Fall, 2022

ME 323 – Mechanics of Materials

Lecture 2 – Review of static equilibrium

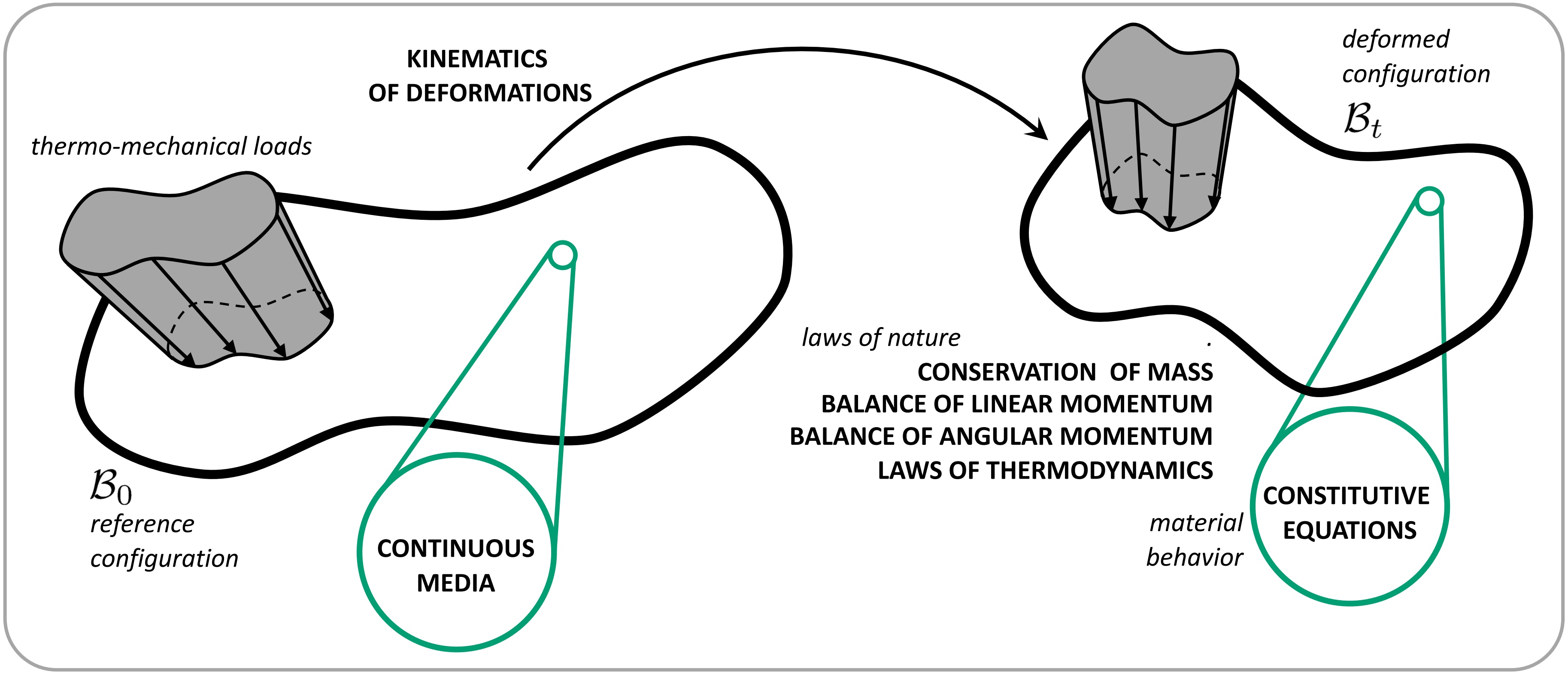
Reading assignment: Ch.2 lecturebook

Office hours: Monday, Wednesday and Friday: 4:30 to 5 p.m. (EST)
in WALC 3121, or in room ME 3061M (by request)

