

Fall, 2020

# ME 323 – Mechanics of Materials

## Lecture 7 – Axial deformation (cont.)

Reading assignment: 3.1–3.9

KEEP A MASK WITH  
YOU AT ALL TIMES



PROTECT  
PURDUE



Mechanical Engineering

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# Axial deformation

## Axial deformation (summary)

- Geometry of the solid body: straight, slender member with cross section that is either constant or that changes slowly along the length of the member.
- Kinematic assumptions: cross sections, which are plane and are perpendicular to the axis before deformation, remain plane and remain perpendicular to the axis after deformation. In addition, cross sections do not rotate about the axis.

**Strain:**  $\epsilon(x) = \frac{du(x)}{dx} = \epsilon_{\text{elastic}} + \epsilon_{\text{thermal}}$

**Elongation:**  $e = \int_0^L \epsilon(x) dx = u(L) - u(0)$

- Material behavior: isotropic linear elastic material; small deformations.

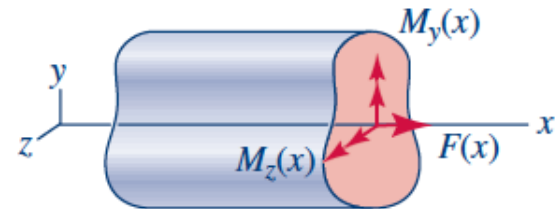
**Homogeneous:**  $\epsilon(x) = \frac{\sigma(x)}{E} + \alpha \Delta T(x)$

- Equilibrium:

**Homogeneous:**  $F(x) = \sigma(x)A(x)$

**Homogeneous, constant cross section, no body forces, thermal load:**  $e = \frac{FL}{AE} + \alpha L \Delta T$

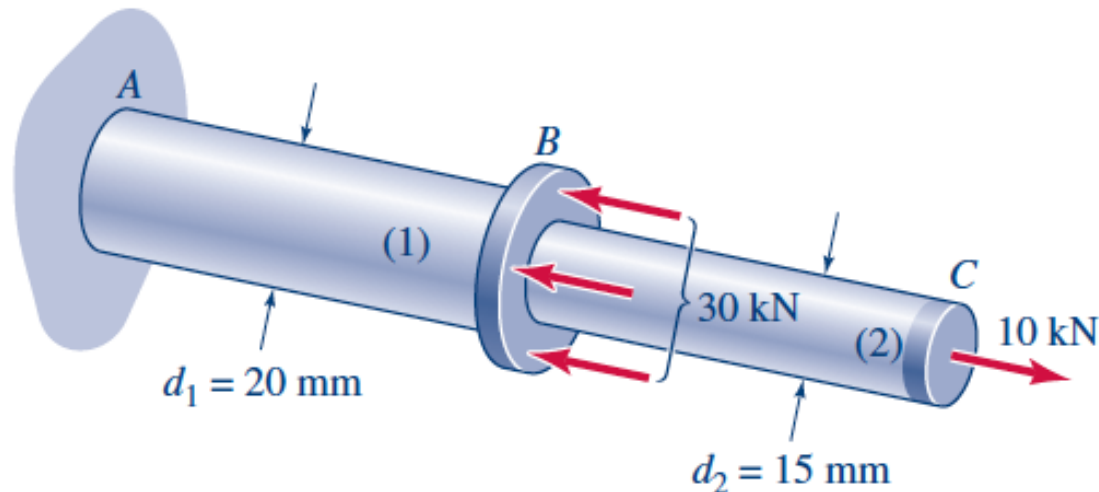
**Homogeneous, loaded with body forces:**  $\frac{dF(x)}{dx} + p(x) = 0$



# Axial deformation

## Example 5 (from Lecture 2 & 6):

Two solid circular rods are welded to a plate at B to form a single rod, as shown in the figure. Consider the 30-kN force at B to be uniformly distributed around the circumference of the collar at B and the 10 kN load at C to be applied at the centroid of the end cross section. Determine the axial stress in each portion of the rod.  $L_1=300\text{mm}$ ,  $L_2=200\text{mm}$ ,  $E_1=600\text{ GPa}$ ,  $E_2=400\text{GPa}$ .



