cost of release goes down as the follower is less able to make use of the information release to improve its intermediate output. Thus $\tilde{\tilde{x}}$ goes up, i.e., the range of x_a for which release is optimal expands. Furthermore, $\tilde{\tilde{x}}$ is decreasing in $\overline{x}_b$: if the follower is more cautious absent release, then the risk that the release tilts the follower towards investment as opposed to deterring it is increased, so the leader releases in a smaller range. Again, the leader wants to invest whether or not release is worthwhile as $x_a \geq \overline{x}_a$. Where $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, which is equivalent to $\overline{x}_b = \overline{\overline{x}}$ here, the release may encourage but can never deter investment by the follower (release cannot raise the follower's investment threshold as $\overline{x}_b = \overline{\overline{x}}$). Thus, the leader does not release but still invests as $x_a \geq \overline{x}_a$.

[20] Note that this encompasses the case where $x_a = \overline{\overline{\overline{x}}} = \overline{x}_a < \overline{\overline{x}}$.

Claim 4 *Suppose* $x_a \in [\overline{\overline{\overline{x}}}, 1)$:

If (i) $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, *which here is equivalent to* $\overline{x}_b < \overline{\overline{x}}$,[21] *then the leader releases if and only if*

$$x_a < \widetilde{\widetilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b\,(1-s)}{s}$$

Whether or not it releases, the leader always invests.

$\widetilde{\widetilde{x}} > \overline{\overline{x}}$ *so the leader must release for some non-empty range* $x_a \in [\overline{\overline{x}}, \widetilde{\widetilde{x}}) \cap [\overline{\overline{\overline{x}}}, 1)$.

$\frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b} < 0$.

$\frac{\partial \widetilde{\widetilde{x}}}{\partial s} < 0$. *As* $s \to 1$, $\widetilde{\widetilde{x}} \to \overline{\overline{x}}$, *i.e., the range for which A releases goes to zero. As* $s \to 0$, $\widetilde{\widetilde{x}} \to \infty$, *so A definitely releases.* $\widetilde{\widetilde{x}} \geq 1 \Leftrightarrow s \leq \widetilde{s}$ *where* $\widetilde{s} = \frac{\overline{\overline{x}} - \overline{x}_b}{1 - \overline{x}_b} \in (0, 1)$, *so the leader always releases if and only if* $s \leq \widetilde{s}$.

If (ii) $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, *which here is equivalent to* $\overline{x}_b = \overline{\overline{x}}$,[22] *then the leader does not release but still invests.*

Proof. See Appendix. ■

In summary, where (i) $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ the leader (a) neither releases nor invests for $x_a \leq \widetilde{x}$, (b) releases and invests for $x_a \in (\widetilde{x}, \widetilde{\widetilde{x}})$ and (c) invests but does not release for $x_a \geq \widetilde{\widetilde{x}}$, where $\widetilde{x} < \overline{x}_a$ and $\widetilde{\widetilde{x}} > \overline{\overline{x}}$.[23] If x_a is too low, even allowing for the deterrent effect of release, the leader will not want to release or invest. In an intermediate range of x_a, the deterrent effect of release makes release followed by investment worthwhile. For x_a too high, release may become too costly as the information passed on to the rival by release becomes too valuable, even though investment remains optimal. As $\widetilde{x} < \overline{x}_a$, there is a positive range in which the leader invests where it would not have done so absent release. $\widetilde{\widetilde{x}}$ is decreasing in s, so as the level of knowledge spillovers falls, the range within which the leader releases expands.

Where (ii) $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$ release is never worthwhile, and the leader thus invests if and only if $x_a \geq \overline{x}_a$. Because either $\overline{x}_b = \overline{\overline{x}}$ or $\overline{x}_b \geq 1$, release cannot raise the follower's investment threshold in a way relevant to its investment decision, so investment cannot

[21] As $\overline{\overline{x}} < 1$ here.

[22] As $\overline{\overline{x}} < 1$ here and $\overline{x}_b \leq \overline{\overline{x}}$ always.

[23] This of course assumes $\overline{x}_a > \overline{\overline{\overline{x}}}$ and $\overline{\overline{x}} < 1$. If $\overline{x}_a = \overline{\overline{\overline{x}}}$, $\widetilde{x}$ is undefined, but from Claims 1 and 3 the leader starts releasing and investing from $x_a = \overline{x}_a = \overline{\overline{\overline{x}}}$. If $\overline{\overline{x}} \geq 1$, $\widetilde{\widetilde{x}}$ is undefined, but Claim 3 tells us that the leader never stops releasing and investing as x_a goes from $\overline{x}_a$ to 1.

be deterred, but the informational content of the release may encourage extra investment by the follower.

4 Equilibrium with Sophisticated Inferences

In the previous section, we assumed that the follower had a fixed or naive inference ρ about the probability that the leader would invest if it did not disclose its intermediate results, leading to a fixed investment threshold for the follower $\overline{x}_b$. We then solved for the leader's optimal behaviour given this fixed inference and threshold. A sophisticated follower's beliefs will not however be fixed at an arbitrary ρ. The sophisticated follower will instead attempt to deduce what the leader's strategy is and react optimally to that. In a Perfect Bayesian Equilibrium (PBE), the follower infers a correct belief distribution over the leader's type and hence probability of investment given the leader's strategy and observed failure to release, while the leader's strategy is optimal given the beliefs and hence strategy of the follower. Note that in the previous section, we already allowed the follower to hold rational inferences following release, namely that the leader must be intending to invest. We continue here with such inferences, so following release from Remark 4 the follower invests if and only if $\widehat{x}_b \geq \overline{\overline{x}}$.[24] Also note that because the leader does not observe the outcome of any decision by the follower before deciding whether to release or not, the leader does not infer any beliefs about the follower's type beyond the initial distribution $x_b \sim U(0,1)$.

Given the leader's optimal strategy following from an arbitrary $\overline{x}_b$, the sophisticated follower will rationally infer a probability ρ^* that the leader will invest if it does not release, leading to an investment threshold $\overline{x}_b^*$.[25] We have a PBE if and only if $\overline{x}_b^* = \overline{x}_b$, where both firms' beliefs are correct and actions optimal given the other's strategy. In this section we find such PBEs, looking at different parameter bounds in turn.

[24] We rule out any possible PBEs in which the leader does not release, but the follower holds off the equilibrium path beliefs following release other than that the leader will invest. Otherwise, off the equilibrium path the follower would believe that the leader is playing a strictly dominated strategy, as the leader is better off not releasing and not investing than releasing and not investing. Release followed by investment, on the other hand, cannot be strictly dominated $\forall x_a$. For example, the leader will release and invest if $\frac{1+x_a}{2} > c$ and the follower desists from investing if and only if the leader releases.

[25] Of course, if given $\overline{x}_b$ the leader's optimal strategy is to always release, no release is off the equilibrium path. Thus, in the absence of release the follower's inferences about the leader's probability of investment are not limited, so we need to consider all possible ρ^* in $[0,1]$.

4.1 Case (a): $2c \leq \alpha$

In this case, from Remarks 1 and 2 $\overline{\overline{x}} = \overline{\overline{\overline{x}}} = 0$. Costs are so low that each firm wishes to invest whatever its rival is doing. Thus the leader will not wish to incur the cost of release as this will not affect the follower's investment decision, but will invest, and the follower will invest also.

Proposition 1 *In the unique PBE the leader does not release, but always invests, and the follower also always invests.*

4.2 Case (b): $c \in (\frac{\alpha}{2}, \alpha)$; $\alpha > \frac{1}{2}$

The second case we look at has $c \in (\frac{\alpha}{2}, \alpha)$ and $\alpha > \frac{1}{2}$. In this case, c is in an intermediate range between $\frac{\alpha}{2}$ and α. This case is consistent with c greater than, equal to, or less than $\frac{1}{2}$, as $\frac{\alpha}{2} < \frac{1}{2} < \alpha$. We start by deriving some useful restrictions which follow from these parameter bounds.

Remark 7 $\overline{\overline{x}} = \frac{2c-\alpha}{\alpha} \in (0, 1)$, *and* $\overline{\overline{x}} \geq \overline{x}_a > \overline{\overline{\overline{x}}} \geq 0$ *so the range* $[\overline{\overline{\overline{x}}}, \overline{x}_a)$ *is non-empty and* $\overline{\overline{x}} > \overline{\overline{\overline{x}}}$.

Proof. See Appendix. ■

We can now show that $\overline{x}_b = \overline{\overline{x}}$ can never form an equilibrium.

Remark 8 *There can be no PBE with* $\overline{x}_b = \overline{x}_b^* = \overline{\overline{x}}$*, so we can assume* $\overline{x}_b < \overline{\overline{x}}$.

Proof. See Appendix. ■

The assumption that $\overline{x}_b < \overline{\overline{x}}$ allows us to derive the following claim, which pins down the leader's optimal strategy for a given $\overline{x}_b$, showing how the leader's disclosure strategy may be non-monotonic in the quality of its intermediate results. $\overline{x}_b < \overline{\overline{x}}$ means that if the follower sees no disclosure, he infers there is a chance the leader is not investing, which forces the leader with middling results to disclose to prove he is investing.

Claim 5 *Given an arbitrary* $\overline{x}_b < \overline{\overline{x}}$, *the leader neither releases nor invests for* $x_a \leq \widetilde{x}$, *releases and invests for* $x_a \in (\widetilde{x}, \widetilde{\widetilde{x}})$ *and does not release but still invests for* $x_a \geq \widetilde{\widetilde{x}}$, *where* $\widetilde{x} = \frac{2c}{\alpha+\overline{x}(1-\alpha)} - 1 \in (\overline{\overline{\overline{x}}}, \overline{x}_a)$, $\widetilde{x} > 0$ *and* $\widetilde{\widetilde{x}} = \frac{\overline{\overline{x}}-\overline{x}_b(1-s)}{s} > \overline{\overline{x}} \geq \overline{x}_a$. $\frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b} < 0$ *and* $\frac{\partial \widetilde{\widetilde{x}}}{\partial s} < 0$.

Proof. See Appendix. ■

We are now in a position to analyze how changes in $\overline{x}_b$ impact on the sophisticated follower's inference ρ^* and investment threshold $\overline{x}_b^*$ when the leader chooses not to release. In the next remark we start by deriving ρ^* as a function of $\widetilde{x}, \widetilde{\widetilde{x}}$ and $\overline{x}_b$, and showing that ρ^* is increasing in $\overline{x}_b$. As $\overline{x}_b$ goes up, $\widetilde{\widetilde{x}}$ falls: for a given $x_a > \overline{\overline{x}}$, disclosure becomes more dangerous as $\overline{x}_b$ rises. The release is more likely to encourage extra investment (where it raises x_b from below the follower's pre-release investment threshold $\overline{x}_b$ to above the post-release investment threshold $\overline{\overline{x}}$) and less likely to to deter investment (where x_b is above $\overline{x}_b$ but $\widehat{x}_b$ remains below $\overline{\overline{x}}$). As $\widetilde{\widetilde{x}}$ falls the leader releases in a smaller range at the top, so observing no release, the sophisticated follower infers that investment by the leader is more likely, i.e., ρ^* rises.

Remark 9 *Where* $\overline{x}_b > \frac{\overline{\overline{x}}-s}{1-s}$, *so* $\widetilde{\widetilde{x}} < 1$, $\rho^* = \frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}} \in (0,1)$, $\frac{\partial \rho^*}{\partial \widetilde{\widetilde{x}}} < 0$ *and* $\frac{d\rho^*}{d\overline{x}_b} > 0$.

Where $\overline{x}_b < \frac{\overline{\overline{x}}-s}{1-s}$, *so* $\widetilde{\widetilde{x}} > 1$, $\rho^* = 0$. *Where* $\overline{x}_b = \frac{\overline{\overline{x}}-s}{1-s}$, *so* $\widetilde{\widetilde{x}} = 1$, $\rho^* = \frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}} = 0$, $\left(\frac{\partial \rho^*}{\partial \widetilde{\widetilde{x}}}\right)^- < 0$ *and* $\left(\frac{d\rho^*}{d\overline{x}_b}\right)^+ > 0$.[26]

Proof. See Appendix. ■

Before seeing how $\overline{x}_b^*$ changes with $\overline{x}_b$, we derive the following useful Remark.

Remark 10 *As either* $s \to 1$ *or* $\overline{x}_b \to \overline{\overline{x}}$, $\widetilde{\widetilde{x}} \to \overline{\overline{x}} < 1$, *so* $\rho^* \to \frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}$ *and* $1-\rho^*(1-\alpha) \to 1 - \left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha) < 2c$. *Furthermore,* $\widetilde{\widetilde{x}}$ *and* $1-\rho^*(1-\alpha)$ *are decreasing in* $\overline{x}_b$. *Thus* $\exists \widetilde{\widetilde{x}}_b \in [\overline{\overline{\overline{x}}}, \overline{\overline{x}})$ *such that* $\widetilde{\widetilde{x}} \leq 1$ *and* $2c \geq 1-\rho^*(1-\alpha)$ *if and only if* $\overline{x}_b \geq \widetilde{\widetilde{x}}_b$, *while* $\widetilde{\widetilde{x}} < 1$ *and* $2c > 1-\rho^*(1-\alpha)$ $\forall \overline{x}_b > \widetilde{\widetilde{x}}_b$. *If* $\overline{x}_b = \widetilde{\widetilde{x}}_b > \overline{\overline{\overline{x}}}$, *then at least one of the inequalities must hold with equality.*

[26] By $\left(\frac{df(x)}{dx}\right)^+$ I mean $\lim_{\gamma \to 0^+} \frac{f(x+\gamma)-f(x)}{\gamma}$, i.e., the slope as x increases, and by $\left(\frac{df(x)}{dx}\right)^-$ I mean $\lim_{\gamma \to 0^-} \frac{f(x+\gamma)-f(x)}{\gamma}$, i.e., the slope as x decreases. Of course, $\left(\frac{df(x)}{dx}\right) = z$ implies that $\left(\frac{df(x)}{dx}\right)^+ = \left(\frac{df(x)}{dx}\right)^- = z$.

Proof. See Appendix. ■

Having seen that ρ^* is increasing in $\overline{x}_b$, we can now show that $\overline{x}_b^*$ also increases as $\overline{x}_b$ ranges from its lower bound of $\overline{\overline{\overline{x}}}$ to its upper bound of $\overline{\overline{x}}$. As $\overline{x}_b$ increases, so does ρ^* from Remark 9, so the follower becomes more cautious if the leader does not release as it infers a higher probability of investment from the leader. Thus the follower's investment threshold $\overline{x}_b^*$ goes up. Letting $\overline{x}_b^*(\overline{x}_b)$ denote $\overline{x}_b^*$ as a function of the value of $\overline{x}_b$, we also find that once it starts rising (which it eventually must do), the function $\overline{x}_b^*(\overline{x}_b)$ is strictly increasing and strictly concave, tending towards a limit strictly below $\overline{\overline{x}}$.

Remark 11 *For any* $\overline{x}_b \in [\overline{\overline{\overline{x}}}, \widetilde{\overline{x}}_b)$, $\frac{d\overline{x}_b^*}{d\overline{x}_b} = 0$, *for* $\overline{x}_b = \widetilde{\overline{x}}_b$, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 0$ *and* $\left(\frac{d\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+}{d\overline{x}_b}\right)^+ < 0$, *and for any* $\overline{x}_b \in (\widetilde{\overline{x}}_b, \overline{\overline{x}})$, $\frac{d\overline{x}_b^*}{d\overline{x}_b} > 0$ *and* $\frac{d^2\overline{x}_b^*}{d(\overline{x}_b)^2} < 0$. *Thus as* $\overline{x}_b$ *rises,* $\overline{x}_b^*(\overline{x}_b)$ *must eventually start increasing, from which point it is strictly increasing and strictly concave.*[27] *As* $\overline{x}_b \to \overline{\overline{x}}$, $\overline{x}_b^* \to \frac{2c}{\left[1-\left(\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right)(1-\alpha)\right]} - 1 < \overline{\overline{x}}$.