PURDUE UNIVERSITY | Mechanical Engineering

Instructor: Prof. Marcial Gonzalez

Theory of deformable bodies – Static equilibrium



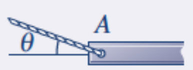
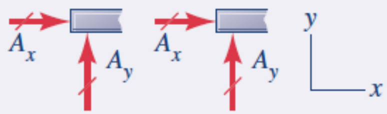
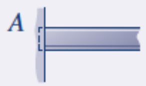
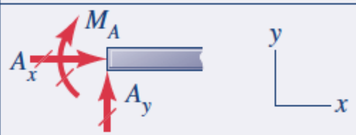
$$\sum \boldsymbol{F} = \mathbf{0} \quad \left(\sum \boldsymbol{M}\right)_O = \mathbf{0}$$

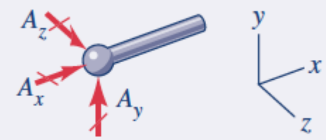
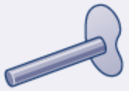
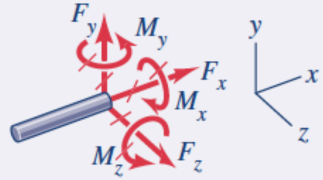
THINGS YOU HAVE TO REMEMBER FROM STATICS

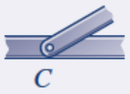
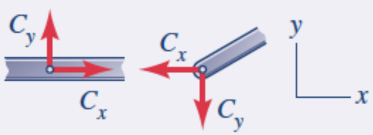
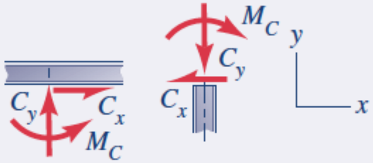
- Number of equations?
- External loads. Can you identify different types?
- Moments about point O, but where is point O?
- Identify internal resultants and support reactions

Theory of deformable bodies – Static equilibrium

Support reactions and connections:

REACTIONS – 2D		
1. Roller support		
2. Cable or rod		
3. Pin support		
4. Cantilever support (fixed end)		

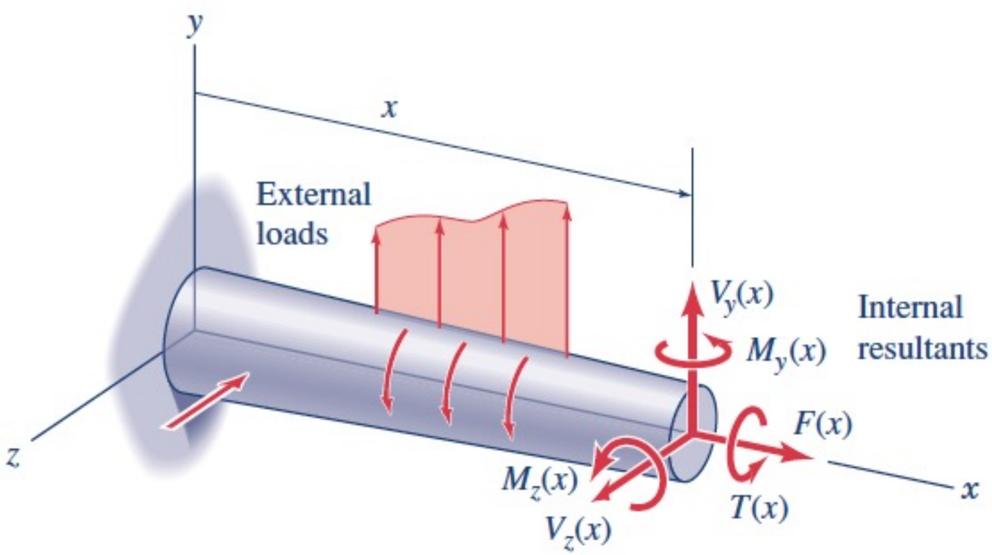
REACTIONS – 3D		
5. Ball joint		
6. Cantilever support (fixed end)		

