Fall, 2022

ME 323 – Mechanics of Materials

Lecture 11 – Torsion (cont.)

Reading assignment: Ch.8 lecturebook

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Torsion

Torsional deformation

- Geometry of the solid body: straight, slender member with circular cross section that changes slowly along the length of the member.
- Kinematic assumptions: the axis remains straight and inextensible. Cross sections, which are plane and are perpendicular to the axis before deformation, remain plane and perpendicular after deformation. Radial lines remain straight and radial as the cross section rotates about the axis

Shear strain $\gamma(x,\rho) = \rho \dfrac{d\phi(x)}{dx}$

Total angle of rotation $\phi = \displaystyle\int_0^L \frac{d\phi(x)}{dx} dx = \phi(L) - \phi(0)$

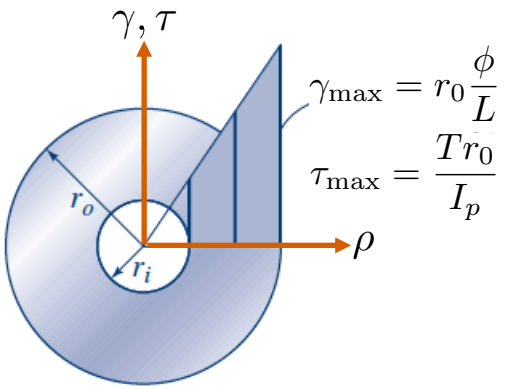
- Material behavior: isotropic linear elastic material; small deformations.

Homogeneous: $\tau(x,\rho) = G\gamma(x,\rho)$

- Equilibrium: (*torque-twist equation*)

Homogeneous: $\dfrac{T(x)}{G\ I_p(x)} = \dfrac{d\phi(x)}{dx} \Rightarrow \tau(x,\rho) = \dfrac{T(x)\rho}{I_p(x)}$

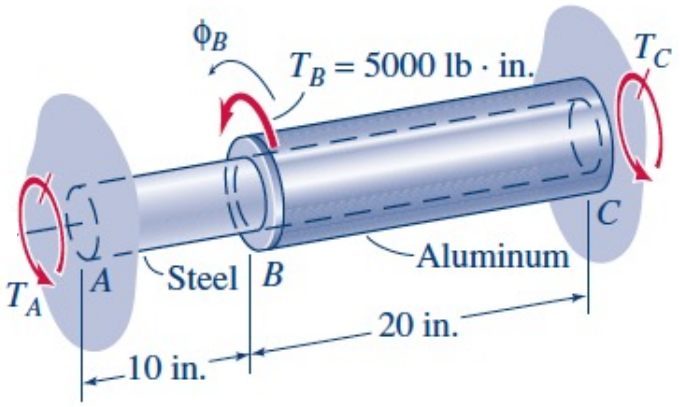
Homogeneous, constant cross section: $\phi = \dfrac{TL}{GI_p} = f_t T$



Torsion

Example 15:

Determine the maximum shear stress in the steel and the maximum shear stress in the aluminum.



statically indeterminate structures

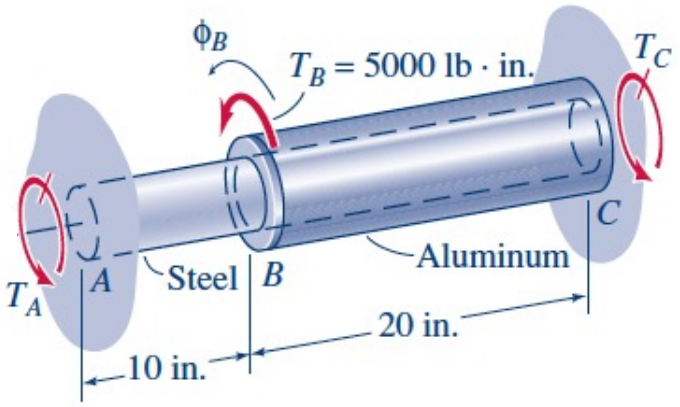
1) Free body diagram
2) Equilibrium equations
3) Torque-twist behavior
4) Compatibility conditions
5) Solve for unknowns

Torsion

Example 16 (Practice problem):

Determine the maximum shear stress in the steel and the maximum shear stress in the aluminum.

Determine the state of stress at four different points in each member.



$$T_1 = -\frac{(L_2/G_2I_{p2})(L_3/G_3I_{p3})}{(L_1/G_1I_{p1})(L_2/G_2I_{p2}) + (L_3/G_3I_{p3})[(L_1/G_1I_{p1}) + (L_2/G_2I_{p2})]}T_B$$

$$T_2 = \frac{(L_1/G_1I_{p1})(L_3/G_3I_{p3})}{(L_1/G_1I_{p1})(L_2/G_2I_{p2}) + (L_3/G_3I_{p3})[(L_1/G_1I_{p1}) + (L_2/G_2I_{p2})]}T_B$$

$$T_3 = \frac{(L_1/G_1I_{p1})(L_2/G_2I_{p2})}{(L_1/G_1I_{p1})(L_2/G_2I_{p2}) + (L_3/G_3I_{p3})[(L_1/G_1I_{p1}) + (L_2/G_2I_{p2})]}T_B$$

Torsion

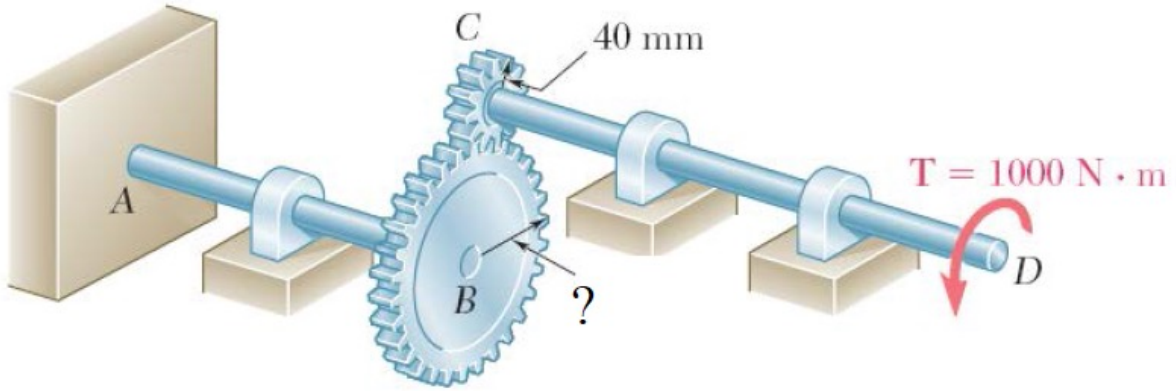
Example 17 (next class)

A torque of magnitude T=1000 Nm is applied at D. The diameter of shaft AB is 67 mm and the diameter of shaft CD is 53 mm. The radius r_B of the gear at B it to be varied.

(a) Determine the value of radius r_B which would cause the magnitudes of the maximum shear stress in the two shafts to be equal.

(b) Determine the total rotation at D.

(c) Indicate the state of stress at 4 points in the cross section of shaft CD.



Torsion

Any questions?