Proof. See Appendix. ■

Finally, we are in a position to determine the qualitative properties of the PBEs. Letting $S^{PBE} = \left\{\overline{x}_b \in [\overline{\overline{\overline{x}}}, \overline{\overline{x}}) | \overline{x}_b^*(\overline{x}_b) = \overline{x}_b\right\}$ denote the set of PBEs, we have the following three propositions.

Proposition 2 *Subcase (i)* $\widetilde{\overline{x}}_b = \overline{\overline{\overline{x}}}$, *and* $\widetilde{\widetilde{x}} < 1$ *and* $2c > 1 - \rho^*(1-\alpha)$ *at* $\overline{x}_b = \overline{\overline{\overline{x}}}$

$\overline{x}_b^*(\overline{\overline{\overline{x}}}) > \overline{\overline{\overline{x}}}$, $\overline{x}_b^*(\overline{x}_b)$ *is everywhere strictly increasing and strictly concave, and* S^{PBE} *has a unique member which lies in* $(\overline{\overline{\overline{x}}}, \overline{\overline{x}})$.

Proof. Suppose that at $\overline{x}_b = \overline{\overline{\overline{x}}}$, $\widetilde{\widetilde{x}} < 1$ and $2c > 1 - \rho^*(1-\alpha)$. Then $\widetilde{\widetilde{x}} < 1 \Rightarrow \rho^* = \frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}} > 0$ from Remark 9. Using Remark 3 and $2c > 1 - \rho^*(1-\alpha)$, $\overline{x}_b^* = \frac{2c}{[1-\rho^*(1-\alpha)]} - 1 > 0$. As $\rho^* > 0$, $\overline{x}_b^* > 2c - 1$. Together with the fact that $\overline{x}_b^* > 0$, we have $\overline{x}_b^* > \min(2c-1, 0)$, so $\overline{x}_b^* > \overline{\overline{\overline{x}}}$.

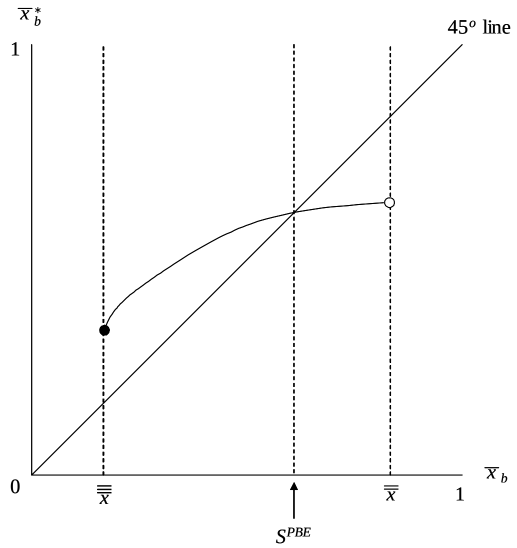
Thus $\overline{x}_b^*(\overline{x}_b)$ starts off strictly above $\overline{\overline{\overline{x}}}$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$. Given $\widetilde{\overline{x}}_b = \overline{\overline{\overline{x}}}$, Remark 11 tells us that $\overline{x}_b^*(\overline{x}_b)$ is everywhere strictly increasing and strictly concave, tending to a limit

[27] Note that the $\overline{x}_b^*$ function is continuous in $\overline{x}_b$ as $\overline{x}_b^*$ is continuous (though kinked) in ρ^*, which in turn is continuous (but kinked) in $\widetilde{\widetilde{x}}$ which itself is continuous in $\overline{x}_b$.

strictly below $\overline{\overline{x}}$ as $\overline{x}_b \to \overline{\overline{x}}$. Where $\overline{x}_b^*(\overline{x}_b)$ crosses the 45^o line in $(\overline{x}_b, \overline{x}_b^*)$ space, $\overline{x}_b^* = \overline{x}_b$ so we have a PBE. The curve starts off above the 45^o line and ends up below it, so by the Intermediate Value Theorem it must cross the 45^o line at least once. Because the curve starts above the 45^o line and is strictly concave, any initial crossing must be from above with $\frac{d\overline{x}_b^*}{d\overline{x}_b} < 1$. Thus by strict concavity the slope must remain below 1 from that point on, so a second crossing is ruled out. Thus S^{PBE} has a unique member which lies in $(\overline{\overline{\overline{x}}}, \overline{\overline{x}})$. ■

This case is illustrated in the following figure.

Figure 1: Unique PBE in Subcase (i)



Proposition 3 *Subcase (ii)* $\tilde{\overline{x}}_b = \overline{\overline{\overline{x}}}$, *and at* $\overline{x}_b = \overline{\overline{\overline{x}}}$, $\tilde{\tilde{x}} = 1$ *or* $2c = 1 - \rho^*(1-\alpha)$

$\overline{x}_b^*(\overline{\overline{\overline{x}}}) = \overline{\overline{\overline{x}}}$ *and* $\overline{x}_b^*(\overline{x}_b)$ *is everywhere strictly increasing and strictly concave.* $\overline{x}_b = \overline{\overline{\overline{x}}} \in S^{PBE}$. *If* $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ *at* $\overline{x}_b = \overline{\overline{\overline{x}}}$, S^{PBE} *has one further member which lies in* $(\overline{\overline{\overline{x}}}, \overline{\overline{x}})$. *If* $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0,1]$ *at* $\overline{x}_b = \overline{\overline{\overline{x}}}$, S^{PBE} *has no further members.*[28]

Proof. $\tilde{\tilde{x}} = 1 \Rightarrow \rho^* = 0$ from Remark 9, so from Remark 3 $\overline{x}_b^* = \overline{\overline{\overline{x}}}$. $2c = 1-\rho^*(1-\alpha)$ implies $c \leq \frac{1}{2}$ as $\rho^* \geq 0$ always, and further implies $\overline{x}_b^* = 0$ from Remark 3. But $c \leq \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 0$, so again $\overline{x}_b^* = \overline{\overline{\overline{x}}}$.

[28] Tedious algebraic manipulation shows that $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ and $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0,1]$ are both possible at $\overline{x}_b = \overline{\overline{\overline{x}}}$ here given appropriate values of α, c and s satisfying the assumed parameter restrictions.

Thus $\overline{x}_b^*(\overline{\overline{\overline{x}}}) = \overline{\overline{\overline{x}}}$ and so $\overline{x}_b = \overline{\overline{\overline{x}}} \in S^{PBE}$. Given $\widetilde{\overline{x}}_b = \overline{\overline{\overline{x}}}$, Remark 11 tells us that $\overline{x}_b^*(\overline{x}_b)$ is everywhere strictly increasing and strictly concave, tending to a limit strictly below $\overline{\overline{x}}$ as $\overline{x}_b \to \overline{\overline{x}}$. If $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$, then the curve starts by rising above the 45^o line, so by the same argument as in the proof of Proposition 2 S^{PBE} has a unique member in $(\overline{\overline{\overline{x}}}, \overline{\overline{x}})$, and hence has two members in total. If $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0, 1]$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$, then curve starts rising immediately, but by strict concavity its slope must remain below 1, so it can never cross the 45^o line for $\overline{x}_b \in (\overline{\overline{\overline{x}}}, \overline{\overline{x}})$. Thus S^{PBE} has a unique member at $\overline{x}_b = \overline{\overline{\overline{x}}}$.

■

The next two figures illustrate the cases with one and two PBEs in subcase (ii).

Figure 2: Two PBEs in Subcase (ii)

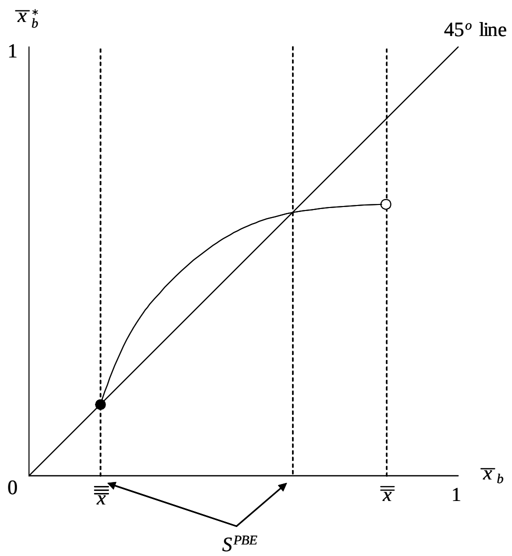
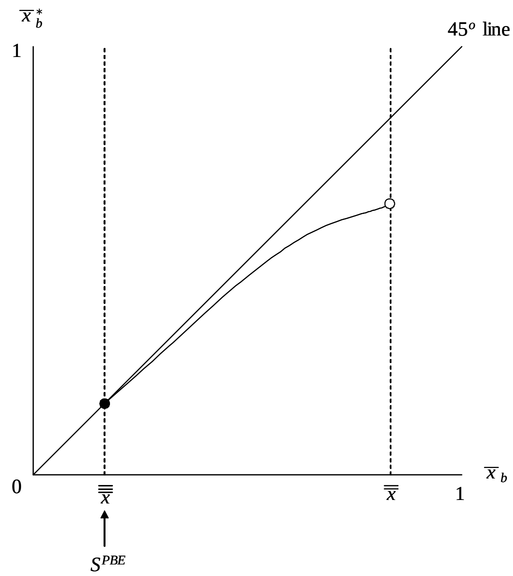


Figure 3: One PBE in Subcase (ii)



Proposition 4 *Subcase (iii)* $\widetilde{\overline{x}}_b > \overline{\overline{\overline{x}}}$

$\overline{x}_b^*(\overline{\overline{\overline{x}}}) = \overline{\overline{\overline{x}}}$ *and* $\frac{d\overline{x}_b^*}{d\overline{x}_b} = 0$ *at* $\overline{x}_b = \overline{\overline{\overline{x}}}$*. The* $\overline{x}_b^*(\overline{x}_b)$ *curve continues at* $\overline{\overline{\overline{x}}}$ *until* $\overline{x}_b$ *reaches* $\widetilde{\overline{x}}_b$*, from which point on* $\overline{x}_b^*(\overline{x}_b)$ *is everywhere strictly increasing and strictly concave.* $\overline{x}_b = \overline{\overline{\overline{x}}} \in S^{PBE}$*, and* S^{PBE} *may have zero, one or two further members lying in* $(\overline{\overline{\overline{x}}}, \overline{\overline{x}})$.[29]

Proof. From Remark 10, $\widetilde{\overline{x}}_b > \overline{\overline{\overline{x}}}$ implies that at $\overline{x}_b = \overline{\overline{\overline{x}}}$, $\widetilde{\widetilde{x}} > 1$ or $2c < 1 - \rho^*(1-\alpha)$. $\widetilde{\widetilde{x}} > 1 \Rightarrow \rho^* = 0$ from Remark 9, so from Remark 3, $\overline{x}_b^* = \overline{\overline{\overline{x}}}$. $2c < 1 - \rho^*(1-\alpha)$ implies

[29] If we were to invoke a simple stability argument, we could rule out the middle of the three PBEs (see Figure 4). Starting from either side of this PBE, a simple myopic best-response dynamic moves away from this PBE towards one of the other two. The other PBEs though are stable.

$c < \frac{1}{2}$ as $\rho^* \geq 0$ always, and further implies $\overline{x}_b^* = 0$ from Remark 3. But $c < \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 0$, so again $\overline{x}_b^* = \overline{\overline{\overline{x}}}$.

Thus $\overline{x}_b^*(\overline{\overline{\overline{x}}}) = \overline{\overline{\overline{x}}}$ and so $\overline{x}_b = \overline{\overline{\overline{x}}} \in S^{PBE}$. Given $\tilde{\overline{x}}_b > \overline{\overline{\overline{x}}}$, Remark 11 tells us that $\frac{d\overline{x}_b^*}{d\overline{x}_b} = 0$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$ and that $\overline{x}_b^*$ will remain at $\overline{\overline{\overline{x}}}$ until $\overline{x}_b$ reaches $\tilde{\overline{x}}_b$ from which point $\overline{x}_b^*(\overline{x}_b)$ is everywhere strictly increasing and strictly concave, tending to a limit strictly below $\overline{\overline{x}}$ as $\overline{x}_b \rightarrow \overline{\overline{x}}$. Once the curve starts rising, it may never reach the 45^o line, in which case $\overline{x}_b = \overline{\overline{\overline{x}}}$ is the unique PBE, it may cross the 45^o line from below, from which point on by the same argument as in the proof of Proposition 2 there must be exactly one further crossing, so S^{PBE} has three members in total, or it may reach the 45^o line tangentially, in which case S^{PBE} has two members. ■

The following three figures illustrate the case with one, two and three PBEs in subcase (iii).

Figure 4: Three PBEs in Subcase (iii)

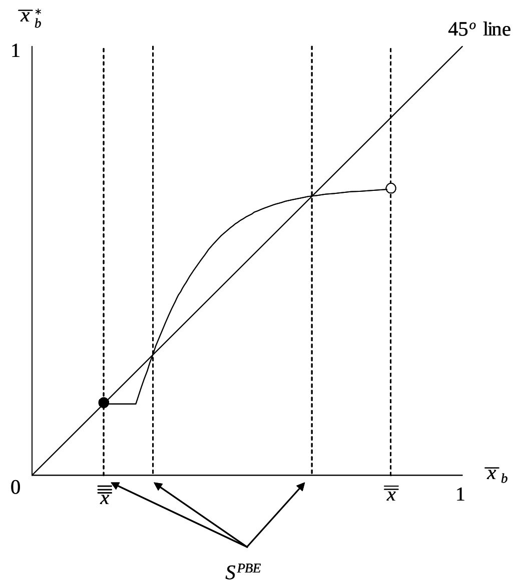


Figure 5: Two PBEs in Subcase (iii)

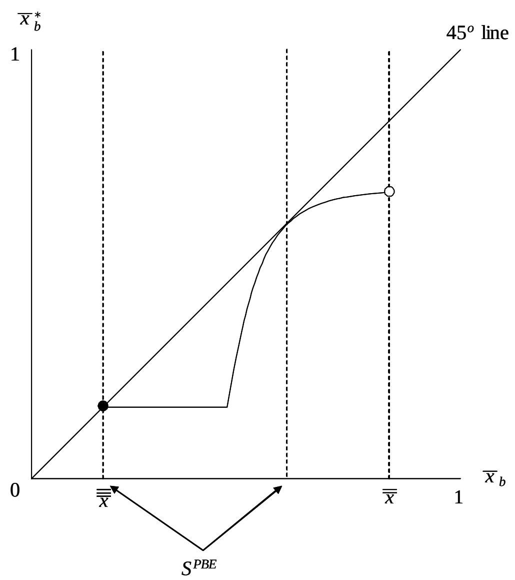
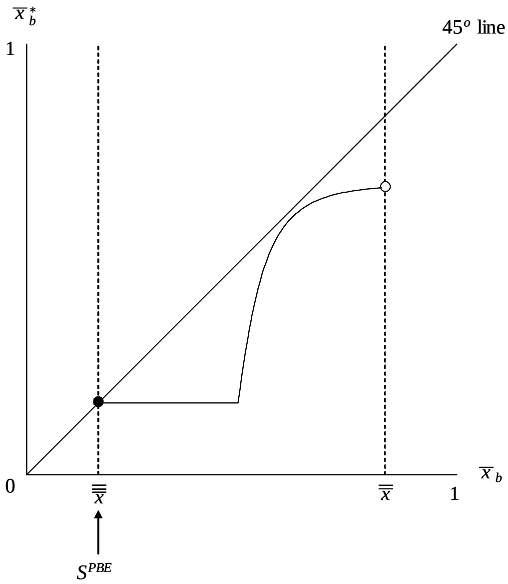


Figure 6: One PBE in Subcase (iii)



For a given level of knowledge spillovers s, $\widetilde{\widetilde{x}}$ is decreasing in $\overline{x}_b^*$ while $\widetilde{x}$ is constant in $\overline{x}_b^*$ (see Claim 5), so where there are multiple equilibria, the range in which the leader releases is smaller for equilibria with higher $\overline{x}_b^*$. As already explained, a higher $\overline{x}_b$ makes release more dangerous for the leader, as the release is more likely to tilt the follower towards investing. Because the leader's release range is smaller, the follower is more wary if the leader does not release, so the follower's release threshold is higher.

