Fall, 2022

ME 323 – Mechanics of Materials

Lecture 3 – Shear stress and strain

Reading assignment: Ch.3 lecturebook

Purdue University | Mechanical Engineering

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General information

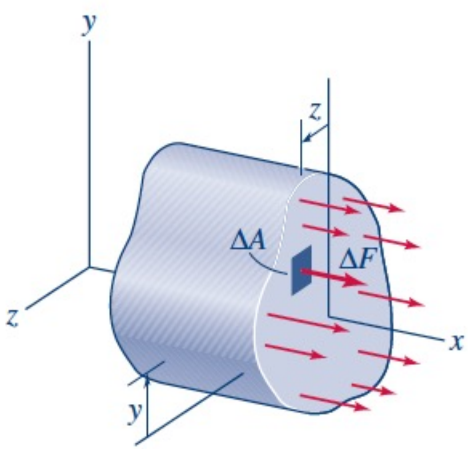
Blog discussion threads:

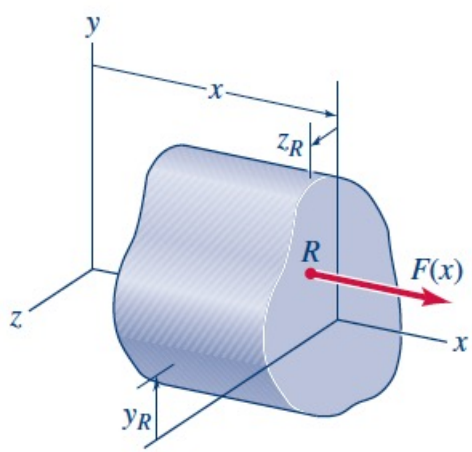
- Please take advantage of these discussion threads to **ask/answer questions** to/from your classmates as you work through the homework assignments and exam preparation in the course.
- The ME 323 teaching team will **frequently monitor** these discussions.
- Please try today and **confirm that you can login**, and let the TAs know if you experience any issues.

Stress and strain

Normal stress:

- Force per (undeformed) unit area, acting perpendicular to a given plane and at a given point.
  (e.g., normal stress at point (x,y,z) on cross section with normal x)





$$\sigma \equiv \lim_{\Delta A_0 \to 0} \frac{\Delta F}{\Delta A_0} = \frac{dF}{dA_0}$$

Resultant normal force:

$$F = \int_{A_0} dF = \int_{A_0} \sigma \; dA_0$$

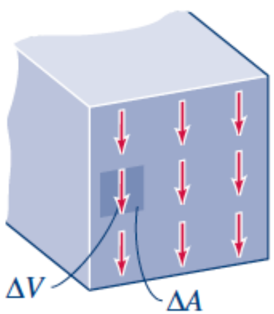
Average normal stress:

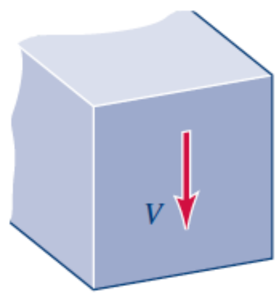
$$\sigma_{\text{avg}} = \frac{F}{A_0}$$

Shear stress and strain

Shear stress:

- Force per (undeformed) unit area, acting tangential to a given plane and at a given point.
  (e.g., shear stress at point (x,y,z) on cross section with normal x)





$$\tau \equiv \lim_{\Delta A_0 \to 0} \frac{\Delta V}{\Delta A_0} = \frac{dV}{dA_0}$$

Resultant shear force:

$$V = \int_{A_0} dV = \int_{A_0} \tau \; dA_0$$

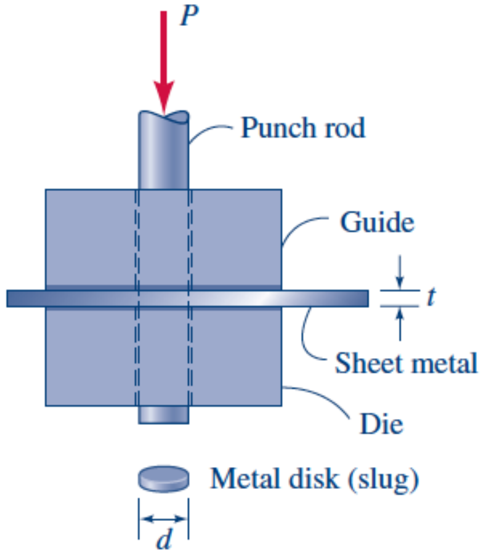
Average shear stress:

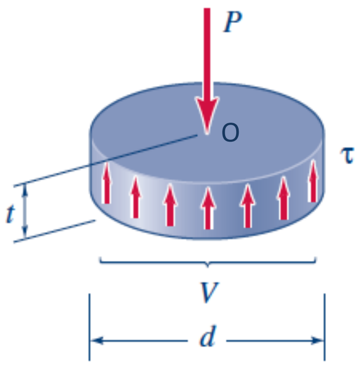
$$\tau_{\mathrm{avg}} = \frac{V}{A_0}$$

Shear stress and strain

Direct shear.

- Direct shear is caused by forces that act parallel to a particular surface, with the direct result of shearing the material at that surface.
  Example: sheet-metal punch





Free Body Diagram

Shear stress and strain

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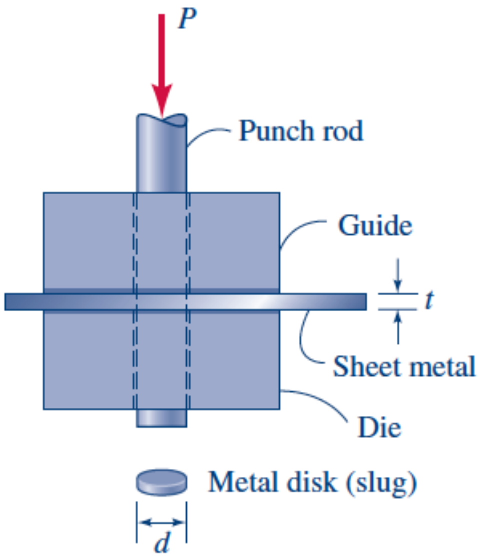

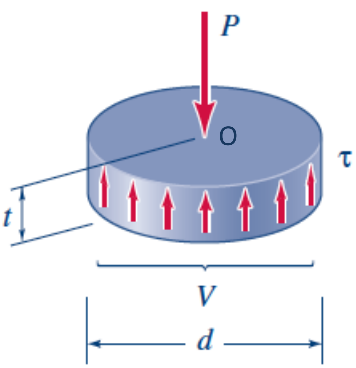

Free Body Diagram

Average shear stress:

$$\tau_{\text{avg}} = \frac{V}{A_0} = \frac{P}{\pi \ d \ t}$$

Note: the body is in equilibrium

$$P = V$$

$$\left(\sum M\right)_O = \mathbf{0}$$

Shear stress and strain

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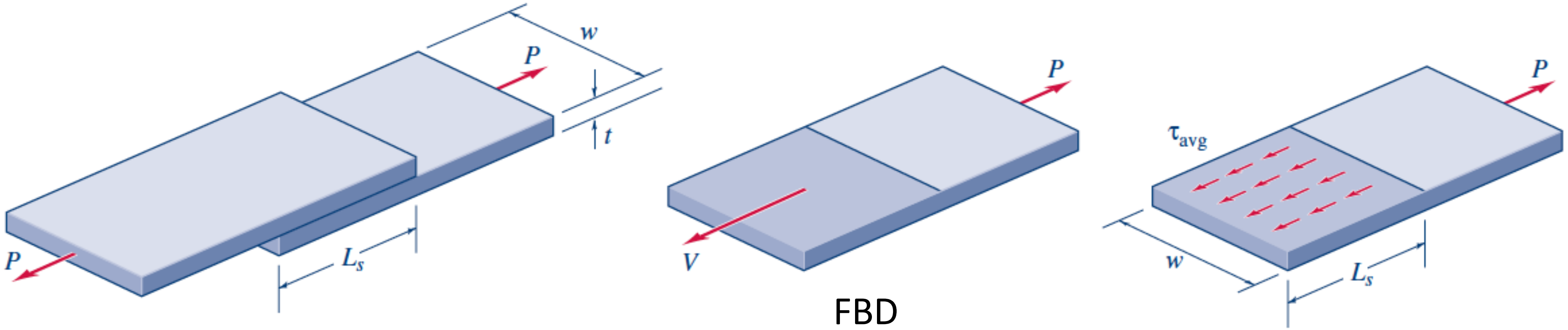
  Example: lap splice