Determine the displacement of end C.  $u_C = e_1 + e_2 = (-3.18 + 2.82)10^{-5}\text{mm}$   
.... easy!  $u_C = -0.36 \times 10^{-5}\text{mm}$

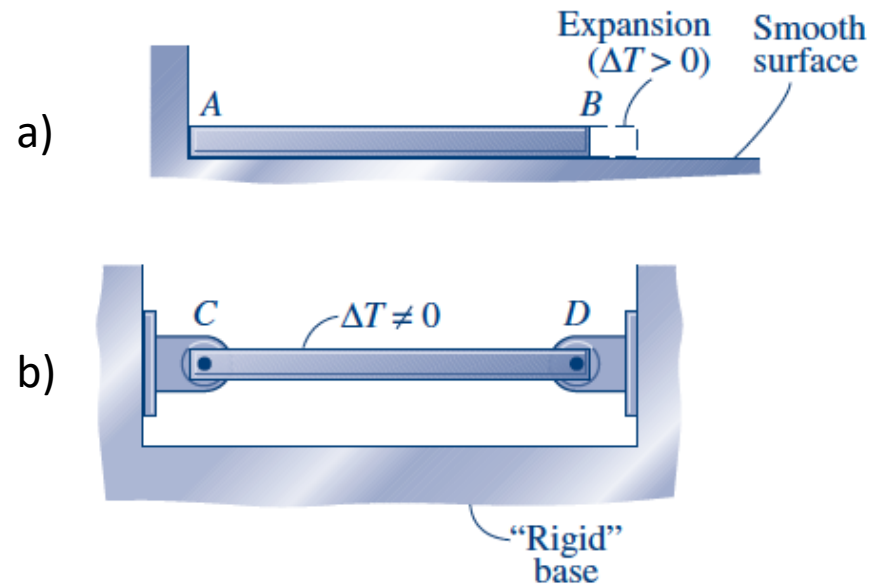
# Axial deformation – Thermal effects

## Example 6:

Thermal load, thermal strain, thermal stress ...

$$e = \frac{FL}{AE} + \alpha L\Delta T$$

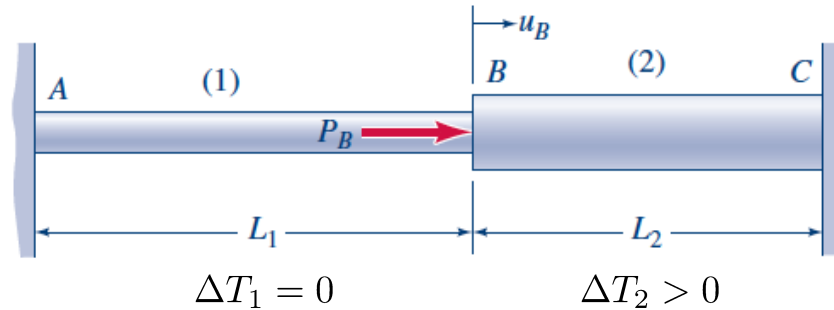
$$\epsilon = \frac{\sigma}{E} + \alpha\Delta T$$



# Axial deformation – Statically indeterminate

## Example 7

Determine the displacement of end B



$$\text{Answer: } F_1 = \frac{P_B - A_2 \alpha_2 \Delta T_2 E_2}{1 + A_2 L_1 / A_1 L_2}$$