CONNECTIONS – 2D		
7. Pinned connection		
8. Rigid connection (e.g., welded, bolted)		

$$\sum F = 0 \quad \left(\sum M\right)_O = 0$$

Theory of deformable bodies – Static equilibrium

External loads - Internal resultants:



Identify internal resultants:

- Axial force (normal force)
- Shear force
- Torque (twisting moment)
- Bending moment

$$\sum \boldsymbol{F} = \boldsymbol{0} \quad \left(\sum \boldsymbol{M}\right)_O = \boldsymbol{0}$$

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases} \quad \begin{cases} \left(\sum M_x\right)_O = 0 \\ \left(\sum M_y\right)_O = 0 \\ \left(\sum M_z\right)_O = 0 \end{cases}$$

Q: for given geometry, support and loading conditions, are these internal resultants a function of

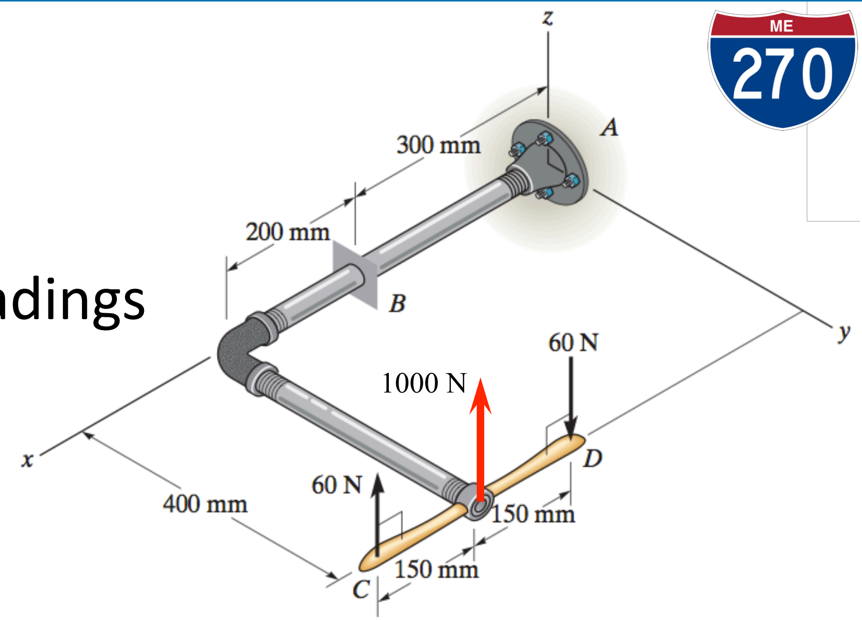
(a) spatial coordinates?

(b) material/cross section properties?

Theory of deformable bodies – Static equilibrium

Example 1:

If the pipe is fixed to the wall at A, determine the resultant internal loadings acting on the cross section at B.



Answer:
$$
\begin{aligned}
F_x &= 0\text{N} \\
F_y &= 0\text{N} \\
F_z &= -1000\text{N} \\
M_x &= -400\text{Nm} \\
M_y &= 218\text{Nm} \\
M_z &= 0\text{Nm}
\end{aligned}
$$

Any questions?

Lecture 2 – Normal stress and strain

Reading assignment: 2.1 – 2.3

News:

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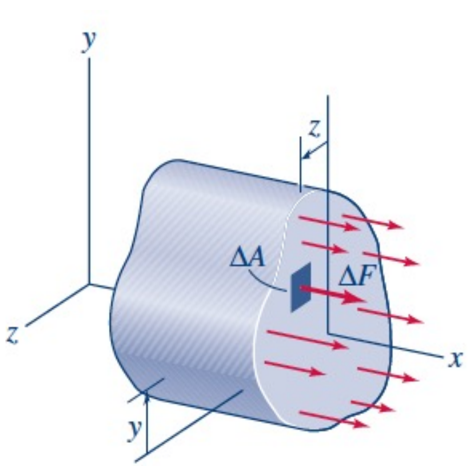
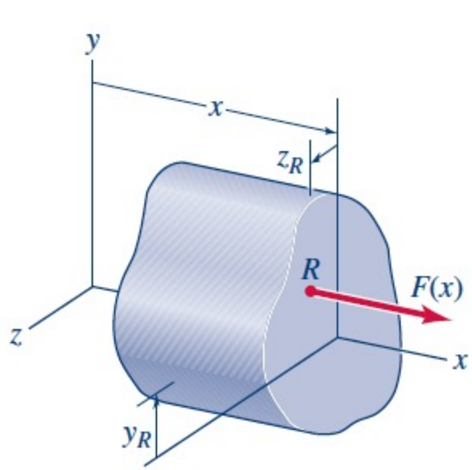
Stress and strain

Normal stress:

- Force per (undeformed) unit area, acting perpendicular to a given plane and at a given point.
  (e.g., normal stress at point (x,y,z) on cross section with normal x)





$$\sigma \equiv \lim_{\Delta A_0 \to 0} \frac{\Delta F}{\Delta A_0} = \frac{dF}{dA_0}$$

Resultant normal force:

$$F = \int_{A_0} dF = \int_{A_0} \sigma \; dA_0$$

Average normal stress:

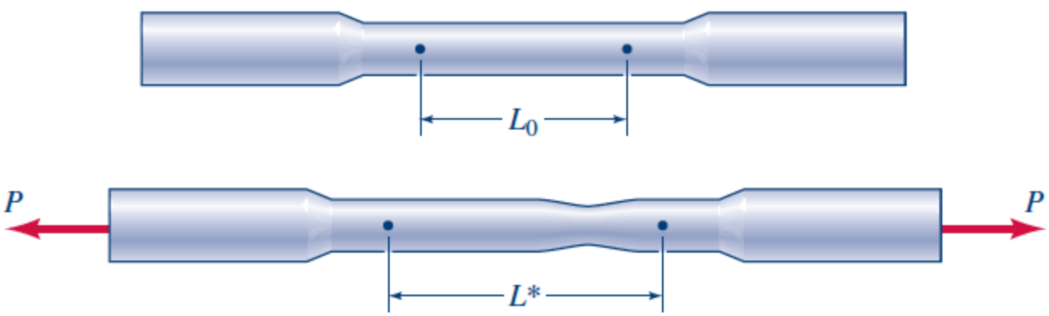
$$\sigma_{\text{avg}} = \frac{F}{A_0}$$

Q: is the normal stress always uniform over the cross section?
Q: if it is uniform, where is the resultant force then located?
Q: what is the sign convention for tension and compression?

Stress and strain

Extensional (engineering) strain:

- Elongation/shortening per (undeformed) unit length that goes with the normal stress.
  (e.g., extensional strain at point (x,y,z) in the x-direction)



$$\epsilon \equiv \frac{L_t - L_0}{L_0}$$

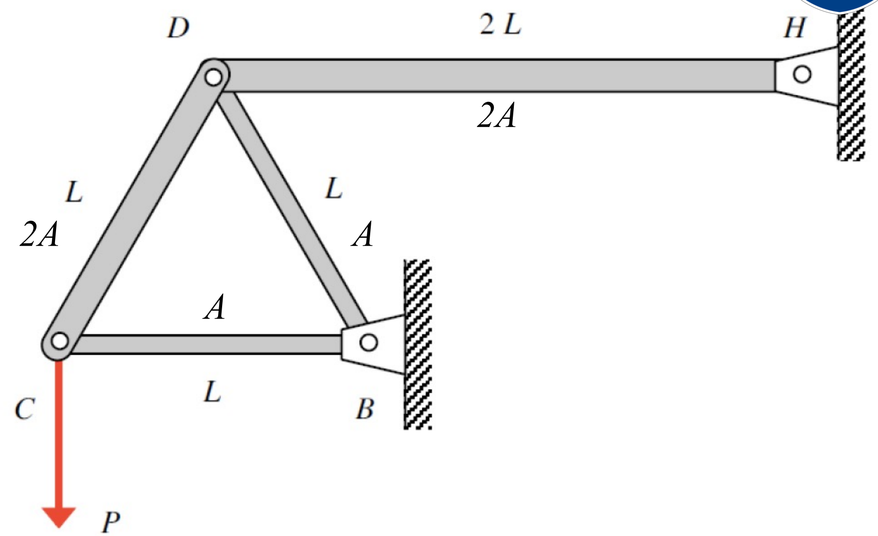
Q: is the extensional engineering strain always uniform over the cross section?
Q: is the extensional engineering strain always uniform along the length of the object?
Q: if it is uniform, does a plane cross section remain plane after deformation?
Q: what is the sign convention for tension and compression?
Q: are there other definitions of strain?

