Fall, 2021

ME 323 – Mechanics of Materials

Lecture 7 – Axial deformation (cont.)

Reading assignment: 3.1 – 3.9

PURDUE UNIVERSITY | Mechanical Engineering

Instructor: Prof. Marcial Gonzalez

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Axial deformation

Axial deformation (summary)

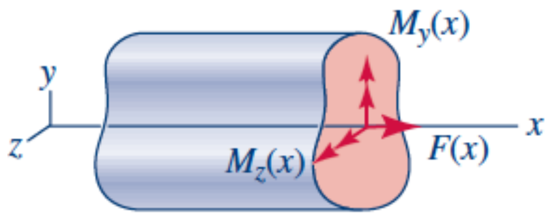
- Geometry of the solid body: straight, slender member with cross section that is either constant or that changes slowly along the length of the member.
- Kinematic assumptions: cross sections, which are plane and are perpendicular to the axis before deformation, remain plane and remain perpendicular to the axis after deformation. In addition, cross sections do not rotate about the axis.

  Strain: $\epsilon(x) = \dfrac{du(x)}{dx} = \epsilon_{\text{elastic}} + \epsilon_{\text{thermal}}$

  Elongation: $e = \displaystyle\int_0^L \epsilon(x)dx = u(L) - u(0)$

- Material behavior: isotropic linear elastic material; small deformations.

  Homogeneous: $\epsilon(x) = \dfrac{\sigma(x)}{E} + \alpha \Delta T(x)$



- Equilibrium:

  Homogeneous: $F(x) = \sigma(x)A(x)$

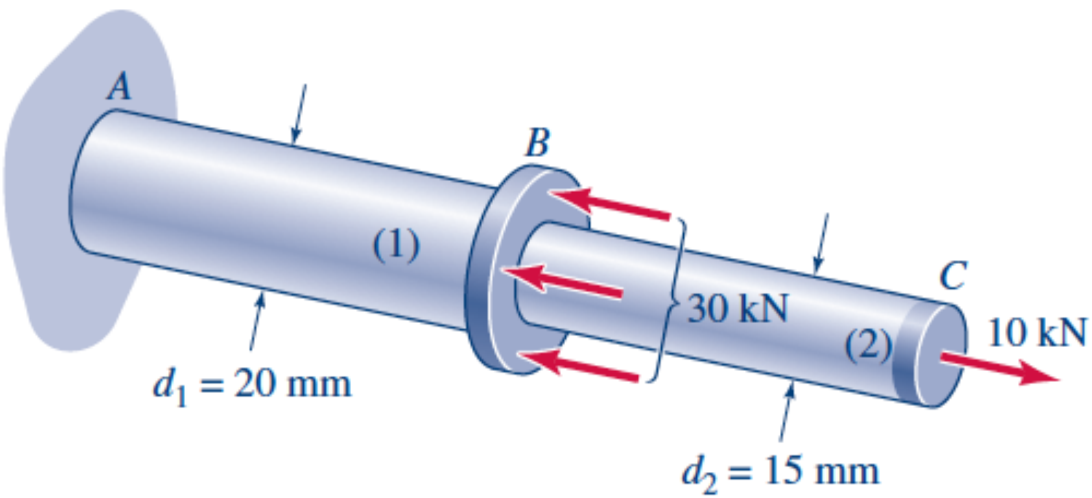
  Homogeneous, constant cross section, no body forces, thermal load: $e = \dfrac{FL}{AE} + \alpha L \Delta T$

  Homogeneous, loaded with body forces: $\dfrac{dF(x)}{dx} + p(x) = 0$

Axial deformation

Example 5 (from Lecture 2 & 6):

Two solid circular rods are welded to a plate at B to form a single rod, as shown in the figure. Consider the 30-kN force at B to be uniformly distributed around the circumference of the collar at B and the 10 kN load at C to be applied at the centroid of the end cross section. Determine the axial stress in each portion of the rod. L_1=300mm, L_2=200mm, E_1=600 GPa, E_2=400GPa.



Determine the displacement of end C. $u_C = e_1 + e_2 = (-3.18 + 2.82)10^{-5}\text{mm}$

.... easy! $u_C = -0.36 \times 10^{-5}\text{mm}$

Axial deformation – Thermal effects

Example 6:

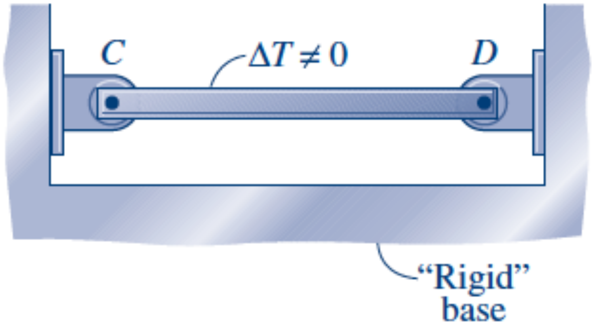
Thermal load, thermal strain, thermal stress …

$$e = \frac{FL}{AE} + \alpha L \Delta T$$

$$\epsilon = \frac{\sigma}{E} + \alpha \Delta T$$

a)

