Fall, 2022

ME 323 – Mechanics of Materials

Lecture 8 – Axial deformation (cont.)

Reading assignment: Ch.6-Ch.7 lecturebook

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Axial deformation

Axial deformation (summary)

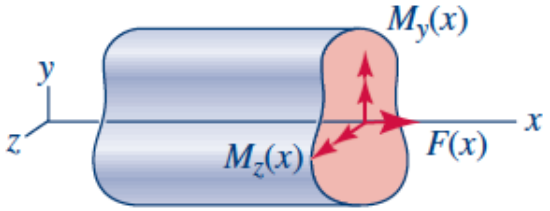
- Geometry of the solid body: straight, slender member with cross section that is either constant or that changes slowly along the length of the member.
- Kinematic assumptions: cross sections, which are plane and are perpendicular to the axis before deformation, remain plane and remain perpendicular to the axis after deformation. In addition, cross sections do not rotate about the axis.

Strain: $\epsilon(x) = \dfrac{du(x)}{dx} = \epsilon_{\text{elastic}} + \epsilon_{\text{thermal}}$

Elongation: $e = \displaystyle\int_0^L \epsilon(x)dx = u(L) - u(0)$

- Material behavior: isotropic linear elastic material; small deformations.

Homogeneous: $\epsilon(x) = \dfrac{\sigma(x)}{E} + \alpha\Delta T(x)$



- Equilibrium:

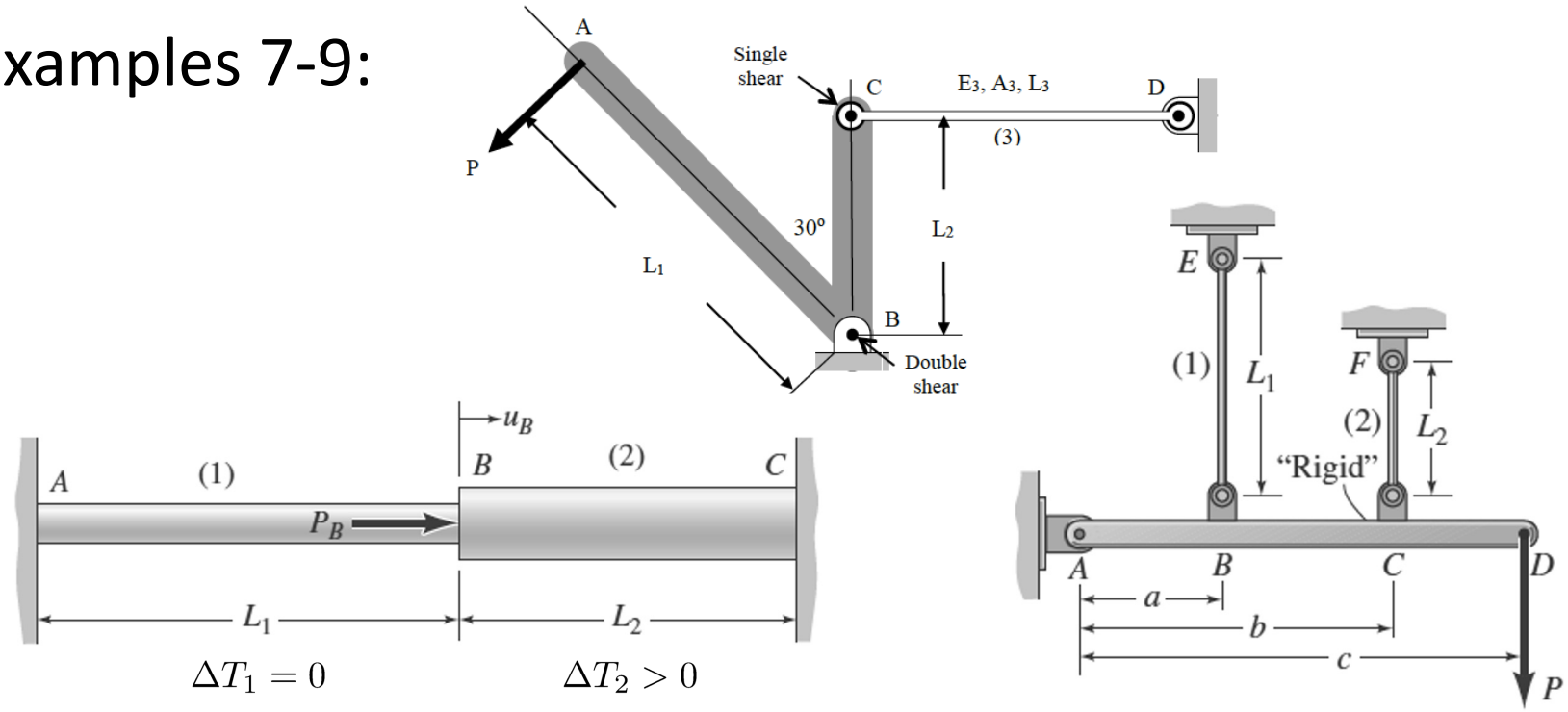
Homogeneous: $F(x) = \sigma(x)A(x)$

Homogeneous, constant cross section, no body forces, thermal load: $e = \dfrac{FL}{AE} + \alpha L\Delta T$

Homogeneous, loaded with body forces: $\dfrac{dF(x)}{dx} + p(x) = 0$

Axial deformation – Statically indeterminate

Examples 7-9:



statically indeterminate structures

1) Free body diagram
2) Equilibrium equations
3) Force-displacement behavior
4) Compatibility conditions, Geometry of deformations
5) Solve for unknowns

Axial deformation – Statically indeterminate

Example 9

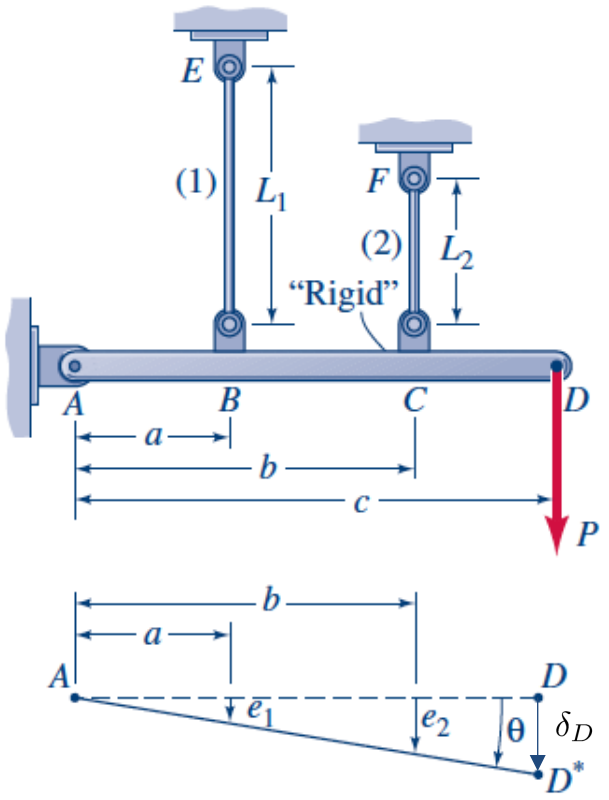
Determine the (small) vertical displacement of B, C and D.