We can now analyze how S^{PBE} changes as the level of knowledge spillovers s changes. We start by deriving the following remark, which tells us that the $\overline{x}_b^*(\overline{x}_b)$ curve shifts up as s rises where $\overline{x}_b \geq \widetilde{\widetilde{x}}_b$. As s goes up, then for a given $\overline{x}_b$ the leader releases in a smaller range at the top, i.e., $\widetilde{\widetilde{x}}$ falls. Observing no release, the follower then infers that investment is more likely, so ρ^* and hence $\overline{x}_b^*$ rise.

Remark 12 *Where $\overline{x}_b > \widetilde{\widetilde{x}}_b$, $\frac{d\rho^*}{ds} > 0$ and hence $\frac{d\overline{x}_b^*}{ds} > 0$, i.e., the $\overline{x}_b^*(\overline{x}_b)$ curve shifts up as s rises.*

Where $\overline{x}_b = \widetilde{\widetilde{x}}_b > \overline{\overline{\overline{x}}}$, $\left(\frac{d\overline{x}_b^*}{ds}\right)^+ > 0$ and $\left(\frac{d\overline{x}_b^*}{ds}\right)^- = 0$.

Where $\overline{x}_b = \widetilde{\widetilde{x}}_b = \overline{\overline{\overline{x}}}$, in subcase (i) $\frac{d\overline{x}_b^*}{ds} > 0$ while in subcase (ii) $\left(\frac{d\overline{x}_b^*}{ds}\right)^+ > 0$ and $\left(\frac{d\overline{x}_b^*}{ds}\right)^- = 0$.

Where $\overline{x}_b < \widetilde{\widetilde{x}}_b$, $\frac{d\overline{x}_b^*}{ds} = 0$.

Proof. See Appendix. ■

The following remark is also of use, showing how we move from subcases (iii) to (ii) to (i) as s rises.

Remark 13 *There exists a* $\widetilde{\widetilde{s}} \in (0,1)$ *such that:*

Where $s > \widetilde{\widetilde{s}}$ *subcase (i) from Proposition 2 applies.*

Where $s = \widetilde{\widetilde{s}}$*, subcase (ii) from Proposition 3 applies.*

Where $s < \widetilde{\widetilde{s}}$*, subcase (iii) from Proposition 4 applies.*

Proof. See Appendix. ■

The next proposition looks at how S^{PBE} changes as s falls towards $\widetilde{\widetilde{s}}$.

Proposition 5 *As* $s \to 1$, $\overline{x}_b^*(\overline{x}_b)$ *tends to a flat line at* $\frac{2c}{1-\left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha)} - 1 < \overline{\overline{x}}$*, and the unique PBE tends to* $\frac{2c}{1-\left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha)} - 1$*. As* s *falls towards* $\widetilde{\widetilde{s}}$*, the* $\overline{x}_b^*(\overline{x}_b)$ *curve, which is decreasing, tends towards the* $\overline{x}_b^*(\overline{x}_b)$ *curve for* $s = \widetilde{\widetilde{s}}$*. Thus the value of the unique PBE falls continuously. If for* $s = \widetilde{\widetilde{s}}$, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0,1]$ *at* $\overline{x}_b = \overline{\overline{\overline{x}}}$*, then as* $s \to \widetilde{\widetilde{s}}$ *the unique PBE tends towards* $\overline{\overline{\overline{x}}}$*, the unique PBE where* $s = \widetilde{\widetilde{s}}$*. If for* $s = \widetilde{\widetilde{s}}$, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ *at* $\overline{x}_b = \overline{\overline{\overline{x}}}$*, the unique PBE tends towards the higher of the two PBEs where* $s = \widetilde{\widetilde{s}}$*.*

Proof. As $s \to 1$, $1 - \rho^*(1-\alpha) \to 1 - \left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha) < 2c$ from Remark 10. Thus, using Remark 3, $\overline{x}_b^* \to \frac{2c}{\left[1-\left(\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right)(1-\alpha)\right]} - 1$, which from Remark 11 is strictly smaller than $\overline{\overline{x}}$. So $\overline{x}_b^*(\overline{x}_b)$ is tending to a flat line, and the unique PBE is tending towards the intersection of this flat line and the 45^o line, which of course occurs at $\frac{2c}{1-\left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha)} - 1$.

From Remarks 12 and 13, for $s > \widetilde{\widetilde{s}}$ we are in subcase (i) and $\frac{d\overline{x}_b^*}{ds} > 0$. Thus as s falls, the $\overline{x}_b^*(\overline{x}_b)$ curve shifts down, and the unique intersection of $\overline{x}_b^*(\overline{x}_b)$ and the 45^o line shifts to the left. Hence the value of the unique PBE falls continuously. As $s \to \widetilde{\widetilde{s}}$ from above, the $\overline{x}_b^*(\overline{x}_b)$ curve tends towards the $\overline{x}_b^*(\overline{x}_b)$ curve for $s = \widetilde{\widetilde{s}}$.[30] The rest of the proposition follows immediately, as illustrated in Figures 7 and 8. ■

The next two figures illustrate how S^{PBE} changes as s falls towards $\widetilde{\widetilde{s}}$. In Figure 7, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0,1]$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$ and $s = \widetilde{\widetilde{s}}$. The left arrow shows how the value of the unique

[30] The argument depends on the continuity of $\overline{x}_b^*(\overline{x}_b)$ in s, which follows from the continuity of all the relevant underlying functions.

PBE gradually decreases, eventually reaching $\overline{\overline{\overline{x}}}$. In Figure 8, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$ and $s = \tilde{\tilde{s}}$, in which case the value of the unique PBE decreases, but only as far as the higher of the two PBEs at $s = \tilde{\tilde{s}}$.

Figure 7

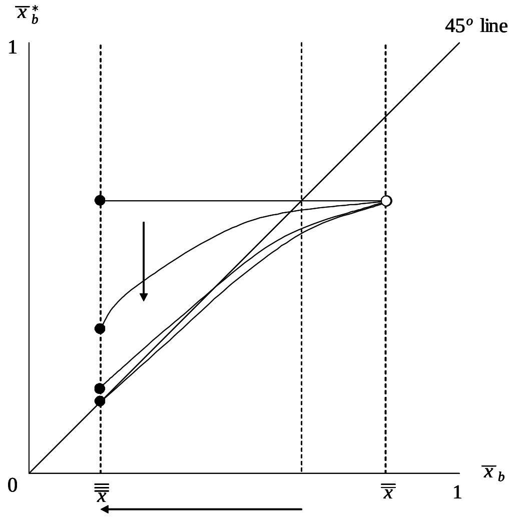
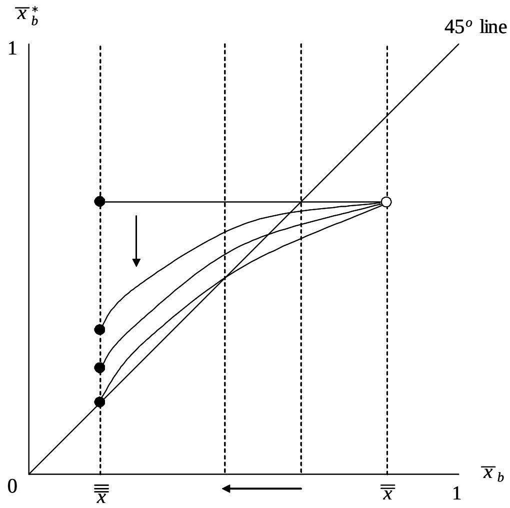


Figure 8



As s falls here there are two self-reinforcing effects on $\tilde{\tilde{x}}$, both causing $\tilde{\tilde{x}}$ to rise, so the leader releases in a larger range at the top. First, there is the direct effect of the decrease on $\tilde{\tilde{x}}$ from Claim 5. As the level of knowledge spillovers falls for a fixed $\overline{x}_b$, the leader will release in a larger range. Secondly, there is the indirect effect of the fall in s on the value of the follower's equilibrium inferences. As s falls, the equilibrium value of $\overline{x}_b$ moves to the left which again from Claim 5 results in an increase in $\tilde{\tilde{x}}$. A decrease in the equilibrium value of $\overline{x}_b$ lowers the cost of release. The follower is less cautious absent release, so release is less likely to tilt the follower towards investing.

The final proposition in this section looks at how S^{PBE} changes as s rises towards $\tilde{\tilde{s}}$.

Proposition 6 *For small enough s, $\overline{x}_b = \overline{\overline{\overline{x}}}$ is the unique PBE. As s rises towards $\tilde{\tilde{s}}$, the $\overline{x}_b^*(\overline{x}_b)$ curve, which is rising in s, tends towards the $\overline{x}_b^*(\overline{x}_b)$ curve for $s = \tilde{\tilde{s}}$. Thus, if for $s = \tilde{\tilde{s}}$, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0,1]$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$, $\overline{x}_b = \overline{\overline{\overline{x}}}$ remains the unique PBE as s rises towards $\tilde{\tilde{s}}$. Alternatively, if for $s = \tilde{\tilde{s}}$, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$, then as s rises towards $\tilde{\tilde{s}}$,* *eventually the $\overline{x}_b^*(\overline{x}_b)$ curve becomes tangential to and then breaches the 45^o line between $\overline{\overline{\overline{x}}}$ and the higher PBE at $s = \widetilde{\widetilde{s}}$. As s rises beyond the tangency point, the lower of the PBEs in $(\overline{\overline{\overline{x}}}, \overline{\overline{x}})$ gradually falls, tending to $\overline{\overline{\overline{x}}}$ as $s \to \widetilde{\widetilde{s}}$, while the higher one gradually rises, tending to the higher PBE at $s = \widetilde{\widetilde{s}}$.*

Proof. From Remark 13, where $s < \widetilde{\widetilde{s}}$ we are in subcase (iii) where $\widetilde{\widetilde{x}}_b > \overline{\overline{\overline{x}}}$.

For small enough s, the equilibrium is unique at $\overline{\overline{\overline{x}}}$. If $\widetilde{\widetilde{x}}_b$ is sufficiently close to $\overline{\overline{x}}$, then $\overline{x}_b^*(\overline{x}_b)$ cannot rise as far as the 45^o line, given that $\overline{x}_b^*(\overline{x}_b)$ is everywhere increasing beyond $\widetilde{\widetilde{x}}_b$ with a limit strictly below $\overline{\overline{x}}$ from Remark 11. Because $\widetilde{\widetilde{x}} \to \infty$ as $s \to 0$, we can set $\widetilde{\widetilde{x}}_b$ arbitrarily close to $\overline{\overline{x}}$.

From Remark 12 $\frac{d\overline{x}_b^*}{ds} > 0$ for $\overline{x}_b > \widetilde{\widetilde{x}}_b$, $\left(\frac{d\overline{x}_b^*}{ds}\right)^+ > 0$ for $\overline{x}_b = \widetilde{\widetilde{x}}_b$ and $\frac{d\overline{x}_b^*}{ds} = 0$ for $\overline{x}_b < \widetilde{\widetilde{x}}_b$. We can show that $\frac{d\widetilde{\widetilde{x}}_b}{ds} < 0$ for $s < \widetilde{\widetilde{s}}$. As s and $\overline{x}_b$ change, changes in ρ^* operate purely through changes in $\widetilde{\widetilde{x}}$. Thus if $\widetilde{\widetilde{x}}$ stays constant, so does and $1 - \rho^*(1-\alpha)$. Now from Claim 5 $\widetilde{\widetilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s}$, so $d\widetilde{\widetilde{x}} = \frac{\partial \widetilde{\widetilde{x}}}{\partial s} ds + \frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b} d\overline{x}_b$. From Claim 5 $\frac{\partial \widetilde{\widetilde{x}}}{\partial s} < 0$ and $\frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b} < 0$. Thus as s changes, to keep $\widetilde{\widetilde{x}}$, and hence also $1 - \rho^*(1-\alpha)$, constant at their values at any initial $(\overline{x}_b)_1$, $\overline{x}_b$ must change in the opposite direction to the change in s to $(\overline{x}_b)_2$.[31] Thus $\widetilde{\widetilde{x}}_b$ changes in the opposite direction to the change in s. Furthermore, $\widetilde{\widetilde{x}}_b \to \overline{\overline{\overline{x}}}$ as $s \to \widetilde{\widetilde{s}}$ from below. An infinitesimal decrease in s below $\widetilde{\widetilde{s}}$ will require just an infinitesimal increase in $\overline{x}_b$ to restore $\widetilde{\widetilde{x}}$ hence $1 - \rho^*(1-\alpha)$ to their original values, so given $\widetilde{\widetilde{x}}_b = \overline{\overline{\overline{x}}}$ at $s = \widetilde{\widetilde{s}}$, $\widetilde{\widetilde{x}}_b \to \overline{\overline{\overline{x}}}$ as $s \to \widetilde{\widetilde{s}}$.

Thus as s rises, the $\overline{x}_b^*(\overline{x}_b)$ curve shifts up for $\overline{x}_b \geq \widetilde{\widetilde{x}}_b$ and $\widetilde{\widetilde{x}}_b$ falls so the range in which $\overline{x}_b^*(\overline{x}_b)$ is increasing gets bigger. Furthermore $\widetilde{\widetilde{x}}_b \to \overline{\overline{\overline{x}}}$ as $s \to \widetilde{\widetilde{s}}$. Thus as $s \to \widetilde{\widetilde{s}}$ from below, the $\overline{x}_b^*(\overline{x}_b)$ curve tends continuously towards the curve at $s = \widetilde{\widetilde{s}}$.[32] The rest of the proposition follows immediately, as illustrated in Figures 9 and 10. ■

The next two figures illustrate how S^{PBE} changes as s rises towards $\widetilde{\widetilde{s}}$. In Figure 9, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ \in (0,1]$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$ and $s = \widetilde{\widetilde{s}}$, so the unique PBE remains $\overline{\overline{\overline{x}}}$. In Figure 10, $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ > 1$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$ and $s = \widetilde{\widetilde{s}}$. The arrows to the right and to the left show how as s increases, once the $\overline{x}_b^*(\overline{x}_b)$ curve breaches the point of tangency with the 45^o line the

[31] Formally, $d\overline{x}_b = -\left(\frac{\partial \widetilde{\widetilde{x}}}{\partial s} / \frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b}\right) ds$.

[32] Again, the argument depends on the continuity of $\overline{x}_b^*(\overline{x}_b)$ in s.

higher PBE increases, tending towards the higher PBE at $s = \widetilde{\widetilde{s}}$, while the lower PBE decreases, tending towards $\overline{\overline{\overline{x}}}$ as s increases.