$$u_B = F_1 \frac{L_1}{A_1 E_1}$$

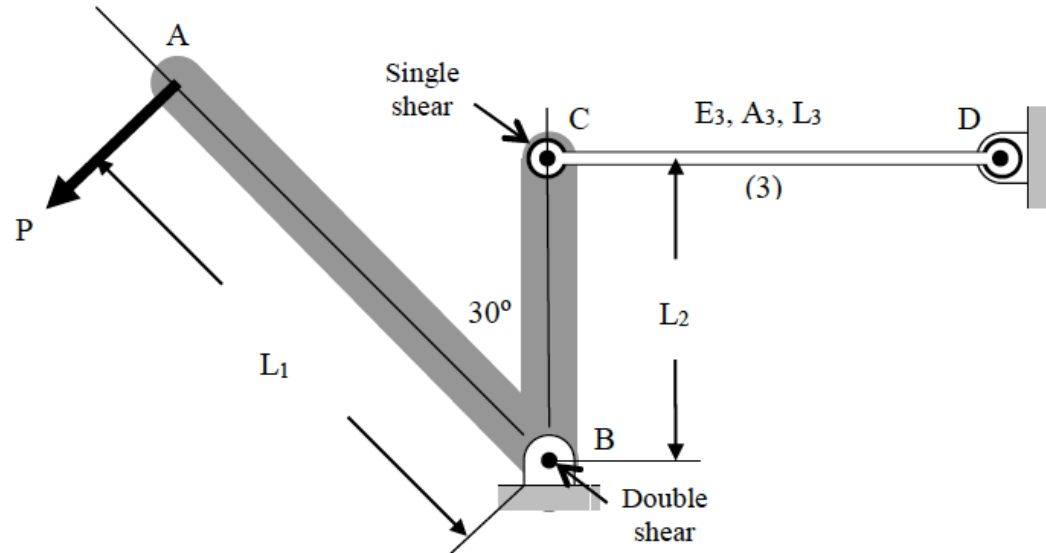
**statically  
indeterminate  
structures**

- 1) Free body diagram
- 2) Equilibrium equations
- 3) Force-displacement behavior
- 4) Compatibility conditions, Geometry of deformations
- 5) Solve for unknowns

# Axial deformation

## Example 8 (review)

Determine the elongation of member 3 and the reactions at support B.



# Axial deformation – Statically indeterminate

## Example 9:

Determine the (small) vertical displacement of B, C and D.

Recall:

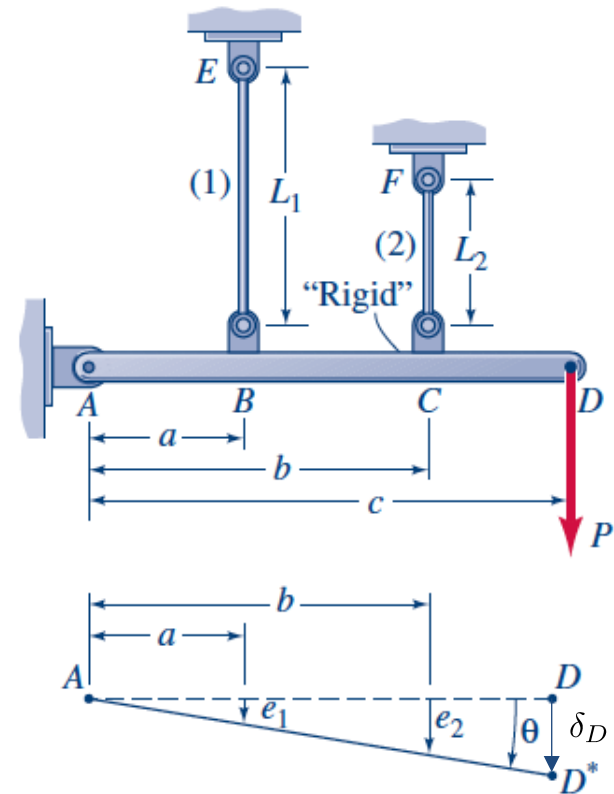
$$e = fF = \frac{L}{AE}F$$

For a small angle of rotation and member AD rigid:

$$\theta \approx \tan(\theta) = \frac{e_1}{a} = \frac{e_2}{b} = \frac{\delta_D}{c}$$

**statically  
indeterminate  
structures**

- 1) Free body diagram
- 2) Equilibrium equations
- 3) Force-displacement behavior
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- 5) Solve for unknowns



Answer:

$$F_1 = \frac{acL_2/A_2E_2}{a^2L_2/A_2E_2 + b^2L_1/A_1E_1}P$$

$$F_2 = \frac{bcL_1/A_1E_1}{a^2L_2/A_2E_2 + b^2L_1/A_1E_1}P$$

# Axial deformation

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Any questions?