

Fall, 2021

# ME 323 – Mechanics of Materials

## Lecture 12 – Torsion (cont.)

Reading assignment: 4.4 – 4.7

KEEP A MASK WITH  
YOU AT ALL TIMES



**PROTECT  
PURDUE**



Mechanical Engineering

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# Torsion

## Torsional deformation

- Geometry of the solid body: straight, slender member with circular cross section that changes slowly along the length of the member.
- Kinematic assumptions: the axis remains straight and inextensible. Cross sections, which are plane and are perpendicular to the axis before deformation, remain plane and perpendicular after deformation. Radial lines remain straight and radial as the cross section rotates about the axis

Shear strain  $\gamma(x, \rho) = \rho \frac{d\phi(x)}{dx}$

Total angle of rotation  $\phi = \int_0^L \frac{d\phi(x)}{dx} dx = \phi(L) - \phi(0)$

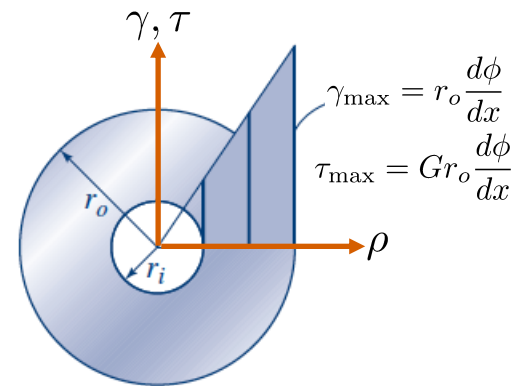
- Material behavior: isotropic linear elastic material; small deformations.

Homogeneous:  $\tau(x, \rho) = G\gamma(x, \rho)$

- Equilibrium: (torque-twist equation)

Homogeneous:  $\frac{T(x)}{G I_p(x)} = \frac{d\phi(x)}{dx} \Rightarrow \tau(x, \rho) = \frac{T(x)\rho}{I_p(x)}$

Homogeneous, constant cross section:  $\phi = \frac{TL}{GI_p} = f_t T$

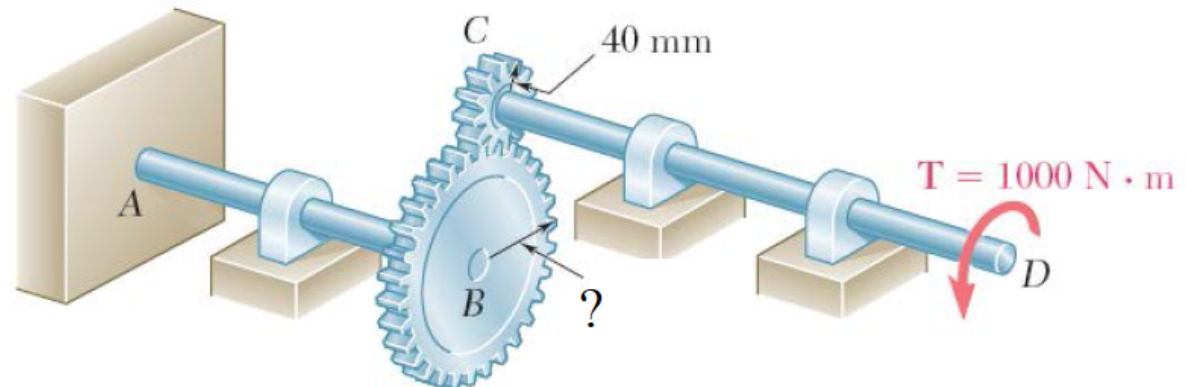


# Torsion

## Example 17

A torque of magnitude  $T=1000 \text{ Nm}$  is applied at D. The diameter of shaft AB is 67 mm and the diameter of shaft CD is 53 mm. The radius  $r_B$  of the gear at B it to be varied.

- Determine the value of radius  $r_B$  which would cause the magnitudes of the maximum shear stress in the two shafts to be equal.
- Determine the total rotation at D.
- Indicate the state of stress at 4 points in the cross section of shaft CD.



# Torsion

## Example 18 (Practice problem):

A tubular shaft of length  $L$  is made of a steel alloy with  $\tau_{\text{allow}}$  and  $G$ . The total angle of rotation can not exceed  $\phi_{\text{allow}}$ , determine the diameters for given  $T$ .