Figure 9

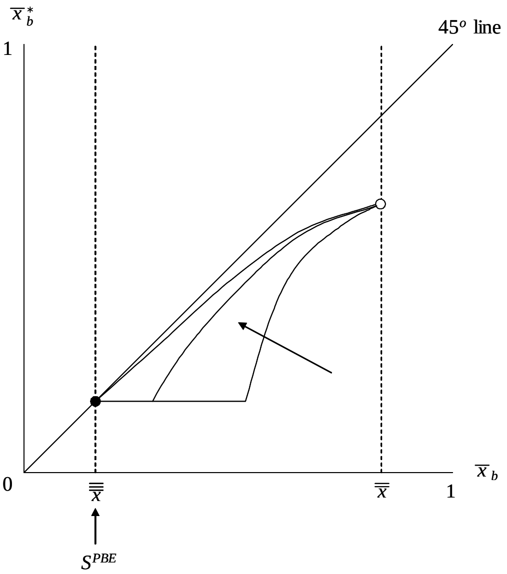
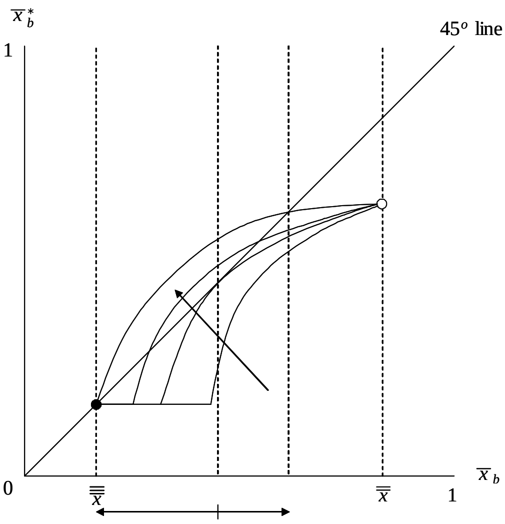


Figure 10



In summary, where spillovers are very low there is a unique equilibrium with $\overline{x}_b^* = \overline{\overline{\overline{x}}}$. The leader always discloses if it invests as the signalling effect outweighs the information transmission effect. However, with poor intermediate results, the leader abandons the project and so does not bother incurring the disclosure cost. Thus seeing no disclosure, the follower infers no investment from the leader, and so will want to invest itself so long as $x_b \geq \overline{\overline{\overline{x}}}$.[33] For higher spillovers, the leader will be more inclined to use secrecy for very good intermediate results to avoid helping the rival too much. Observing no release, the follower becomes more cautious, and so requires a higher x_b before it invests, permitting equilibria with $\overline{x}_b^* > \overline{\overline{\overline{x}}}$. For spillovers close to but below $\widetilde{\widetilde{s}}$ both types of equilibria may exist. (Whether the leader wishes to rely on secrecy will depend on the follower's inferences from no disclosure, which explains how more than one equilibrium can exist). As spillovers increase, the leader will use more and more secrecy, and the follower will therefore become more and more cautious following no disclosure - for spillovers above $\widetilde{\widetilde{s}}$, there will be a unique equilibrium, in which $\overline{x}_b^*$ is greater than $\overline{\overline{\overline{x}}}$ and is rising in s.

[33] Where c is low, there may be equilibria with $\overline{x}_b^* = \overline{\overline{\overline{x}}}$ in which the leader makes some use of secrecy, but the follower still wants to invest $\forall x_b$ following no disclosure (where $\widetilde{\widetilde{x}} < 1$, so $\rho^* > 0$, but $2c \leq 1 - \rho^*(1 - \alpha)$.

4.3 Case (c): $c \in (\frac{\alpha}{2}, \frac{1}{2}]$ and $\alpha \leq \frac{1}{2}$

The third case we look at has $c \in (\frac{\alpha}{2}, \frac{1}{2}]$ and $\alpha \leq \frac{1}{2}$. The case thus subsumes two possible subcases: (i) $c \in (\frac{\alpha}{2}, \alpha)$ and $\alpha \leq \frac{1}{2}$, and (ii) $c \in \left[\alpha, \frac{1}{2}\right]$. In subcase (i), as in case (b), c is in an intermediate range between $\frac{\alpha}{2}$ and α, but now the degree of post-innovation competition is strong as $\alpha \leq \frac{1}{2}$. In subcase (ii), c is large relative to α, but yet is constrained to be no greater than $\frac{1}{2}$.

From Remark 1, $2c > \alpha \Rightarrow \overline{\overline{x}} = \frac{2c-\alpha}{\alpha} > 0$. From Remark 2, $c \leq \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 0$. These useful restrictions are summarized below.

Remark 14 $\overline{\overline{x}} = \frac{2c-\alpha}{\alpha} > 0$; $\overline{\overline{\overline{x}}} = 0$

We can now pin down the leader's behaviour for any $\overline{x}_b$. The next remark tells us that the leader always wants to invest in this case. This may appear surprising in comparison to case (b) where α is higher, so the post-innovation level of competition is weaker, but yet the leader does not always invest (from Claim 5, there is a strictly positive range for which the leader neither invests nor releases in that case).[34] The explanation comes from the indirect effect of α on $\overline{\overline{x}}$. The direct effect of α on $\overline{x}_a$ and $\widetilde{x}$ is as expected: a lower α, i.e., stronger post-innovation competition, raises both $\overline{x}_a$ and $\widetilde{x}$.[35] However, a lower α also raises $\overline{\overline{x}}$. This makes release and investment more attractive, as disclosure, which raises the follower's investment threshold to $\overline{\overline{x}}$ is more likely to deter the follower. Thus, where $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, even for poor intermediate results the leader wants to disclose and invest, while for very good results the leader will invest but may rely on secrecy. For $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, i.e., for pessimistic follower beliefs, disclosure is not required and the leader just invests. Effectively, stronger product market competition makes the follower sufficiently cautious (if it thinks the leader is investing) that the leader always wants to invest for any inferences the follower might be drawing, disclosing if this is required to make the follower believe it is investing and disclosure is not too costly.

[34] Letting $\alpha_b > \frac{1}{2}$ and $\alpha_c \leq \frac{1}{2}$ be the competition parameters in the two cases, we can see that $c \in (\frac{\alpha_b}{2}, \frac{1}{2}]$ is compatible with both cases (b) and (c).

[35] See Remark 5 and Claim 2.

Remark 15 *The leader always invests.*

Where $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, *the leader never releases.*

Where $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ *and* $\overline{\overline{x}} < 1$, *the leader releases if and only if* $x_a < \tilde{\tilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s}$.

Where $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ *and* $\overline{\overline{x}} \geq 1$, *the leader always releases.*

Proof. See Appendix. ■

We can now analyze the set of PBEs. The proposition below shows that $\overline{x}_b = \overline{\overline{x}}$ forms an equilibrium, in contrast to case (b) where this was ruled out. We can have a PBE here because, as explained above, if the leader does not want to disclose it still wants to invest everywhere here, so the follower infers that the rival is definitely investing absent release, destroying any incentive to disclose and hence justifying $\overline{x}_b^* = \overline{\overline{x}}$. In case (b), the follower allocated a strictly positive probability to the leader not investing, ruling out this PBE.

Proposition 7 *There is a PBE with* $\overline{x}_b = \overline{\overline{x}}$ *in which the leader always invests, but does not release.*

Proof. Suppose $\overline{x}_b = \overline{\overline{x}}$. This entails $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, so from Remark 15 the leader always invests, but never releases. Seeing no release, the follower infers $\rho^* = 1$, which from Remark 3 gives $\overline{x}_b^* = \overline{\overline{x}}$ so we have a PBE. ■

The next two claims find all the other possible PBEs. We cannot have PBEs with $\overline{x}_b < \overline{\overline{x}}$ in which the leader might or might not release depending on x_a, as we had in case (b). Remark 15 tells us that the leader must always invest in such a putative PBE, but then seeing no release the follower infers the leader is definitely investing so $\overline{x}_b = \overline{\overline{x}}$. We do find PBEs in which the leader always releases and invests, where the follower's arbitrary off the equilibrium path beliefs following no release are that the probability the leader has invested is small enough. However, we do not find these PBEs convincing. Given there is a single, well-defined equilibrium (with $\overline{x}_b = \overline{\overline{x}}$) in which the leader does not release, if the follower observes no release it seems much more reasonable that it infers that the leader thinks it is playing the unique no release equilibrium and hence is definitely investing. Such a forwards-induction type reasoning process implies that $\overline{x}_b = \overline{\overline{x}}$ forms the unique equilibrium $\forall s$.

Claim 6 *Where $\overline{\overline{x}} < 1$, there can be no PBE with $\overline{x}_b \in (\frac{\overline{\overline{x}}-s}{1-s}, \overline{\overline{x}})$.*[36] *In particular, where $s > \overline{\overline{x}}$, $\overline{x}_b = \overline{\overline{x}}$ is the unique PBE. Any $\overline{x}_b \in \left[0, \frac{\overline{\overline{x}}-s}{1-s}\right]$ can form a PBE in which the leader always invests and releases, with appropriately constructed but arbitrary off the equilibrium path follower beliefs following no release by the leader.*

Proof. See Appendix. ■

Claim 7 *Where $\overline{\overline{x}} \geq 1$, there can be no PBE with $\overline{x}_b \in [1, \overline{\overline{x}})$. Any $\overline{x}_b \in [0, 1)$ can form a PBE in which the leader always invests and releases, with appropriately constructed but arbitrary off the equilibrium path follower beliefs following no release by the leader.*

Proof. See Appendix. ■

In summary, in the unique reasonable PBE, the leader always invests, but never discloses. Strong competition makes the follower sufficiently cautious that the leader will always invest for any follower inferences. Thus in equilibrium, disclosure cannot make the follower more likely to believe the leader has invested, destroying any incentive to disclose. The leader is able to use its first-mover advantage to force many of the follower types out of the market. Note that where $c > \frac{3\alpha}{4}$, so $\overline{\overline{x}} > \frac{1}{2}$, more than half the follower types are forced out, and where $c \geq \alpha$, so $\overline{\overline{x}} \geq 1$, all the follower types are forced out.

4.4 Case (d): $c \geq \alpha$ and $c > \frac{1}{2}$

The final case has $c \geq \alpha$ and $c > \frac{1}{2}$. Costs are high both relative to α and in absolute terms. We start by deriving some useful restrictions following from these parameter bounds.

Remark 16 $\overline{\overline{x}} = \frac{2c-\alpha}{\alpha} \geq 1$; $\overline{\overline{\overline{x}}} = 2c - 1 \in (0, 1)$; $\overline{x}_a > 0$; $\overline{x}_b^* = \frac{2c}{1-\rho^*(1-\alpha)} - 1 > 0$

[36] Note that $\frac{\overline{\overline{x}}-s}{1-s} < \overline{\overline{x}}$ here as $\overline{\overline{x}} < 1 \Rightarrow \overline{\overline{x}}s < s \Rightarrow \overline{\overline{x}} - \overline{\overline{x}}s > \overline{\overline{x}} - s$.

Proof. See Appendix. ■

Next we identify the leader's behaviour for any $\overline{x}_b$. In comparison to case (c), we see that for low x_a the leader does not want to invest - this is driven directly by the higher c in this case. For $\overline{x}_b < 1$, the leader always wants to disclose where it invests, which contrasts with the results in case (b). This follows because $\overline{\overline{x}} \geq 1$, so $\widehat{x}_b < \overline{\overline{\overline{x}}}$, which means the release can never encourage the follower to extra investment, but following disclosure the follower definitely does not invest given $\overline{\overline{x}} \geq 1$.

Remark 17 *Where $\overline{x}_b \geq 1$, the leader never releases, and invests if and only if $x_a \geq \overline{\overline{\overline{x}}} > 0$.*

Where $\overline{x}_b < 1$, the leader invests and releases $\forall x_a > \overline{\overline{\overline{x}}} > 0$, and otherwise neither invests nor releases.

Proof. See Appendix. ■

Where $\overline{x}_b < 1$, if the leader invests it also releases. Hence if the follower sees no release it infers the leader is not investing, so $\overline{x}_b^* = \overline{\overline{\overline{x}}}$. Hence we have a PBE with $\overline{x}_b = \overline{\overline{\overline{x}}}$. We cannot have a PBE with $\overline{x}_b = \overline{\overline{x}} \geq 1$, as seeing no release, the follower would infer that there is a chance that $x_a \in (0, \overline{\overline{\overline{x}}})$ where the leader would not invest, so $\overline{x}_b^* < \overline{\overline{x}}$. However, where $\alpha \leq \frac{1}{2}$, which implies $\overline{\overline{x}} > 1$, there is a PBE with $\overline{x}_b \in [1, \overline{\overline{x}})$ in which the leader never releases.

Proposition 8 *There is a PBE with $\overline{x}_b = \overline{\overline{\overline{x}}}$ in which the leader invests and releases $\forall x_a > \overline{\overline{\overline{x}}}$, and otherwise neither invests nor releases. There are no other PBEs with $\overline{x}_b < 1$. If and only if $\alpha \leq \frac{1}{2}$, which implies $\overline{\overline{x}} > 1$, there is one further PBE, with $\overline{x}_b = \frac{2c}{1-(2-2c)(1-\alpha)} - 1 \in [1, \overline{\overline{x}})$ in which the leader never releases, and invests if and only if $x_a \geq \overline{\overline{\overline{x}}}$.*

Proof. Suppose $\overline{x}_b \in [0, 1)$. From Remark 17, the leader invests and releases $\forall x_a > \overline{\overline{\overline{x}}}$, and otherwise neither invests nor releases. From Remark 16 $\overline{\overline{\overline{x}}} > 0$, so if the follower sees no release, it infers $\rho^* = 0$. From Remark 3 this gives $\overline{x}_b^* = \overline{\overline{\overline{x}}}$. Thus $\overline{x}_b = \overline{\overline{\overline{x}}}$ is the unique equilibrium for $\overline{x}_b < 1$.

Suppose $\overline{x}_b \in [1, \overline{\overline{x}}]$. From Remark 17, the leader never releases, and invests if and only if $x_a \geq \overline{\overline{\overline{x}}} = 2c - 1 > 0$. Thus if the follower sees no release, it infers $\rho^* = 1 - \overline{\overline{\overline{x}}} = 2 - 2c < 1$, so $\overline{x}_b^* = \frac{2c}{1-(2-2c)(1-\alpha)} - 1$ from Remark 16. Using $2c - 1 > 0$, we can show $\overline{x}_b^* < \overline{\overline{x}}$:

$$\begin{aligned}
\frac{2c}{1-(2-2c)(1-\alpha)} - 1 &< \frac{2c}{\alpha} - 1 \Leftrightarrow \\
\alpha &< 1 - (2 - 2\alpha - 2c + 2c\alpha) \Leftrightarrow \\
2c\alpha - \alpha &< 2c - 1 \Leftrightarrow \\
\alpha(2c-1) &< 2c - 1 \Leftrightarrow \alpha < 1
\end{aligned}$$

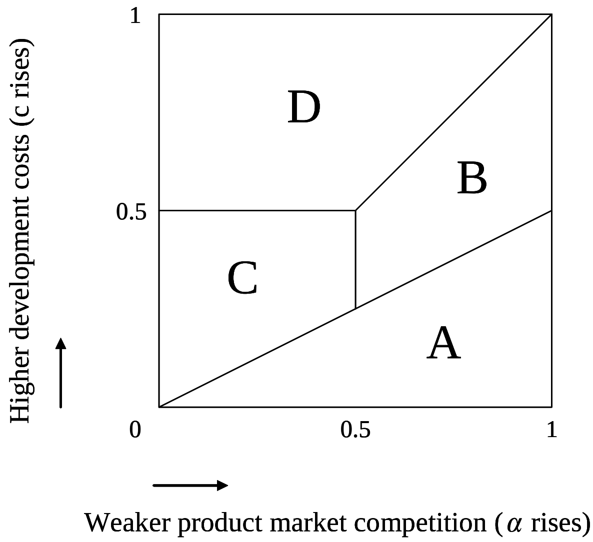
which always holds by assumption. We can further show $\overline{x}_b^* \geq 1 \Leftrightarrow \alpha \leq \frac{1}{2}$ as follows:

$$\begin{aligned}
\frac{2c}{1-(2-2c)(1-\alpha)} - 1 &\geq 1 \Leftrightarrow \\
2c &\geq 2\left[1 - (2-2c)(1-\alpha)\right] \Leftrightarrow \\
c &\geq 1 - (2 - 2\alpha - 2c + 2c\alpha) \Leftrightarrow \\
c &\geq 2\alpha - 1 - (2\alpha - 1)c + c \Leftrightarrow \\
0 &\geq (2\alpha - 1)(1 - c) \Leftrightarrow \alpha \leq \frac{1}{2}
\end{aligned}$$

Also, $\alpha \leq \frac{1}{2} \Rightarrow c > \alpha$ as $c > \frac{1}{2}$, which implies $\overline{\overline{x}} = \frac{2c}{\alpha} - 1 > 1$. In conclusion, where $\alpha > \frac{1}{2}$, $\overline{x}_b^* < 1$ so we have a contradiction and hence no PBE, but where $\alpha \leq \frac{1}{2}$ we have a PBE with $\overline{x}_b^* = \frac{2c}{1-(2-2c)(1-\alpha)} - 1 \in [1, \overline{\overline{x}})$. ■

In summary, the leader abandons the project unless intermediate results are good enough. There is an equilibrium in which it always discloses where it invests, irrespective of the level of spillovers - high costs ensure disclosure always puts the rival off. With sufficiently strong competition, there is also a second equilibrium in which the leader does not need to disclose because the follower's inferences are sufficiently pessimistic in the absence of disclosure.