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  <u>Example</u>: lap splice

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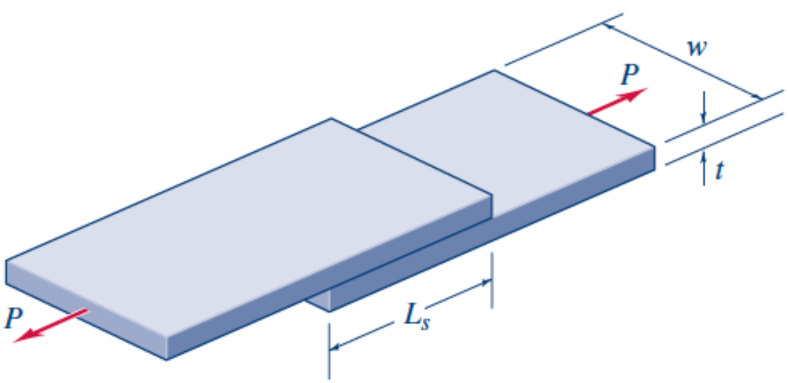
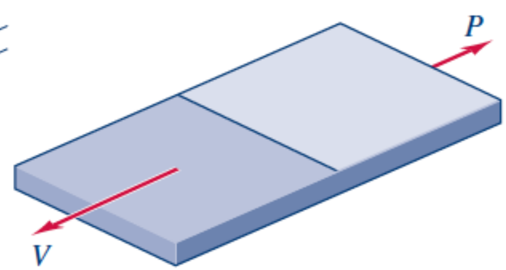
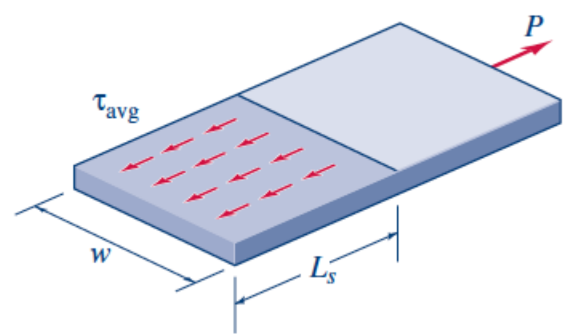






FBD

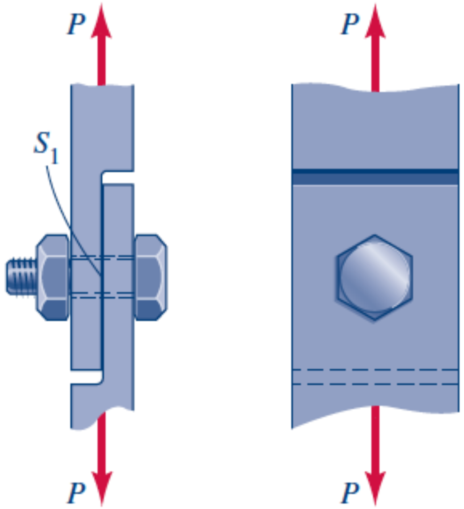
Note: the body is in equilibrium

$$P = V$$

Shear stress and strain

Direct shear.

- Direct shear is caused by forces that act parallel to a particular surface, with the direct result of shearing the material at that surface.
  <u>Example</u>: bolted joint



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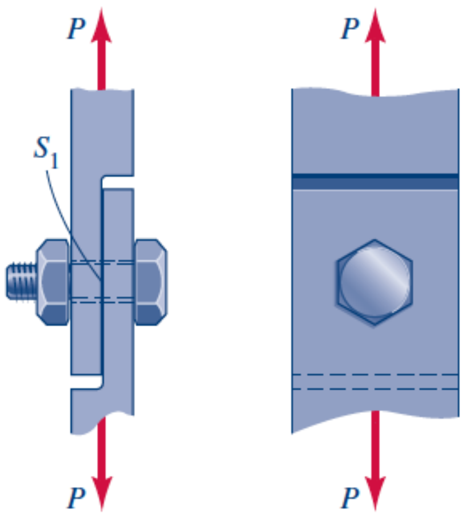
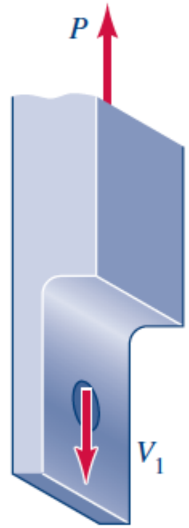