Recall:

$$e = fF = \frac{L}{AE}F$$

For a small angle of rotation and member AD rigid:

$$\theta \approx \tan(\theta) = \frac{e_1}{a} = \frac{e_2}{b} = \frac{\delta_D}{c}$$



statically indeterminate structures

1) Free body diagram
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Answer:

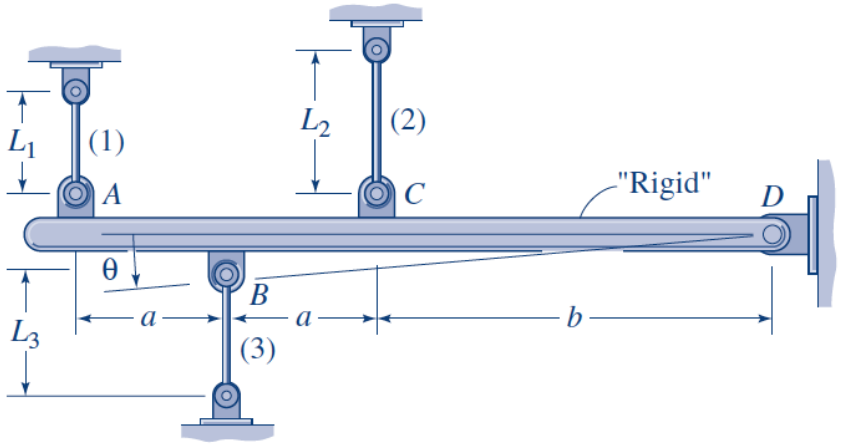
$$F_1 = \frac{acL_2/A_2E_2}{a^2L_2/A_2E_2 + b^2L_1/A_1E_1}P$$

$$F_2 = \frac{bcL_1/A_1E_1}{a^2L_2/A_2E_2 + b^2L_1/A_1E_1}P$$

Axial deformation – Statically indeterminate

Example 10

Determine the axial stress induced in the three elastic rods by the cooling of rod (3).

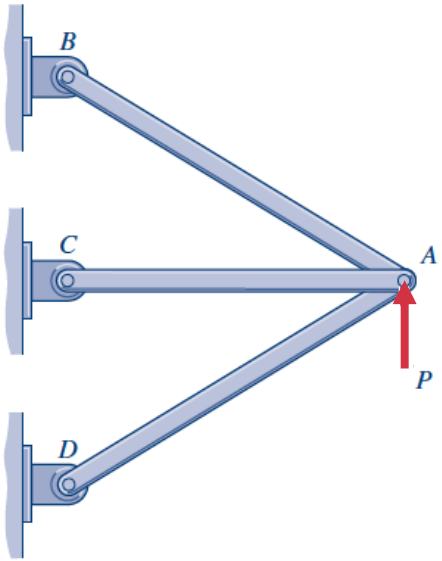


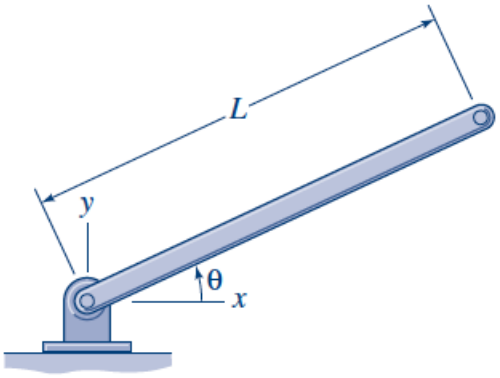
Axial deformation – Statically indeterminate

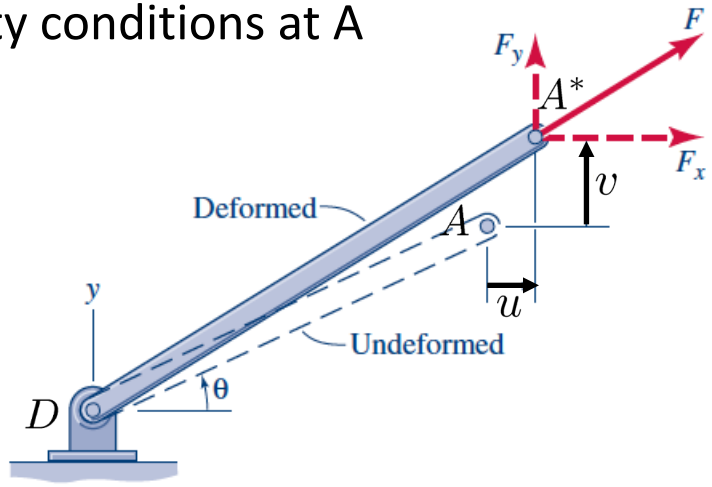
Analysis of planar trusses

Goal: study the deformation of trusses under external forces and thermal loads.