4.5 Diagrammatic Representation of Results



Case (a): The leader always invests and never discloses. The follower also invests everywhere.

Case (b): For poor intermediate results (for $x_a \leq \widetilde{x}$ with $\widetilde{x} > 0$ and $\widetilde{x} > \overline{\overline{\overline{x}}}$), the leader abandons the project. For middling results, the leader invests and discloses. For good results the leader invests, relying on secrecy unless the level of knowledge spillovers are low. Following disclosure, the follower's investment threshold is $\overline{\overline{x}} < 1$, but if the leader fails to disclose it is in $[\overline{\overline{\overline{x}}}, \overline{\overline{x}})$. As spillovers rise, the leader uses more secrecy and the follower's threshold following no disclosure generally rises.

Case (c): The leader always invests and never discloses. The follower's investment threshold therefore equals $\overline{\overline{x}}$, forcing many if not all of the follower types out of the market.

Case (d): The leader invests for sufficiently good intermediate results (for $x_a > \overline{\overline{\overline{x}}} > 0$), always disclosing in that case. Following disclosure, the follower abandons the project. Absent disclosure, the follower's investment threshold is $\overline{\overline{\overline{x}}}$. If $\alpha \leq \frac{1}{2}$, there is a second equilibrium in which the leader makes the same investment decision, but never discloses, forcing all follower types out of the market.

Comparing these cases, we see that as product market competition rises, the leader tends to invest more often. Note that for a constant c, $\overline{\overline{\overline{x}}}$ does not change in α. Suppose $c > \frac{1}{2}$. Then as competition rises, we move from case (b), in which the leader invests for $x_a > \widetilde{x} > \overline{\overline{\overline{x}}} > 0$, to case (d), in which the leader invests for $x_a > \overline{\overline{\overline{x}}}$. Suppose $c \in (\frac{1}{4}, \frac{1}{2}]$. Then as competition rises we move from the uninteresting case (a), where both firms always invest regardless of the other firm's play, to case (b), where the leader only invests for $x_a > \widetilde{x} > 0$, and then to case (c), where the leader always invests. Where $c \in (0, \frac{1}{4}]$, we move from (a) to (c), so the leader always invests.

5 Conclusion

Disclosure of intermediate research results is a common phenomenon with many possible motivations. In this paper, we have analyzed the use of disclosure as a strategic deterrence device. In particular, we have looked at the incentives of a leading firm in a research competition to release its intermediate R&D results so as to use its first-mover advantage to try to induce a rival to exit the competition. Despite the risk that the disclosure may in fact help the rival to catch up (the information transmission effect), we have learned that for weak product market competition or for high development costs (cases (b) and (d)), disclosure will often be a profitable strategy because disclosure signals commitment to the research project (the signalling effect). We were also able to discover how the disclosure rule varies with the level of knowledge spillovers. With strong product market competition, on the other hand, disclosure is not employed so long as development costs are not too high (cases (a) and (c)). In case (c) where competition is strong and development costs are in an intermediate range, disclosure is not necessary in equilibrium as the follower's beliefs absent disclosure are sufficiently pessimistic that the leader can deter most (if not all) rival types without the need to actually release! We also found that in general, the leader invests more often where product market competition is stronger. The various cases were summarized diagrammatically in Section 4.5.

Our model is more general than the small amount of existing literature on the issue (Lichtman et al. (2000) and Jansen (2004(b)). In particular, we allow the firms' intermediate results to be drawn from a continuous distribution, allowing us to find a richer set of results, including in case (b) an intuitively appealing non-monotonicity of disclosure in the level of the intermediate results. Our model is also differentiated from the patent race literature more generally in that the firms are not competing for a unique prize of fixed value. Instead, the value of the prize from innovating depends stochastically on the level of the intermediate results and on whether the rival also innovates.

Appendix

Proof of Remark 1. If j is investing, i should invest if and only if

$$\begin{aligned} E[v_i|x_i]\,\alpha - c &\geq 0 \Leftrightarrow \left(\frac{1+x_i}{2}\right)\alpha \geq c \Leftrightarrow \\ x_i &\geq \frac{2c}{\alpha} - 1 \end{aligned}$$

Where $2c > \alpha$, $\frac{2c}{\alpha} - 1 > 0$ so i should invest if and only if $x_i \geq \overline{\overline{x}} = \frac{2c}{\alpha} - 1$. Where $2c \leq \alpha$, $\frac{2c}{\alpha} - 1 \leq 0$. Thus i always invests and $\overline{\overline{x}} = 0$. As $c \to \alpha$, $\overline{\overline{x}} \to 1$. Where $c \geq \alpha$, $\overline{\overline{x}} \geq 1$, so i never invests. ∎

Proof of Remark 2. If j is not investing, i should invest if and only if

$$\begin{aligned} E[v_i|x_i] - c &\geq 0 \Leftrightarrow \left(\frac{1+x_i}{2}\right) \geq c \Leftrightarrow \\ x_i &\geq 2c - 1 \end{aligned}$$

Where $c > \frac{1}{2}$, $2c - 1 \in (0,1)$, so i should invest if and only if $x_i \geq \overline{\overline{\overline{x}}} = 2c - 1$. Where $c \leq \frac{1}{2}$, $2c - 1 \leq 0$. Thus i always invests and $\overline{\overline{\overline{x}}} = 0$. As $c \to 1$, $\overline{\overline{\overline{x}}} \to 1$. ∎

Proof of Remark 3. B should invest if and only if

$$\begin{aligned} E[v_b|x_b]\{\rho\alpha + (1-\rho)\} - c &\geq 0 \Leftrightarrow \\ \left(\frac{1+x_b}{2}\right)\{1 - \rho(1-\alpha)\} - c &\geq 0 \Leftrightarrow \\ x_b &\geq \frac{2c}{\{1-\rho(1-\alpha)\}} - 1 \end{aligned}$$

Where $2c \geq 1 - \rho(1-\alpha)$, $\frac{2c}{\{1-\rho(1-\alpha)\}} - 1 \geq 0$ so B should invest if and only if $x_b \geq \overline{x}_b = \frac{2c}{\{1-\rho(1-\alpha)\}} - 1$. If $2c > 1 - \rho(1-\alpha)$, $\frac{2c}{\{1-\rho(1-\alpha)\}} - 1 > 0$, while if $2c = 1 - \rho(1-\alpha)$, $\frac{2c}{\{1-\rho(1-\alpha)\}} - 1 = 0$. Where $2c < 1 - \rho(1-\alpha)$, $\frac{2c}{\{1-\rho(1-\alpha)\}} - 1 < 0$. Thus B always invests and $\overline{x}_b = 0$. Referring back to Remarks 1 and 2, it is clear that for $\rho = 0$, $\overline{x}_b = \overline{\overline{\overline{x}}}$, and for $\rho = 1$, $\overline{x}_b = \overline{\overline{x}}$. Finally, $\overline{x}_b$ is increasing in ρ, so as ρ ranges from 0 to 1, $\overline{x}_b$ increases from $\overline{\overline{\overline{x}}}$ to $\overline{\overline{x}}$. ∎

Proof of Remark 5. For $\overline{x}_b \in [0,1]$, A should invest if and only if

$$\begin{aligned} E\left[v_a|x_a\right]\{(1-\overline{x}_b)\alpha+\overline{x}_b\}-c &\geq 0 \Leftrightarrow \\ \left(\frac{1+x_a}{2}\right)\{\alpha+\overline{x}_b(1-\alpha)\}-c &\geq 0 \Leftrightarrow \\ x_a &\geq \frac{2c}{\{\alpha+\overline{x}_b(1-\alpha)\}}-1 \end{aligned}$$

For $\overline{x}_b > 1$, the probability that B invests is 0, so A effectively faces $\overline{x}_b = 1$. Now

$$\frac{2c}{\{\alpha+\overline{x}_b(1-\alpha)\}}-1>0 \Leftrightarrow 2c-\alpha>\overline{x}_b(1-\alpha) \Leftrightarrow \overline{x}_b<\frac{2c-\alpha}{1-\alpha} \tag{2}$$

(i) $2c \leq \alpha \Rightarrow \frac{2c-\alpha}{1-\alpha} \leq 0 \Rightarrow \overline{x}_b \geq \frac{2c-\alpha}{1-\alpha}$ which implies $\frac{2c}{\{\alpha+\overline{x}_b(1-\alpha)\}}-1 \leq 0$ from (2). Thus, A always invests and $\overline{x}_a = 0$. From Remarks 1 and 2, given $2c \leq \alpha$, which implies $c \leq \frac{1}{2}, \overline{\overline{x}} = \overline{\overline{\overline{x}}} = 0$.

(ii) $2c \in (\alpha, 1]$, so $\frac{2c-\alpha}{1-\alpha} \in (0,1]$. For $\overline{x}_b \in [0, \frac{2c-\alpha}{1-\alpha})$, $\overline{x}_a = \frac{2c}{\{\alpha+\overline{x}_b(1-\alpha))\}}-1>0$ from (2). For $\overline{x}_b \geq \frac{2c-\alpha}{1-\alpha}, \frac{2c}{\{\alpha+\overline{x}_b(1-\alpha)\}}-1 \leq 0$ from (2), so A always invests and $\overline{x}_a = 0$. This equals $\overline{\overline{\overline{x}}}$ from Remark 2 given $c \leq \frac{1}{2}$.

(iii) For $c > \frac{1}{2}$, $2c > 1$, so $\frac{2c-\alpha}{1-\alpha} > 1$. Thus for $\overline{x}_b \in [0,1)$, $\overline{x}_b < \frac{2c-\alpha}{1-\alpha}$ and so $\overline{x}_a = \frac{2c}{\{\alpha+\overline{x}_b(1-\alpha))\}}-1>0$ from (2). For $\overline{x}_b \geq 1, \overline{x}_a = 2c-1>0$, which from Remark 2 equals $\overline{\overline{\overline{x}}}$ given $c > \frac{1}{2}$.

In all cases, for $\overline{x}_b \geq 1$, $\overline{x}_a = \overline{\overline{\overline{x}}}$ from above. For $\overline{x}_b = 0$, in case (i) $\overline{x}_a = \overline{\overline{x}}$ from above, and in cases (ii) and (iii), $\overline{x}_a = \frac{2c}{\alpha}-1$ which equals $\overline{\overline{x}}$ from Remark 1 given $2c > \alpha$. $\overline{x}_a$ is everywhere decreasing in $\overline{x}_b$, so as $\overline{x}_b$ ranges from 1 to 0, $\overline{x}_a$ increases from $\overline{\overline{\overline{x}}}$ to $\overline{\overline{x}}$. ■

Proof of Remark 6. We first note that $x_b < \overline{\overline{x}} \Leftrightarrow \widehat{x}_b < \overline{\overline{x}}$. $(\Rightarrow)$ If $x_a \leq x_b$, $\widehat{x}_b = x_b < \overline{\overline{x}}$.[37] If $x_a > x_b$, so $\widehat{x}_b > x_b$, then $\widehat{x}_b < x_a < \overline{\overline{x}}$, as $x_b < x_a \Rightarrow x_b\,(1-s) < x_a\,(1-s) \Rightarrow \widehat{x}_b = x_b + s(x_a - x_b) < x_a$. $(\Leftarrow)$ $x_b \leq \widehat{x}_b$.

Suppose (a) $x_b < \overline{x}_b$. Then whether the leader releases or not, the follower will not invest. $x_b < \overline{x}_b$, so absent release the follower fails to invest. Also, $x_b < \overline{\overline{x}}$ as from Remark 3, $\overline{x}_b \leq \overline{\overline{x}}$. Thus, $\widehat{x}_b < \overline{\overline{x}}$ from above, so the follower does not invest following

[37] Refer back to (1) for the rule governing the updating of intermediate results.

release either.

Suppose (b) $x_b \in [\overline{x}_b, \overline{\overline{x}})$. Because $x_b \geq \overline{x}_b$, the follower invests if the leader fails to release. Again, $x_b < \overline{\overline{x}} \Rightarrow \widehat{x}_b < \overline{\overline{x}}$, so following release, investment by the follower is deterred.

Suppose (c) $x_b \geq \overline{\overline{x}}$. Then the follower invests whether or not the leader releases. $\widehat{x}_b \geq x_b \geq \overline{\overline{x}}$ so the follower invests following release. Also, $x_b \geq \overline{x}_b$ as $\overline{\overline{x}} \geq \overline{x}_b$, so the follower also invests if the leader fails to release.

In cases (a) and (c) the release therefore does not alter the follower's investment decision, while in case (b), the follower's investment is deterred. Where (i) $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, $[\overline{x}_b, \overline{\overline{x}}) \cap (0,1)$ is of strictly positive length, so with strictly positive probability release deters investment and otherwise makes no difference. Where (ii) $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, $[\overline{x}_b, \overline{\overline{x}}) \cap (0,1) = \varnothing$, so the release makes no difference. ■

Proof of Claim 2. Remark 6 applies as $x_a < \overline{\overline{x}}$ ($x_a < \overline{x}_a$ here and $\overline{x}_a \leq \overline{\overline{x}}$ always).