FBD

Note: the body is in equilibrium

$$P = V_1$$

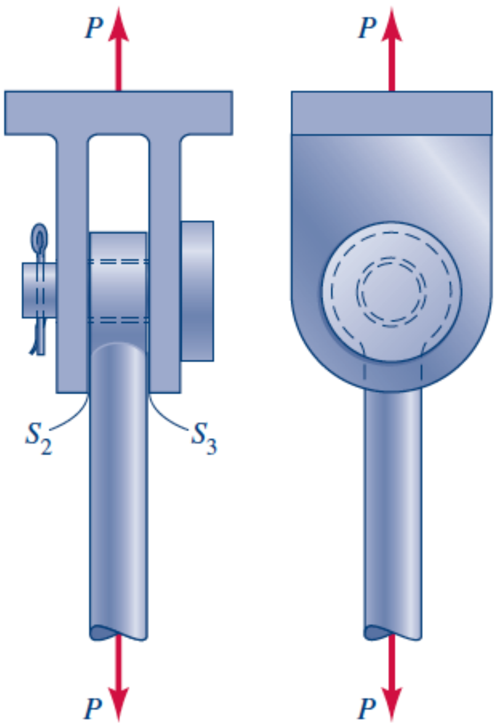
Average shear stress:

$$\tau_{\text{avg}} = \frac{V_1}{A_b} = \frac{4\ P}{\pi d_b^2}$$

Shear stress and strain

Direct shear.

- Direct shear is caused by forces that act parallel to a particular surface, with the direct result of shearing the material at that surface.
  Example: pinned joint



Shear stress and strain

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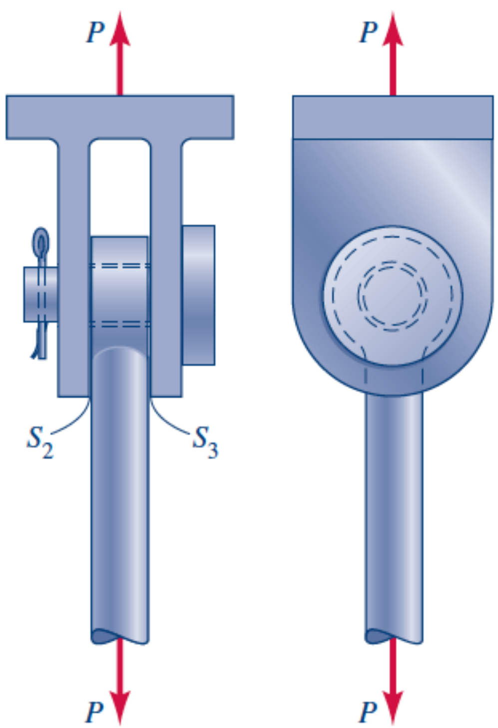
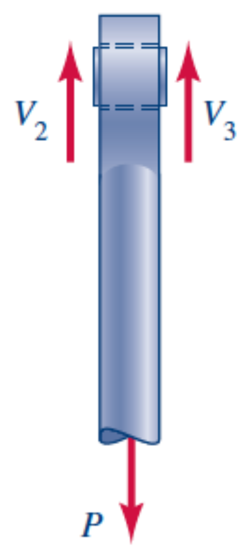
  <u>Example</u>: pinned joint





FBD

Average shear stress:

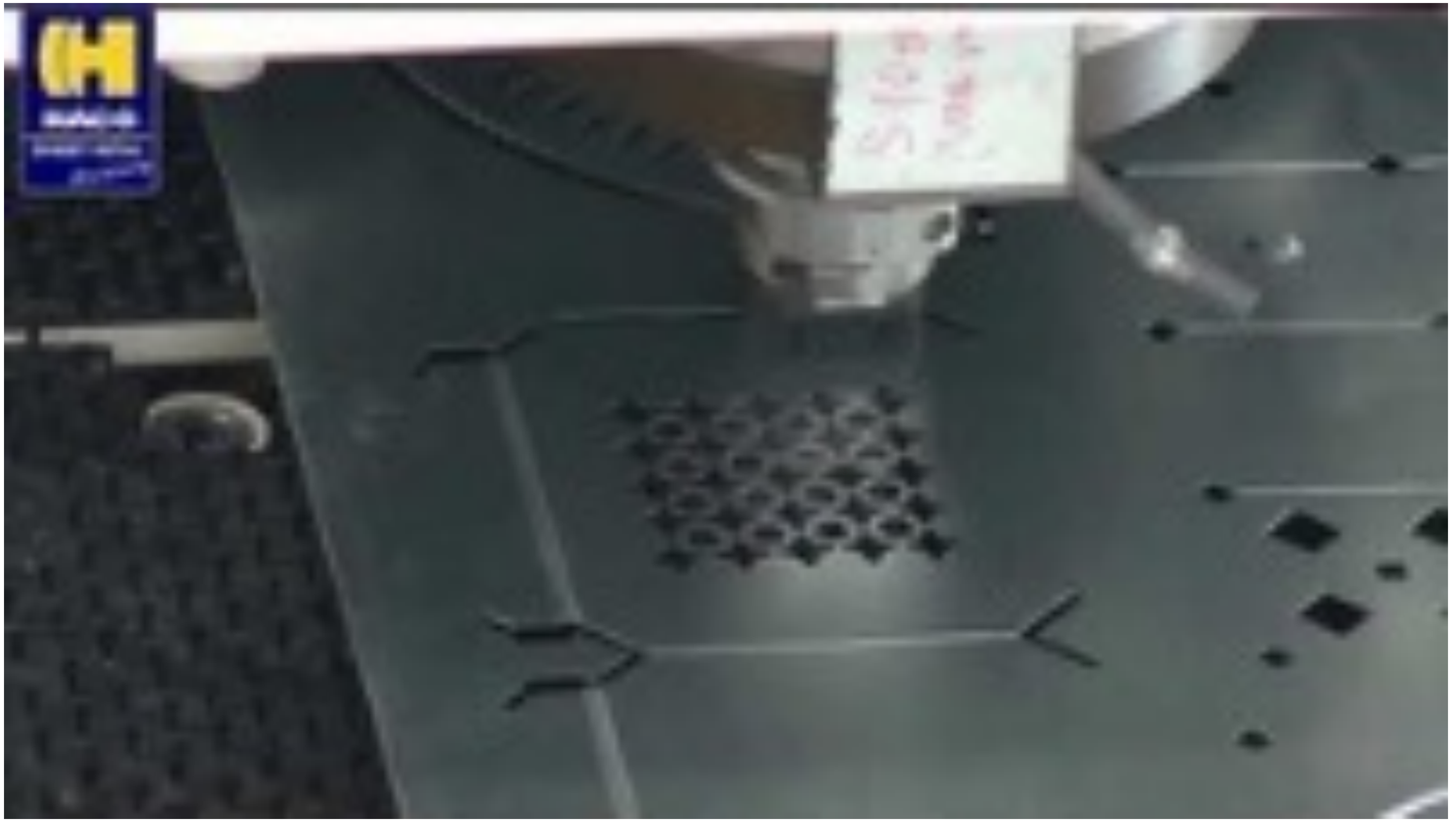
$$\tau_{\text{avg}} = \frac{V_2}{A_p} = \frac{V_3}{A_p} = \frac{2\ P}{\pi d_p^2}$$

$$V_1 = V_2 = P/2$$

Note: the body is in equilibrium

Shear stress and strain

Direct shear. Heavy duty CNC punching machine.