Yes, for example, the true strain: $\epsilon_{\text{true}} = \ln\left(\frac{L_t}{L_0}\right) = \ln(1+\epsilon)$

Normal stress

Example 2 (review):

Determine the stress in members BC, BD, CD, and DH. State whether each member is in tension or compression.



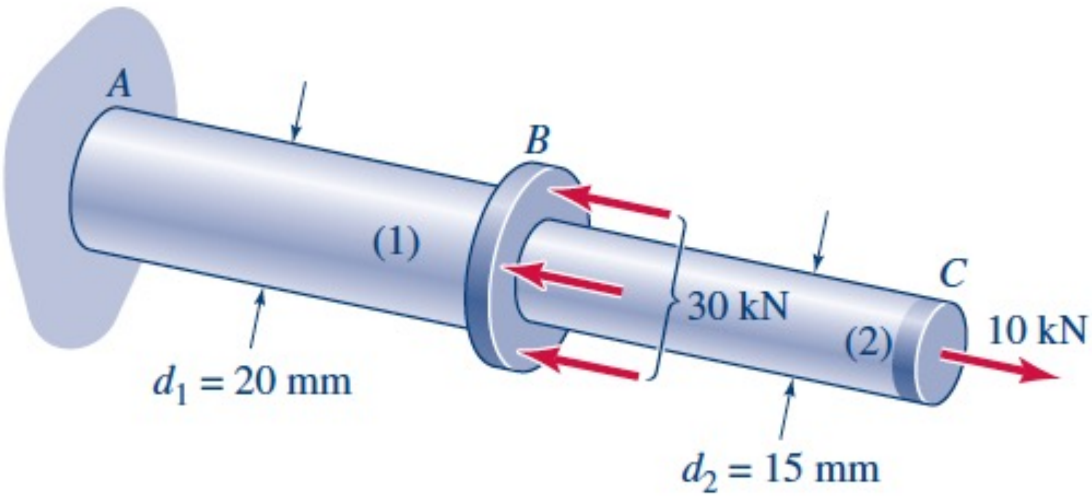
Answer:

$$\begin{aligned} \sigma_{CB} &= P/A\sqrt{3} \quad \text{(C))} \\ \sigma_{CD} &= P/A\sqrt{3} \quad \text{(T)} \\ \sigma_{DB} &= 2P/A\sqrt{3} \quad \text{(C)} \\ \sigma_{DH} &= P/A\sqrt{3} \quad \text{(T)} \end{aligned}$$

Normal stress

Example 3:

Two solid circular rods are welded to a plate at B to form a single rod, as shown in the figure. Consider the 30 kN force at B to be uniformly distributed around the circumference of the collar at B and the 10 kN load at C to be applied at the centroid of the end cross section. Determine the axial stress in each portion of the rod.



Answer:
$$\begin{aligned} \sigma_1 &= 63.7 \text{ MPa} \quad (\text{C}) \\ \sigma_2 &= 56.6 \text{ MPa} \quad (\text{T}) \end{aligned}$$

Stress and strain

Any questions?