Where (i) $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, Remark 6 tells us that the release deters investment by the follower with positive probability but never encourages extra investment. Absent any release and hence deterrence, investment is not optimal (as $x_a < \overline{x}_a$), so incurring the ε cost of release will be worthwhile if and only if post-release investment is strictly optimal. Following release, from Remark 4 the follower will invest if and only if $\widehat{x}_b \geq \overline{\overline{x}}$. But we noted in the proof of Remark 6 that for $x_a < \overline{\overline{x}}$, $x_b < \overline{\overline{x}} \Leftrightarrow \widehat{x}_b < \overline{\overline{x}}$, so B will invest if and only if $x_b \geq \overline{\overline{x}}$. Thus for $\overline{\overline{x}} \leq 1$, post-release investment is strictly optimal for A if and only if

$$\begin{aligned} E[v_a|x_a]\left\{\Pr\left[x_b \geq \overline{\overline{x}}\right]\alpha + \Pr\left[x_b < \overline{\overline{x}}\right]\right\} - c &> 0 \Leftrightarrow \\ \left(\frac{1+x_a}{2}\right)\left\{\left(1-\overline{\overline{x}}\right)\alpha + \overline{\overline{x}}\right\} &> c \Leftrightarrow \\ x_a &> \widetilde{x} = \frac{2c}{\alpha + \overline{\overline{x}}(1-\alpha)} - 1 \end{aligned}$$

Thus, for $x_a > \widetilde{x}$ the leader should incur the ε cost of release and then invest. Where $\overline{\overline{x}} > 1$, the follower's post-release probability of investment is zero, so $\widetilde{x} = 2c - 1$, the same as for $\overline{\overline{x}} = 1$.

Where (ii) $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, Remark 6 tells us that the release does not alter the follower's investment decision. Thus, the leader will not pay the cost of release, and will not invest as $x_a < \overline{x}_a$.

In case (i), $\overline{x}_a > \overline{\overline{\overline{x}}} \Rightarrow \overline{x}_a = \frac{2c}{\{\alpha+\overline{x}_b(1-\alpha))\}} - 1 > 0$ from Remark 5. Thus, $\widetilde{x} < \overline{x}_a$ as $\min(\overline{\overline{x}}, 1) > \overline{x}_b$. Also $\widetilde{x} < 1$. We've just shown $\widetilde{x} < \overline{x}_a$ and $\overline{x}_a \leq \overline{\overline{x}}$ always, so if $\overline{\overline{x}} \leq 1$, $\widetilde{x} < 1$. If $\overline{\overline{x}} > 1$, $\widetilde{x} = 2c - 1 < 1$. So the range in which the leader releases and invests, $x_a \in (\widetilde{x}, \overline{x}_a) \cap (0, 1)$, is non-empty. Clearly, $\frac{\partial \widetilde{x}}{\partial \overline{\overline{x}}} \leq 0$.

Next we prove $\max(c, \alpha) \leq \frac{1}{2} \Rightarrow \widetilde{x} \leq \overline{\overline{\overline{x}}} = 0$ and $\max(c, \alpha) > \frac{1}{2}$ implies $\widetilde{x} > \overline{\overline{\overline{x}}}$ for $\overline{\overline{x}} < 1$ and $\widetilde{x} = \overline{\overline{\overline{x}}}$ for $\overline{\overline{x}} \geq 1$.

First, suppose $\overline{\overline{x}} \geq 1$, so $c \geq \alpha$ and $\widetilde{x} = 2c - 1$. $\max(c, \alpha) \leq \frac{1}{2} \Rightarrow c \leq \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 0$. Thus $\widetilde{x} \leq \overline{\overline{\overline{x}}}$ as $2c - 1 \leq 0$. $\max(c, \alpha) > \frac{1}{2} \Rightarrow c > \frac{1}{2}$ as $c \geq \alpha$. Thus $\overline{\overline{\overline{x}}} = 2c - 1 = \widetilde{x}$.

Second, suppose $\overline{\overline{x}} < 1$, so $c < \alpha$ and $\widetilde{x} = \frac{2c}{\alpha+\overline{\overline{x}}(1-\alpha)} - 1$. $c \leq \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 0$. Thus, where $c \leq \frac{1}{2}$, $\widetilde{x} \leq \overline{\overline{\overline{x}}} \Leftrightarrow \frac{2c}{\alpha+\overline{\overline{x}}(1-\alpha)} - 1 \leq 0 \Leftrightarrow \frac{2c-\alpha}{1-\alpha} \leq \overline{\overline{x}}$. But $\overline{\overline{x}} > 0$ as $0 = \overline{\overline{\overline{x}}} \leq x_a < \overline{x}_a$ here and $\overline{x}_a \leq \overline{\overline{x}}$ always, so we can replace $\overline{\overline{x}}$ with $\frac{2c}{\alpha} - 1$ to give $\widetilde{x} \leq \overline{\overline{\overline{x}}} \Leftrightarrow \frac{2c-\alpha}{1-\alpha} \leq \frac{2c-\alpha}{\alpha} \Leftrightarrow \alpha \leq \frac{1}{2}$. Thus $\max(c, \alpha) \leq \frac{1}{2} \Rightarrow \widetilde{x} \leq \overline{\overline{\overline{x}}} = 0$ and if $c \leq \frac{1}{2}$ but $\alpha > \frac{1}{2}$, $\widetilde{x} > \overline{\overline{\overline{x}}}$. To show $\max(c, \alpha) > \frac{1}{2} \Rightarrow \widetilde{x} > \overline{\overline{\overline{x}}}$ we need to further show $c > \frac{1}{2} \Rightarrow \widetilde{x} > \overline{\overline{\overline{x}}}$. Where $c > \frac{1}{2}$, $\overline{\overline{\overline{x}}} = 2c - 1$. Thus $\widetilde{x} > \overline{\overline{\overline{x}}} \Leftrightarrow \frac{2c}{\alpha+\overline{\overline{x}}(1-\alpha)} - 1 > 2c - 1 \Leftrightarrow 1 > \alpha + \overline{\overline{x}}(1 - \alpha) \Leftrightarrow 1 > \overline{\overline{x}}$ which we have supposed here.

Clearly as $\overline{\overline{x}} \to 1$ from below, $\widetilde{x} \to \overline{\overline{\overline{x}}}$ from above, so where $\max(c, \alpha) > \frac{1}{2}$ the release and investment range for $\overline{\overline{x}} < 1$ tends to that for $\overline{\overline{x}} \geq 1$. ■

Proof of Claim 3. Where (i) $\overline{x}_b < \min(\overline{\overline{x}}, 1)$, from Remark 6 release deters investment with strictly positive probability and otherwise makes no difference. Before the information release, investment was profitable for A, as $x_a \geq \overline{x}_a$. Thus, after the release it must be even more so as for some types investment is deterred. Thus A will want to incur the ε cost of information release and invest.

Where (ii) $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$, from Remark 6 the release makes no difference to the follower's investment decision. Hence the leader will not incur the the ε cost of release, but still invests as $x_a \geq \overline{x}_a$. ■

Proof of Claim 4. Where $x_b \geq \overline{\overline{x}}$, the follower invests whether or not the leader releases. $\widehat{x}_b \geq x_b \geq \overline{\overline{x}}$ so the follower invests following release. Also, $x_b \geq \overline{x}_b$ as $\overline{\overline{x}} \geq \overline{x}_b$, so the follower also invests if the leader fails to release.

Where $x_b < \overline{\overline{x}}$, release may deter or encourage investment. If the leader does not release, the follower invests if and only if $x_b \geq \overline{x}_b$. Following release, $\widehat{x}_b = x_b + s(x_a - x_b) > x_b$ as $x_b < \overline{\overline{x}} \leq x_a$. Thus, the follower invests if and only if

$$\begin{aligned} \widehat{x}_b &= x_b + s(x_a - x_b) \geq \overline{\overline{x}} \Leftrightarrow \\ x_b(1-s) &\geq \overline{\overline{x}} - sx_a \Leftrightarrow \\ x_b &\geq \frac{\overline{\overline{x}} - sx_a}{1-s} \end{aligned}$$

Where $\frac{\overline{\overline{x}}-sx_a}{1-s} > \overline{x}_b$, the release deters innovation in the range of strictly positive length $x_b \in [\overline{x}_b, \frac{\overline{\overline{x}}-sx_a}{1-s}) \cap (0,1)$,[38] but makes no difference for the other possible values of x_b. Where $\frac{\overline{\overline{x}}-sx_a}{1-s} < \overline{x}_b$, the release encourages innovation which would not have otherwise occurred in the range $x_b \in [\frac{\overline{\overline{x}}-sx_a}{1-s}, \overline{x}_b) \cap (0,1)$, but makes no difference for the other possible values of x_b. Where $\frac{\overline{\overline{x}}-sx_a}{1-s} = \overline{x}_b$, the release makes no difference.

Before any information release, investment is profitable for A, as $x_a \geq \overline{\overline{x}} \geq \overline{x}_a$. Thus incurring the ε cost of release is worthwhile if and only if $\frac{\overline{\overline{x}}-sx_a}{1-s} > \overline{x}_b$, so that the release deters innovation with strictly positive probability but never encourages it, making investment strictly more profitable. From above, where $\frac{\overline{\overline{x}}-sx_a}{1-s} \leq \overline{x}_b$ the release never deters investment by the follower and may encourage it, so release is not worthwhile. Thus, the leader will release if and only if

$$\begin{aligned} \frac{\overline{\overline{x}} - sx_a}{1-s} &> \overline{x}_b \Leftrightarrow \\ \overline{\overline{x}} - sx_a &> \overline{x}_b(1-s) \Leftrightarrow \\ x_a &< \widetilde{\widetilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s} \end{aligned}$$

Now, $\widetilde{\widetilde{x}} > \overline{\overline{x}} \Leftrightarrow \frac{\overline{\overline{x}}-\overline{x}_b(1-s)}{s} > \overline{\overline{x}} \Leftrightarrow \overline{\overline{x}} > \overline{x}_b$. Thus for $\overline{x}_b < \overline{\overline{x}}$, $\widetilde{\widetilde{x}} > \overline{\overline{x}}$ so the leader must release for some non-empty range $x_a \in [\overline{\overline{x}}, \widetilde{\widetilde{x}}) \cap [\overline{\overline{x}}, 1)$. For $\overline{x}_b = \overline{\overline{x}}$, $\widetilde{\widetilde{x}} = \overline{\overline{x}}$, so the leader never releases (release can only encourage but never deter investment). In both cases, the leader always invests. It will clearly do so following release, and even if does not

[38] Note $\frac{\overline{\overline{x}}-sx_a}{1-s} \leq \overline{\overline{x}}$ as $x_a \geq \overline{\overline{x}}$.

release it still invests as $x_a \geq \overline{x}_a$.

$\frac{\partial \tilde{\tilde{x}}}{\partial \overline{x}_b} = \frac{-(1-s)}{s} < 0$.

In case (i), $\frac{\partial \tilde{\tilde{x}}}{\partial s} = -\overline{\overline{x}} s^{-2} + \overline{x}_b s^{-2} < 0$ as $\overline{x}_b < \overline{\overline{x}}$. As $s \to 1$, $\tilde{\tilde{x}} \to \overline{\overline{x}}$. As $s \to 0$, $\tilde{\tilde{x}} \to \infty$. Furthermore

$$\begin{aligned} \tilde{\tilde{x}} &= \frac{\overline{\overline{x}} - \overline{x}_b (1-s)}{s} \geq 1 \Leftrightarrow \\ \overline{\overline{x}} - \overline{x}_b + \overline{x}_b s &\geq s \Leftrightarrow s(1-\overline{x}_b) \leq \overline{\overline{x}} - \overline{x}_b \Leftrightarrow \\ s &\leq \tilde{s} = \frac{\overline{\overline{x}} - \overline{x}_b}{1 - \overline{x}_b} \end{aligned}$$

Note $\tilde{s} \in (0,1)$ as $\overline{\overline{x}} < 1$, and $\overline{\overline{x}} > \overline{x}_b$ in case (i). ■

Proof of Remark 7. From Remark 1, $2c > \alpha \Rightarrow \overline{\overline{x}} = \frac{2c-\alpha}{\alpha} > 0$ and $c < \alpha \Rightarrow \overline{\overline{x}} < 1$. From Remark 5, $\overline{\overline{x}} \geq \overline{x}_a$ always.

From Remark 3, $\overline{x}_b \leq \overline{\overline{x}}$. Also, $\alpha > \frac{1}{2} \Rightarrow \alpha > 1 - \alpha \Rightarrow \overline{\overline{x}} = \frac{2c-\alpha}{\alpha} < \frac{2c-\alpha}{1-\alpha}$, so $\overline{x}_b < \frac{2c-\alpha}{1-\alpha}$. Then if $c \leq \frac{1}{2}$, so $2c \in (\alpha, 1]$, from Remark 5 $\overline{x}_a > 0$. But from Remark 2, $c \leq \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 0$, so $\overline{x}_a > \overline{\overline{\overline{x}}} = 0$. If $c > \frac{1}{2}$, then from Remark 5 $\overline{x}_a = \frac{2c}{\{\alpha + \overline{x}_b(1-\alpha))\}} - 1$ as $\overline{x}_b \leq \overline{\overline{x}} < 1$. From Remark 2, $c > \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 2c - 1 > 0$. So $c > \frac{1}{2} \Rightarrow \overline{x}_a > \overline{\overline{\overline{x}}} > 0$ as $\overline{x}_b < 1$ ($\overline{x}_a$ is strictly decreasing in $\overline{x}_b$ and equals $2c - 1$ at $\overline{x}_b = 1$). ■

Proof of Remark 8. $\overline{x}_b = \overline{\overline{x}} \Rightarrow \overline{x}_b \geq \min(\overline{\overline{x}}, 1)$ which from Claims 1 to 4 and Remark 5 implies that the leader never releases and invests if and only if $x_a \geq \overline{x}_a$. Thus, if the follower sees no release, it infers $\rho^* = 1 - \overline{x}_a$. But $\overline{x}_a > 0$ from Remark 7, so $\rho^* < 1$. From Remark 3, $\overline{x}_b^* = \frac{2c}{\{1-\rho^*(1-\alpha)\}} - 1$ (which is strictly increasing in ρ^* and equals $\frac{2c}{\alpha} - 1$ at $\rho^* = 1$) or 0, so $\rho^* < 1 \Rightarrow \overline{x}_b^* < \overline{\overline{x}}$ given $\overline{\overline{x}} = \frac{2c}{\alpha} - 1 > 0$ (from Remark 7). ■

Proof of Claim 5. $\overline{x}_b < \overline{\overline{x}}$ together with $\overline{\overline{x}} < 1$ from Remark 7 imply $\overline{x}_b < \min(\overline{\overline{x}}, 1)$.

From Claim 1, the leader neither releases nor invests for $x_a < \overline{\overline{\overline{x}}}$.

From Remark 7 the range $[\overline{\overline{\overline{x}}}, \overline{x}_a)$ is non-empty. Where $x_a \in [\overline{\overline{\overline{x}}}, \overline{x}_a)$, from Claim 2 and the fact that $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ and $\overline{\overline{x}} < 1$ the leader releases and invests if $x_a > \tilde{x} = \frac{2c}{\alpha + \overline{\overline{x}}(1-\alpha)} - 1$, and otherwise neither releases nor invests. Claim 2 also tells us that

$\widetilde{x} < \overline{x}_a$ and, using $\alpha > \frac{1}{2} \Rightarrow \max(c, \alpha) > \frac{1}{2}$, that $\widetilde{x} > \overline{\overline{\overline{x}}}$, which further implies $\widetilde{x} > 0$ as $\overline{\overline{\overline{x}}} \geq 0$.

Claim 3 together with $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ imply that the leader continues to release and invest for $x_a \in [\overline{x}_a, \overline{\overline{x}})$.

$\overline{\overline{x}} < 1$ implies the range $x_a \in [\overline{\overline{x}}, 1)$ is non-empty. Claim 4 together with $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ imply that in this range the leader releases if and only if $x_a < \widetilde{\widetilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s}$, but always invests, and that $\widetilde{\widetilde{x}} > \overline{\overline{x}}$, which is always weakly greater than $\overline{x}_a$.