1) FBD + Equilibrium
2) Force-displacement behavior
3) Compatibility conditions at A







$$e_{DA} = D\bar{A}^* - D\bar{A} \approx u\cos(\theta_{DA}) + v\sin(\theta_{DA})$$ with *D* fixed

(angle measured at D from x-axis to the member counterclockwise)

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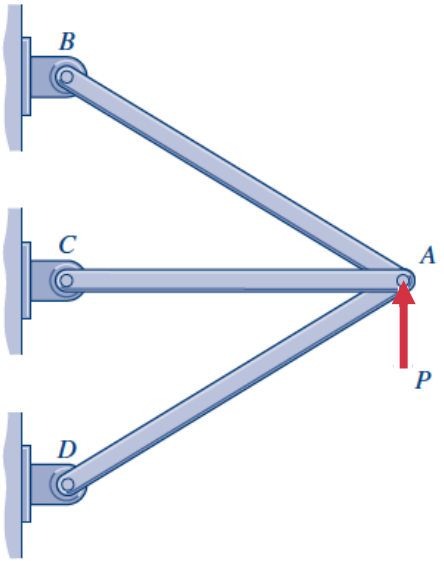

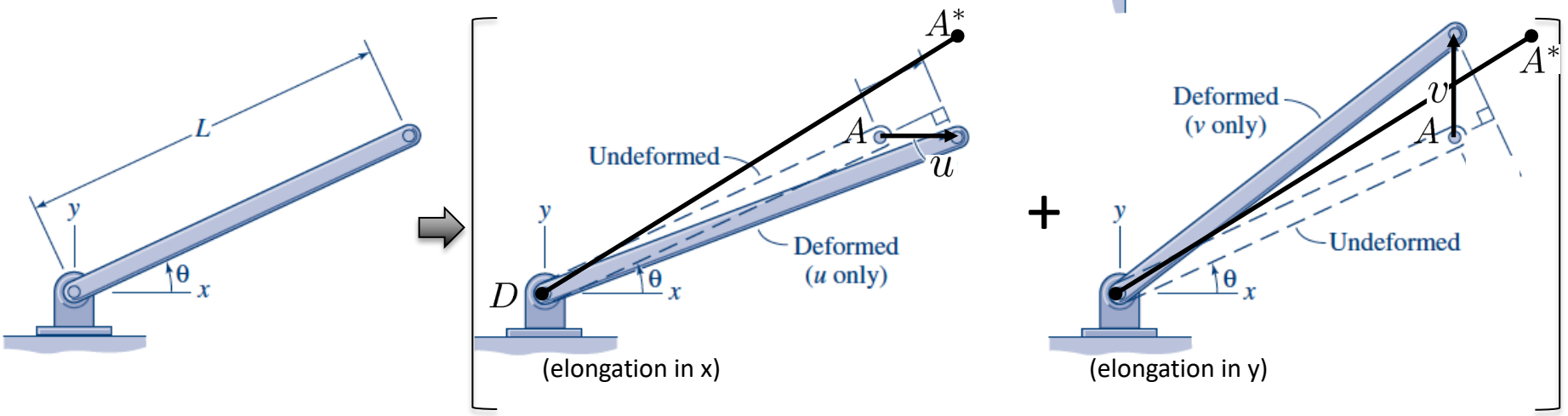

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(angle measured at D from x-axis to the member counterclockwise)