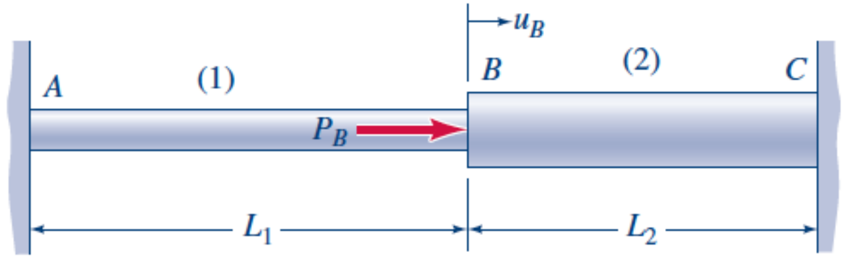
b)



Axial deformation – Statically indeterminate

Example 7

Determine the displacement of end B



$\Delta T_1 = 0$ $\qquad$ $\Delta T_2 > 0$

Answer: $F_1 = \frac{P_B - A_2 \alpha_2 \Delta T_2 E_2}{1 + A_2 L_1 / A_1 L2}$

$$u_B = F_1 \frac{L_1}{A_1 E_1}$$

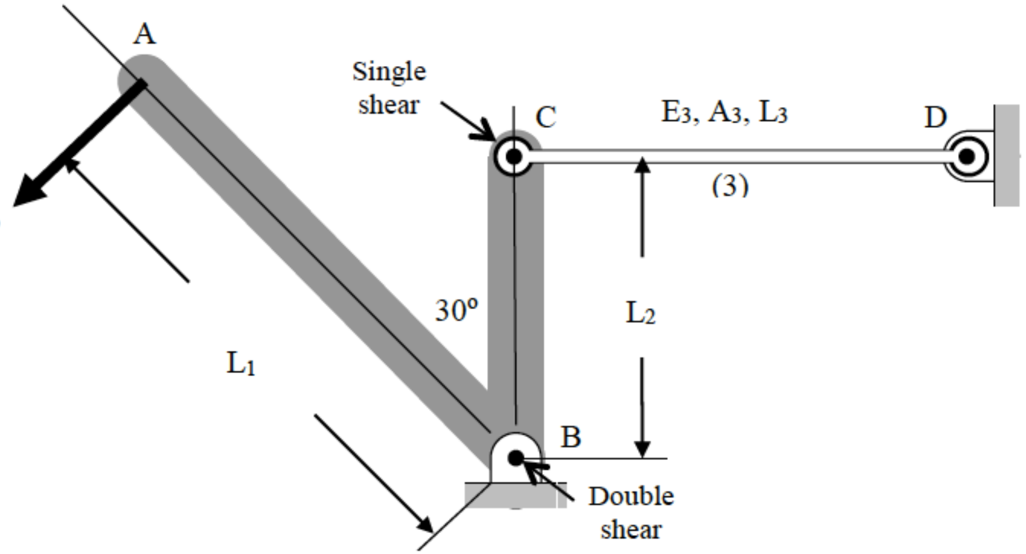
statically indeterminate structures

1) Free body diagram
2) Equilibrium equations
3) Force-displacement behavior
4) Compatibility conditions, Geometry of deformations
5) Solve for unknowns

Axial deformation

Example 8 (review)

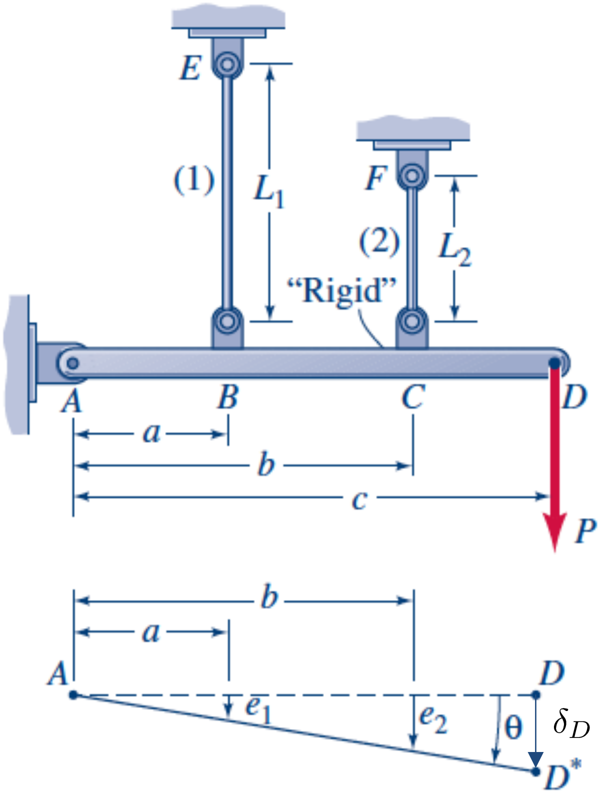
Determine the elongation of member 3 and the reactions at support B.



Axial deformation – Statically indeterminate

Example 9:

Determine the (small) vertical displacement of B, C and D.



Recall:

$$e = fF = \frac{L}{AE}F$$

For a small angle of rotation and member AD rigid:

$$\theta \approx \tan(\theta) = \frac{e_1}{a} = \frac{e_2}{b} = \frac{\delta_D}{c}$$

statically indeterminate structures

1) Free body diagram
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5) Solve for unknowns

Answer:

$$F_1 = \frac{acL_2/A_2E_2}{a^2L_2/A_2E_2+b^2L_1/A_1E_1}P$$

$$F_2 = \frac{bcL_1/A_1E_1}{a^2L_2/A_2E_2+b^2L_1/A_1E_1}P$$

Axial deformation

Any questions?