$\frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b}$ and $\frac{\partial \widetilde{\widetilde{x}}}{\partial s} < 0$ from Claim 4. ■

Proof of Remark 9. First we show that $\widetilde{\widetilde{x}} \lesseqgtr 1 \Leftrightarrow \overline{x}_b \gtreqless \frac{\overline{\overline{x}} - s}{1-s}$:

$$\widetilde{\widetilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b\,(1-s)}{s} \lesseqgtr 1 \Leftrightarrow \overline{\overline{x}} - \overline{x}_b\,(1-s) \lesseqgtr s \Leftrightarrow \overline{x}_b \gtreqless \frac{\overline{\overline{x}} - s}{1-s}$$

Where $\widetilde{\widetilde{x}} < 1$, $\rho^* = \frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}}$ as the probability that the leader invests if it does not release is the ratio of the range of x_a for which it invests if it does not release (from $\widetilde{\widetilde{x}}$ to 1) to the range of x_a for which it does not release (from 0 to $\widetilde{x}$ and from $\widetilde{\widetilde{x}}$ to 1). But $\widetilde{x} > 0$ from Claim 5, so $\frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}} \in (0, 1)$ for $\widetilde{\widetilde{x}} < 1$. Where $\widetilde{\widetilde{x}} \geq 1$, the probability of investment following a failure to release is zero, as the leader only releases for $x_a < \widetilde{x}$ where it does not intend to invest. Thus $\rho^* = 0$. Note that at $\widetilde{\widetilde{x}} = 1$, $\rho^* = 0 = \frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}}$.

Now $\frac{d\rho^*}{d\overline{x}_b} = \frac{\partial \rho^*}{\partial \widetilde{\widetilde{x}}} . \frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b}$. For $\widetilde{\widetilde{x}} < 1$

$$\begin{aligned} \frac{\partial \rho^*}{\partial \widetilde{\widetilde{x}}} &= (1-\widetilde{\widetilde{x}})(-1)(1-\widetilde{\widetilde{x}}+\widetilde{x})^{-2}(-1) + (1-\widetilde{\widetilde{x}}+\widetilde{x})^{-1}(-1) \\ &= \frac{(1-\widetilde{\widetilde{x}}) - (1-\widetilde{\widetilde{x}}+\widetilde{x})}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2} = \frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2} < 0 \end{aligned}$$

as $\widetilde{x} > 0$ and $\widetilde{\widetilde{x}} \leq 1$. Also, from Claim 5 $\frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b} = \frac{-(1-s)}{s} < 0$. Thus $\frac{d\rho^*}{d\overline{x}_b} = \frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2} . \frac{-(1-s)}{s} > 0$. Where $\widetilde{\widetilde{x}} = 1$ then $\rho^* = \frac{1-\widetilde{\widetilde{x}}}{1-\widetilde{\widetilde{x}}+\widetilde{x}}$ at $\widetilde{\widetilde{x}} = 1$ and as $\widetilde{\widetilde{x}}$ falls below 1, so $\left(\frac{\partial \rho^*}{\partial \widetilde{\widetilde{x}}}\right)^- = \frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2} < 0$ and hence $\left(\frac{d\rho^*}{d\overline{x}_b}\right)^+ = \frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2} . \frac{-(1-s)}{s} > 0$. ■

Proof of Remark 10. From Claim 5, $\widetilde{\widetilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s}$. Thus as $s \to 1$ or $\overline{x}_b \to \overline{\overline{x}}$, $\widetilde{\widetilde{x}} \to \overline{\overline{x}}$. As $\overline{\overline{x}} < 1$ from Remark 7, $\rho^* \to \frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}$ using Remark 9. Thus $1 - \rho^*(1-\alpha) \to 1 - \left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha)$. We can show that $2c > 1 - \left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha)$ as follows, noting that $\overline{\overline{x}} = \frac{2c-\alpha}{\alpha} > 0$ from Remark 7, that $\widetilde{x} = \frac{2c}{\alpha+\overline{\overline{x}}(1-\alpha)} - 1 > 0$ from Claim 5, and that $(1-\overline{\overline{x}}+\widetilde{x}) > 0$ as $\widetilde{x} > 0$ and $\overline{\overline{x}} < 1$:

$$\begin{aligned}
2c &> 1 - \left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha) \Leftrightarrow \\
(1-\overline{\overline{x}})(1-\alpha) &> (1-\overline{\overline{x}}+\widetilde{x})(1-2c) \Leftrightarrow \\
(1-\overline{\overline{x}})\left[(1-\alpha)-(1-2c)\right] &> \widetilde{x}(1-2c) \Leftrightarrow \\
(1-\overline{\overline{x}})(2c-\alpha) &> \left[\frac{2c}{\alpha+\overline{\overline{x}}(1-\alpha)} - 1\right](1-2c) \Leftrightarrow \\
(1-\overline{\overline{x}})(2c-\alpha)[\alpha+\overline{\overline{x}}(1-\alpha)] &> \left[2c-\alpha-\overline{\overline{x}}(1-\alpha)\right](1-2c) \Leftrightarrow \\
\left[1-\left(\frac{2c-\alpha}{\alpha}\right)\right](2c-\alpha)[\alpha+\left(\frac{2c-\alpha}{\alpha}\right)(1-\alpha)] &> \left[(2c-\alpha)-\left(\frac{2c-\alpha}{\alpha}\right)(1-\alpha)\right](1-2c) \Leftrightarrow \\
\left[\frac{2\alpha-2c}{\alpha}\right](2c-\alpha)[\frac{\alpha^2+2c-\alpha-2c\alpha+\alpha^2}{\alpha}] &> \left(\frac{2c-\alpha}{\alpha}\right)[\alpha-(1-\alpha)](1-2c) \Leftrightarrow \\
(2\alpha-2c)[2\alpha^2+2c-\alpha-2c\alpha] &> \alpha[2\alpha-1](1-2c) \Leftrightarrow \\
4\alpha^3+4c\alpha-2\alpha^2-4c\alpha^2-4c\alpha^2-4c^2+2c\alpha+4c^2\alpha &> \alpha(2\alpha-1-4c\alpha+2c) \Leftrightarrow \\
4\alpha^3+6c\alpha-2\alpha^2-8c\alpha^2-4c^2+4c^2\alpha &> 2\alpha^2-\alpha-4c\alpha^2+2c\alpha \Leftrightarrow \\
4\alpha^3+4c\alpha-4\alpha^2-4c\alpha^2-4c^2+4c^2\alpha+\alpha &> 0 \Leftrightarrow \\
4\alpha^3-4\alpha^2+\alpha+4c\alpha-4c^2-4c\alpha^2+4c^2\alpha &> 0 \Leftrightarrow \\
\alpha(4\alpha^2-4\alpha+1)+4c(\alpha-c-\alpha^2+c\alpha) &> 0 \Leftrightarrow \\
\alpha(2\alpha-1)^2+4c(1-\alpha)(\alpha-c) &> 0
\end{aligned}$$

which must be true as $\alpha > c$ by assumption.

From Claim 5 $\frac{\partial\widetilde{\widetilde{x}}}{\partial\overline{x}_b} < 0$, so $\widetilde{\widetilde{x}}$ is decreasing in $\overline{x}_b$. From Remark 9 ρ^* is weakly increasing in $\overline{x}_b$, so $1-\rho^*(1-\alpha)$ is weakly decreasing $\overline{x}_b$. Thus $\widetilde{\widetilde{x}}$ and $1-\rho^*(1-\alpha)$ are decreasing in $\overline{x}_b$, tending to a limit below 1 and $2c$ respectively as $\overline{x}_b \to \overline{\overline{x}}$. It follows that $\exists\widetilde{\overline{x}}_b \in [\overline{\overline{\overline{x}}}, \overline{x})$ such that for $\overline{x}_b \in [\overline{\overline{\overline{x}}}, \overline{x})$, $\widetilde{\widetilde{x}} \leq 1$ and $2c \geq 1-\rho^*(1-\alpha)$ if and only if $\overline{x}_b \geq \widetilde{\overline{x}}_b$. Note that this argument depends on the continuity of $\widetilde{\widetilde{x}}$ and ρ^* in $\overline{x}_b$. Clearly, given this continuity, unless we are at a boundary point with $\widetilde{\overline{x}}_b = \overline{\overline{\overline{x}}}$, then at least one of the inequalities must hold with equality at $\overline{x}_b = \widetilde{\overline{x}}_b$. Furthermore $\frac{\partial\widetilde{\widetilde{x}}}{\partial\overline{x}_b} < 0 \Rightarrow \widetilde{\widetilde{x}} < 1$ $\forall\overline{x}_b > \widetilde{\overline{x}}_b$. Also from Remark 9, $\left(\frac{d\rho^*}{d\overline{x}_b}\right)^+ > 0$ at $\widetilde{\widetilde{x}} = 1$ and $\left(\frac{d\rho^*}{d\overline{x}_b}\right) > 0$ for $\widetilde{\widetilde{x}} < 1$, so $1-\rho^*(1-\alpha)$ strictly falls as $\overline{x}_b$ rises beyond $\widetilde{\overline{x}}_b$. Thus $2c > 1-\rho^*(1-\alpha)$ $\forall\overline{x}_b > \widetilde{\overline{x}}_b$. ■

Proof of Remark 11. Suppose that $\overline{x}_b > \widetilde{\widetilde{x}}_b$. From Remark 10 $\widetilde{\widetilde{x}} < 1$ and $2c > 1 - \rho^*(1-\alpha)$. Using Remark 3 and $2c > 1 - \rho^*(1-\alpha)$, $\overline{x}_b^* = \frac{2c}{[1-\rho^*(1-\alpha)]} - 1$. Now $\frac{\partial \overline{x}_b^*}{\partial \rho^*} = 2c(-1)[1-\rho^*(1-\alpha)]^{-2}(-1)(1-\alpha) > 0$ as $\rho^* \in [0,1]$. Also, $\frac{d\rho^*}{d\overline{x}_b} = \frac{\partial \rho^*}{\partial \widetilde{\widetilde{x}}}.\frac{\partial \widetilde{\widetilde{x}}}{\partial \overline{x}_b} = \frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2}.\frac{-(1-s)}{s} > 0$ from Remark 9 and its proof. Thus $\frac{d\overline{x}_b^*}{d\overline{x}_b} = \frac{\partial \overline{x}_b^*}{\partial \rho^*}.\frac{d\rho^*}{d\overline{x}_b} > 0$. Furthermore $\frac{d^2\overline{x}_b^*}{d(\overline{x}_b)^2} < 0$:

$$\begin{aligned}\frac{d^2\overline{x}_b^*}{d(\overline{x}_b)^2} &= \frac{d\left(\frac{2c(1-\alpha)}{[1-\rho^*(1-\alpha)]^2}.\frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2}.\frac{-(1-s)}{s}\right)}{d\overline{x}_b} \\ &= 2c(1-\alpha).\widetilde{x}.\frac{(1-s)}{s}.(-2)\left\{[1-\rho^*(1-\alpha)](1-\widetilde{\widetilde{x}}+\widetilde{x})\right\}^{-3}.\frac{d\left\{[1-\rho^*(1-\alpha)](1-\widetilde{\widetilde{x}}+\widetilde{x})\right\}}{d\overline{x}_b}\end{aligned}$$

Note that $2c(1-\alpha).\widetilde{x}.\frac{(1-s)}{s}.(-2)\left\{[1-\rho^*(1-\alpha)](1-\widetilde{\widetilde{x}}+\widetilde{x})\right\}^{-3} < 0$ as $\widetilde{x}$, $[1-\rho^*(1-\alpha)]$ and $(1-\widetilde{\widetilde{x}}+\widetilde{x})$ are all strictly positive. Thus the sign of the second derivative will the same as the sign of $(-1).\frac{d\left\{[1-\rho^*(1-\alpha)](1-\widetilde{\widetilde{x}}+\widetilde{x})\right\}}{d\overline{x}_b}$, which equals

$$\begin{aligned}&(-1)\left\{(1-\widetilde{\widetilde{x}}+\widetilde{x}).(-1).(1-\alpha).\frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2}.\frac{-(1-s)}{s} + [1-\rho^*(1-\alpha)].(-1).\frac{-(1-s)}{s}\right\} \\ =\ &\frac{(1-s)}{s}.(1-\widetilde{\widetilde{x}}+\widetilde{x})^{-1}.\left\{\widetilde{x}.(1-\alpha) - [1-\rho^*(1-\alpha)].(1-\widetilde{\widetilde{x}}+\widetilde{x})\right\}\end{aligned}$$

But $\frac{(1-s)}{s}.(1-\widetilde{\widetilde{x}}+\widetilde{x})^{-1} > 0$, so this in turn has the same sign as

$$\begin{aligned}&\widetilde{x}.(1-\alpha) - [1-\rho^*(1-\alpha)].(1-\widetilde{\widetilde{x}}+\widetilde{x}) \\ =\ &\widetilde{x}\{(1-\alpha) - 1 + \rho^*(1-\alpha)\} - (1-\widetilde{\widetilde{x}})[1-\rho^*(1-\alpha)]\end{aligned}$$

But $\alpha > \frac{1}{2} \Rightarrow (1-\alpha) < \frac{1}{2}$, which also entails $\rho^*(1-\alpha) < \frac{1}{2}$. Thus $(1-\alpha) - 1 + \rho^*(1-\alpha) < 0$. As $\widetilde{x} > 0$, the first part of the expression is strictly negative. The second part is also negative as $\widetilde{\widetilde{x}} \leq 1$ and $[1-\rho^*(1-\alpha)] > 0$. Thus the above expression, and hence the second derivative, is strictly negative.

Suppose that $\overline{x}_b = \widetilde{\widetilde{x}}_b$. From Remark 10 $\widetilde{\widetilde{x}} \leq 1$ and $2c \geq 1-\rho^*(1-\alpha)$ $\forall \overline{x}_b \geq \widetilde{\widetilde{x}}_b$. From Remark 9 and its proof, $\left(\frac{d\rho^*}{d\overline{x}_b}\right)^+ = \frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2}.\frac{-(1-s)}{s} > 0$.[39] Using Remark 3 and $2c \geq 1 - \rho^*(1-\alpha)$, $\overline{x}_b^* = \frac{2c}{[1-\rho^*(1-\alpha)]} - 1$ for $\overline{x}_b \geq \widetilde{\widetilde{x}}_b$. Thus $\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+ = \frac{2c(1-\alpha)}{[1-\rho^*(1-\alpha)]^2}.\frac{-\widetilde{x}}{(1-\widetilde{\widetilde{x}}+\widetilde{x})^2}.\frac{-(1-s)}{s} >$

[39] Remembering that $\left(\frac{d\rho^*}{d\overline{x}_b}\right) = z \Rightarrow \left(\frac{d\rho^*}{d\overline{x}_b}\right)^+ = z$.

0 given we saw above that $\frac{\partial\left(\frac{2c}{[1-\rho^*(1-\alpha)]}-1\right)}{\partial\rho^*} = \frac{2c(1-\alpha)}{[1-\rho^*(1-\alpha)]^2} > 0$. Finally, the same argument that shows $\frac{d^2\overline{x}_b^*}{d(\overline{x}_b)^2} > 0$ for $\overline{x}_b > \tilde{\tilde{x}}_b$ now shows that $\left(\frac{d\left(\frac{d\overline{x}_b^*}{d\overline{x}_b}\right)^+}{d\overline{x}_b}\right)^+ < 0$, appropriately swapping $\left(\frac{d\rho^*}{d\overline{x}_b}\right)$ with $\left(\frac{d\rho^*}{d\overline{x}_b}\right)^+$.