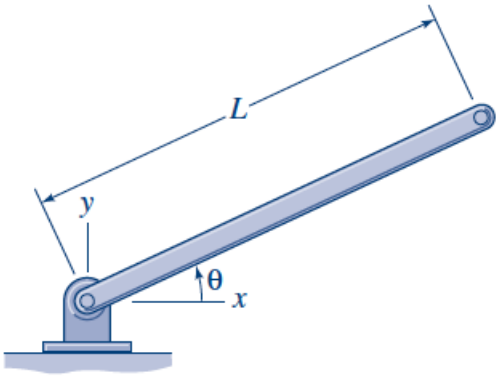
Axial deformation

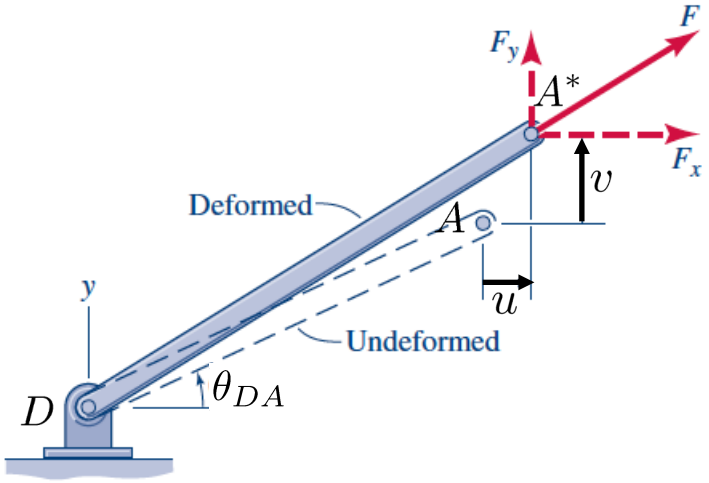
Axial deformation (summary)

- Kinematic assumptions (cont.): in planar truss structure, members deform as follows …

Elongation: $e_{DA} = D\bar{A}^* - D\bar{A} \approx u\cos(\theta_{DA}) + v\sin(\theta_{DA})$ with *D* fixed

(angle measured at D from x-axis to the member counterclockwise)



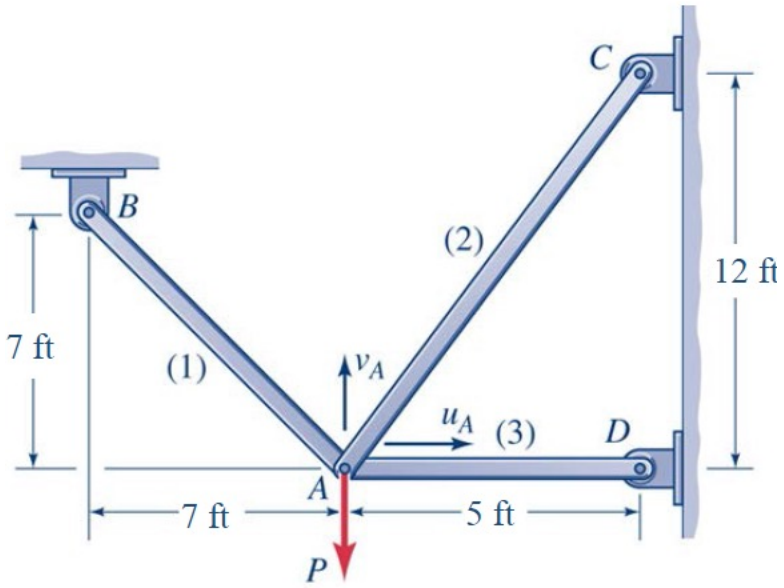


Axial deformation – Statically indeterminate

Example 11

(a) Determine the displacement at A.
(b) Determine the resulting stresses in all three members.

$$\theta_1 = 3\pi/2 + \arctan(1/1)$$
$$\theta_2 = \pi + \arctan(12/5)$$
$$\theta_3 = \pi$$



Axial deformation

Any questions?