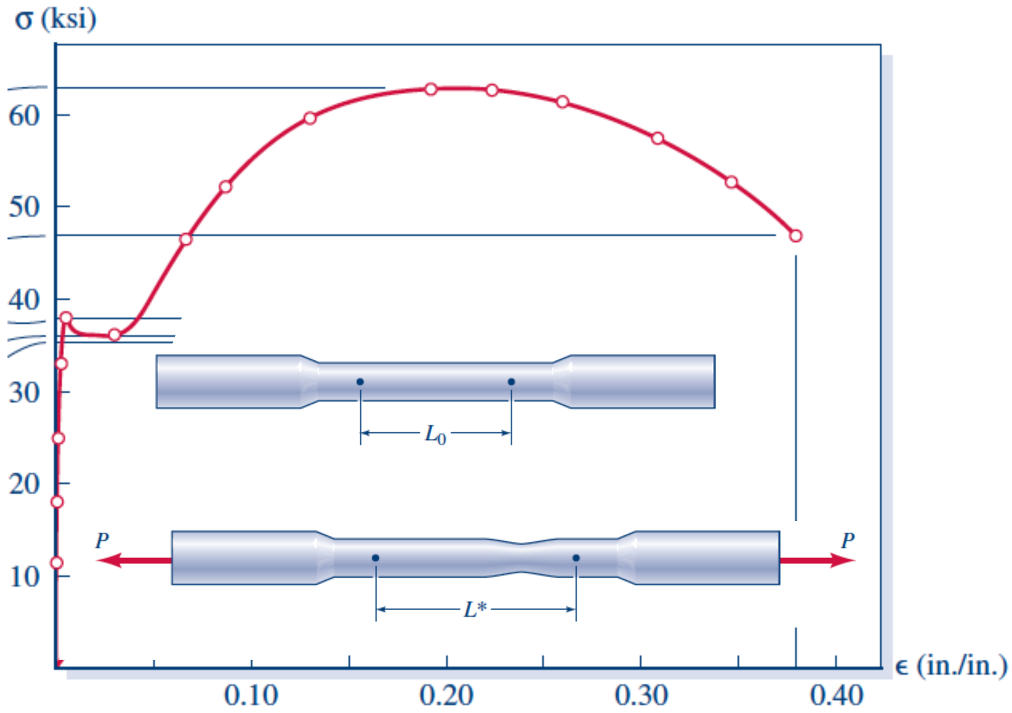


https://youtu.be/pZ--KJ06ykw

Mechanical properties

Material properties from stress-strain diagrams:

- Tensile uniaxial test

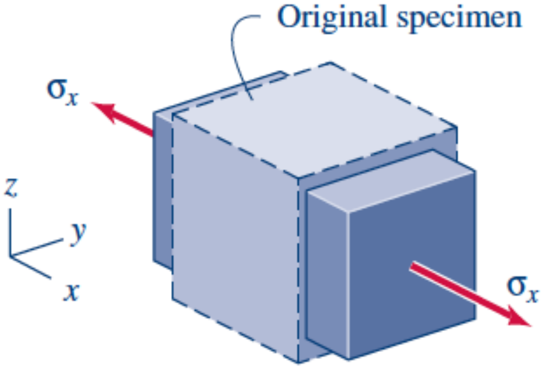


Engineering stress and strain: $\sigma = \dfrac{P}{A_0} \quad \epsilon = \dfrac{\Delta L}{L_0}$

Mechanical properties – Next time …

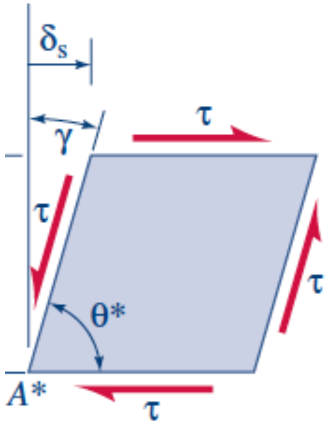
Linear elastic deformations (in homogeneous and isotropic materials)

- Hooke's law:
  - \+ elongation $\sigma_x = E \, \epsilon_x$
  - \+ shear $\tau = G \, \gamma$




S.D. Poisson

- Young's modulus E
- Poisson's ratio $\epsilon_y = \epsilon_z = -\nu \, \epsilon_x$
- Shear modulus $G = \dfrac{E}{2(1+\nu)}$



- Note: only two material constants are needed for the description of linear elastic materials. You will learn other elastic material properties in future lectures (however, all these properties are related)

Stress and strain

Any questions?