Suppose that $\overline{x}_b < \tilde{\tilde{x}}_b$. From Remark 10 either $\tilde{\tilde{x}} > 1$ or $2c < 1-\rho^*(1-\alpha)$. If $\tilde{\tilde{x}} > 1$, then $\frac{d\rho^*}{d\overline{x}_b} = 0$ as $\rho^* = 0$ for $\tilde{\tilde{x}} > 1$ from Remark 9, so $\frac{d\overline{x}_b^*}{d\overline{x}_b} = 0$. If $2c < 1-\rho^*(1-\alpha)$, $\overline{x}_b^* = 0$ from Remark 3, so $\frac{d\overline{x}_b^*}{d\overline{x}_b} = 0$.

From Remark 10 as $\overline{x}_b \to \overline{\overline{x}}$, $1-\rho^*(1-\alpha) \to 1-\left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right](1-\alpha) < 2c$, which last entails $\overline{x}_b^* \to \frac{2c}{\left[1-\left(\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right)(1-\alpha)\right]} - 1$ from Remark 3. $\frac{2c}{\left[1-\left(\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}}\right)(1-\alpha)\right]} - 1 < \frac{2c}{\alpha} - 1 = \overline{\overline{x}}$ given $\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\widetilde{x}} < 1$ (as $\widetilde{x} > 0$). ■

Proof of Remark 12. $\frac{d\rho^*}{ds} = \frac{\partial\rho^*}{\partial\tilde{\tilde{x}}}.\frac{\partial\tilde{\tilde{x}}}{\partial s}$. From Claim 5, $\frac{\partial\tilde{\tilde{x}}}{\partial s} < 0$. From Remark 10 $\overline{x}_b > \tilde{\tilde{x}}_b$ implies that $\tilde{\tilde{x}} < 1$ and $2c > 1-\rho^*(1-\alpha)$. Given $\tilde{\tilde{x}} < 1$, $\frac{\partial\rho^*}{\partial\tilde{\tilde{x}}} < 0$ from Remark 9, so $\frac{d\rho^*}{ds} > 0$. By the same argument as in the proof of Remark 11, $\frac{\partial\overline{x}_b^*}{\partial\rho^*} > 0$ given the two conditions hold. Thus $\frac{d\overline{x}_b^*}{ds} = \frac{\partial\overline{x}_b^*}{\partial\rho^*}.\frac{d\rho^*}{ds} > 0$. If $\overline{x}_b = \tilde{\tilde{x}}_b = \overline{\overline{\overline{x}}}$ but we are in subcase (i), then again $\tilde{\tilde{x}} < 1$ and $2c > 1-\rho^*(1-\alpha)$ so the same argument applies.

Where $\overline{x}_b = \tilde{\tilde{x}}_b > \overline{\overline{\overline{x}}}$, from Remark 10 $\tilde{\tilde{x}} \leq 1$ and $2c \geq 1-\rho^*(1-\alpha)$, with at least one of these inequalities holding with equality. Again $\frac{\partial\tilde{\tilde{x}}}{\partial s} < 0$, and $\left(\frac{\partial\rho^*}{\partial\tilde{\tilde{x}}}\right)^- < 0$ (from Remark 9), so $\left(\frac{d\rho^*}{ds}\right)^+ > 0$ and hence $\left(\frac{d(1-\rho^*(1-\alpha))}{ds}\right)^+ < 0$. Thus as s rises, $2c \geq 1-\rho^*(1-\alpha)$ continues to hold, so from Remark 3 $\overline{x}_b^* = \frac{2c}{[1-\rho^*(1-\alpha)]} - 1$. Hence $\left(\frac{d\overline{x}_b^*}{ds}\right)^+ > 0$ as $1-\rho^*(1-\alpha)$ is falling. If $\tilde{\tilde{x}} = 1$, $\left(\frac{\partial\rho^*}{\partial\tilde{\tilde{x}}}\right)^+ = 0$ as $\rho^* = 0$ $\forall\tilde{\tilde{x}} \geq 1$ from Remark 9. Thus $\left(\frac{d\overline{x}_b^*}{ds}\right)^- = 0$. If $2c = 1-\rho^*(1-\alpha)$ but $\tilde{\tilde{x}} < 1$, $\left(\frac{\partial\rho^*}{\partial\tilde{\tilde{x}}}\right)^+ < 0$ from Remark 9, so $\left(\frac{d\rho^*}{ds}\right)^- > 0$. Thus as s goes down, $1-\rho^*(1-\alpha)$ rises above $2c$, so $\overline{x}_b^*$ stays at zero from Remark 3, i.e., $\left(\frac{d\overline{x}_b^*}{ds}\right)^- = 0$. If $\overline{x}_b = \tilde{\tilde{x}}_b = \overline{\overline{\overline{x}}}$ but we are in subcase (ii), then again $\tilde{\tilde{x}} \leq 1$ and $2c \geq 1-\rho^*(1-\alpha)$, with at least one of these inequalities holding with equality, so the same argument applies.

From Remark 10 $\overline{x}_b < \tilde{\tilde{x}}_b$ implies that $\tilde{\tilde{x}} > 1$ and $2c < 1-\rho^*(1-\alpha)$. Where $\tilde{\tilde{x}} > 1$, $\rho^* = 0$ from Remark 9, so $\frac{\partial\rho^*}{\partial\tilde{\tilde{x}}} = 0$ and hence $\frac{d\overline{x}_b^*}{ds} = 0$. Where $2c < 1-\rho^*(1-\alpha)$, $\overline{x}_b^* = 0$ from Remark 3, so $\frac{\partial\overline{x}_b^*}{\partial\rho^*} = 0$ and hence $\frac{d\overline{x}_b^*}{ds} = 0$. ■

Proof of Remark 13. From Remark 10 and Propositions 2 to 4, we can see that we need to show that $\exists s \in (0,1)$ such that at $\overline{x}_b = \overline{\overline{\overline{x}}}$: $\tilde{\tilde{x}} < 1$ and $2c > 1 - \rho^*(1-\alpha)$ if $s > \tilde{\tilde{s}}$; $\tilde{\tilde{x}} \le 1$ and $2c \ge 1 - \rho^*(1-\alpha)$ at $s = \tilde{\tilde{s}}$, with at least one of the two inequalities holding with equality; and finally $\tilde{\tilde{x}} > 1$ or $2c < 1 - \rho^*(1-\alpha)$ if $s < \tilde{\tilde{s}}$.

From Claim 5 $\tilde{\tilde{x}} = \frac{\overline{\overline{x}} - \overline{\overline{\overline{x}}}(1-s)}{s}$ at $\overline{x}_b = \overline{\overline{\overline{x}}}$. Thus $\tilde{\tilde{x}} \to \infty$ as $s \to 0$ and $\tilde{\tilde{x}} \to \overline{\overline{x}} < 1$ as $s \to 1$. Also from Claim 5 $\frac{\partial \tilde{\tilde{x}}}{\partial s} < 0$. Hence $\exists s_1 \in (0,1)$ such that $\tilde{\tilde{x}} = 1$ at $s = s_1$, $\tilde{\tilde{x}} < 1$ for $s > s_1$ and $\tilde{\tilde{x}} < 1$ for $s < s_1$. Now $\left(\frac{d(1-\rho^*(1-\alpha))}{ds}\right)^+ < 0\ \forall s \ge s_1$ as we have already noted that $\frac{\partial \tilde{\tilde{x}}}{\partial s} < 0$, and $\left(\frac{\partial \rho^*}{\partial \tilde{\tilde{x}}}\right)^- < 0$ for $\tilde{\tilde{x}} \le 1$ from Remark 9. Hence if at s_1, $2c \ge 1 - \rho^*(1-\alpha)$, then $2c > 1 - \rho^*(1-\alpha)\ \forall s > s_1$, and in that case $\tilde{\tilde{s}} = s_1$. If at s_1, $2c < 1 - \rho^*(1-\alpha)$, we know from Remark 10 that $1 - \rho^*(1-\alpha) \to 1 - \left[\frac{1-\overline{\overline{x}}}{1-\overline{\overline{x}}+\tilde{x}}\right](1-\alpha) < 2c$ as $s \to 1$, and from above $\left(\frac{d(1-\rho^*(1-\alpha))}{ds}\right)^+ < 0\ \forall s \ge s_1$. Thus $\exists s_2 \in (s_1, 1)$ such that $2c = 1 - \rho^*(1-\alpha)$ for $s = s_2$, $2c > 1 - \rho^*(1-\alpha)$ for $s > s_2$ and $2c < 1 - \rho^*(1-\alpha)$ for $s \in [s_1, s_2)$, so $\tilde{\tilde{s}} = s_2$. ■

Proof of Remark 15. Suppose first that $\overline{x}_b \ge \min(\overline{\overline{x}}, 1)$. As $\overline{x}_b \le \overline{\overline{x}}$ always, either $\overline{x}_b = \overline{\overline{x}}$ or $\overline{x}_b \in [1, \overline{\overline{x}})$. If $\overline{x}_b = \overline{\overline{x}}$, from Remark 14 $\overline{x}_b = \frac{2c-\alpha}{\alpha}$. $\alpha \le \frac{1}{2} \Rightarrow \alpha \le 1-\alpha \Rightarrow \frac{2c-\alpha}{\alpha} \ge \frac{2c-\alpha}{1-\alpha}$, so $\overline{x}_b \ge \frac{2c-\alpha}{1-\alpha}$. If $\overline{x}_b \in [1, \overline{\overline{x}})$, then again $\overline{x}_b \ge \frac{2c-\alpha}{1-\alpha}$ as $\frac{2c-\alpha}{1-\alpha} \le 1 \Leftrightarrow c \le \frac{1}{2}$. So in either case, from Remark 5 together with $c \in (\frac{\alpha}{2}, \frac{1}{2}]$, $\overline{x}_a = 0$. Finally, from Claims 3 and 4 together with $\overline{x}_b \ge \min(\overline{\overline{x}}, 1)$ and $\overline{x}_a = 0$, the leader never releases but always invests.

Suppose next that $\overline{x}_b < \min(\overline{\overline{x}}, 1)$. If $\overline{x}_a = 0$, Claims 3 and 4 then tell us that for $\overline{\overline{x}} < 1$ the leader always invests, and releases if and only if $x_a < \tilde{\tilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s}$, while for $\overline{\overline{x}} \ge 1$ the leader always invests and releases. If $\overline{x}_a > 0$, so $\overline{x}_a > \overline{\overline{\overline{x}}} = 0$, then $\tilde{x} \le 0$ from Claim 2, using $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ and $\max(c, \alpha) \le \frac{1}{2}$. Then Claims 2 to 4 again give us the result. ■

Proof of Claim 6. Suppose $\overline{x}_b \in (\frac{\overline{\overline{x}}-s}{1-s}, \overline{\overline{x}})$. $\overline{x}_b < \overline{\overline{x}}$ and $\overline{\overline{x}} < 1$ imply $\overline{x}_b < \min(\overline{\overline{x}}, 1)$. $\overline{x}_b > \frac{\overline{\overline{x}}-s}{1-s}$, so $s > \overline{\overline{x}} - \overline{x}_b(1-s)$ and hence $\tilde{\tilde{x}} = \frac{\overline{\overline{x}} - \overline{x}_b(1-s)}{s} < 1$. Thus from Remark 15 the leader always invests, and releases if and only if $x_a < \tilde{\tilde{x}} < 1$. Observing no release, the follower infers that $x_a \in (\tilde{\tilde{x}}, 1)$ and that the leader is still investing, so $\rho^* = 1$ and hence $\overline{x}_b^* = \overline{\overline{x}}$, leading to a contradiction. Thus we cannot have a PBE. In particular, if $s > \overline{\overline{x}}$, $\frac{\overline{\overline{x}}-s}{1-s} < 0$, so $\overline{x}_b \in (\frac{\overline{\overline{x}}-s}{1-s}, \overline{\overline{x}})\ \forall \overline{x}_b \ne \overline{\overline{x}}$, and hence $\overline{x}_b = \overline{\overline{x}}$ is the unique PBE.

Suppose $\overline{x}_b \in \left[0, \frac{\overline{\overline{x}}-s}{1-s}\right]$. As noted above, $\frac{\overline{\overline{x}}-s}{1-s} < \overline{\overline{x}}$, so this entails $\overline{x}_b < \overline{\overline{x}}$, which together with $\overline{\overline{x}} < 1$ gives $\overline{x}_b < \min(\overline{\overline{x}}, 1)$. Now $\overline{x}_b \leq \frac{\overline{\overline{x}}-s}{1-s} \Rightarrow s \leq \overline{\overline{x}} - \overline{x}_b\,(1-s)$ and hence $\tilde{\tilde{x}} = \frac{\overline{\overline{x}}-\overline{x}_b(1-s)}{s} \geq 1$. Thus from Remark 15 the leader always invests and always releases. No release is off the putative equilibrium path, and we can thus attach arbitrary beliefs to the follower on this path. From Remark 3, we can always find a $\rho^* \in [0,1]$ to give $\overline{x}^*_b = \overline{x}_b$ as $\overline{x}^*_b$ ranges continuously from $\overline{\overline{\overline{x}}}$ to $\overline{\overline{x}}$ as ρ^* ranges from 0 to 1. Thus we can construct a PBE with appropriately chosen but arbitrary off the equilibrium path follower beliefs following no release by the leader, in which the leader always invests and releases. ■

Proof of Claim 7. Suppose $\overline{x}_b \in [1, \overline{\overline{x}})$. Then $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$ so from Remark 15 the leader always invests but never releases. Seeing no release, the follower infers $\rho^* = 1$, which from Remark 3 gives $\overline{x}^*_b = \overline{\overline{x}}$ so we have a contradiction and hence there can be no PBE.

Suppose $\overline{x}_b \in [0, 1)$. Then $\overline{x}_b < \min(\overline{\overline{x}}, 1)$ as $\overline{\overline{x}} \geq 1$, so from Remark 15 the leader always invests and releases. As per the proof of Claim 6 for $\overline{x}_b \in \left[0, \frac{\overline{\overline{x}}-s}{1-s}\right]$, we can thus construct a PBE with appropriately chosen but arbitrary off the equilibrium path follower beliefs following no release by the leader. ■

Proof of Remark 16. From Remark 1, $c \geq \alpha \Rightarrow \overline{\overline{x}} = \frac{2c-\alpha}{\alpha} \geq 1$. From Remark 2, $c > \frac{1}{2} \Rightarrow \overline{\overline{\overline{x}}} = 2c - 1 \in (0, 1)$. From Remark 5, $c > \frac{1}{2} \Rightarrow \overline{x}_a > 0$. Finally, $c > \frac{1}{2} \Rightarrow 2c > 1 - \rho^*\,(1-\alpha)$, so from Remark 3 $\overline{x}^*_b = \frac{2c}{1-\rho^*(1-\alpha)} - 1 > 0$. ■

Proof of Remark 17. Suppose first that $\overline{x}_b \geq 1$, which implies $\overline{x}_b \geq \min(\overline{\overline{x}}, 1)$. Given this and $\overline{\overline{x}} \geq 1$, Claims 1 to 3 imply that the leader never releases, and invests if and only if $x_a \geq \overline{x}_a$. From Remark 5, $c > \frac{1}{2}$ and $\overline{x}_b \geq 1$ imply $\overline{x}_a = \overline{\overline{\overline{x}}}$. From Remark 16, $\overline{\overline{\overline{x}}} > 0$.

Suppose second that $\overline{x}_b < 1$. Because $\overline{\overline{x}} \geq 1$, we have $\overline{x}_b < \min(\overline{\overline{x}}, 1)$. This, together with $\overline{\overline{x}} \geq 1$, implies $\tilde{x} = 2c - 1$ from Claim 2. Thus, $\tilde{x} = \overline{\overline{\overline{x}}}$, and from Claims 1 to 3, the leader invests and releases $\forall x_a > \overline{\overline{\overline{x}}}$, and otherwise neither invests nor releases. Again, $\overline{\overline{\overline{x}}} > 0$. ■

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