Fall, 2022

ME 323 – Mechanics of Materials

Lecture 12 – Torsion (cont.)

Reading assignment: Ch.8 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

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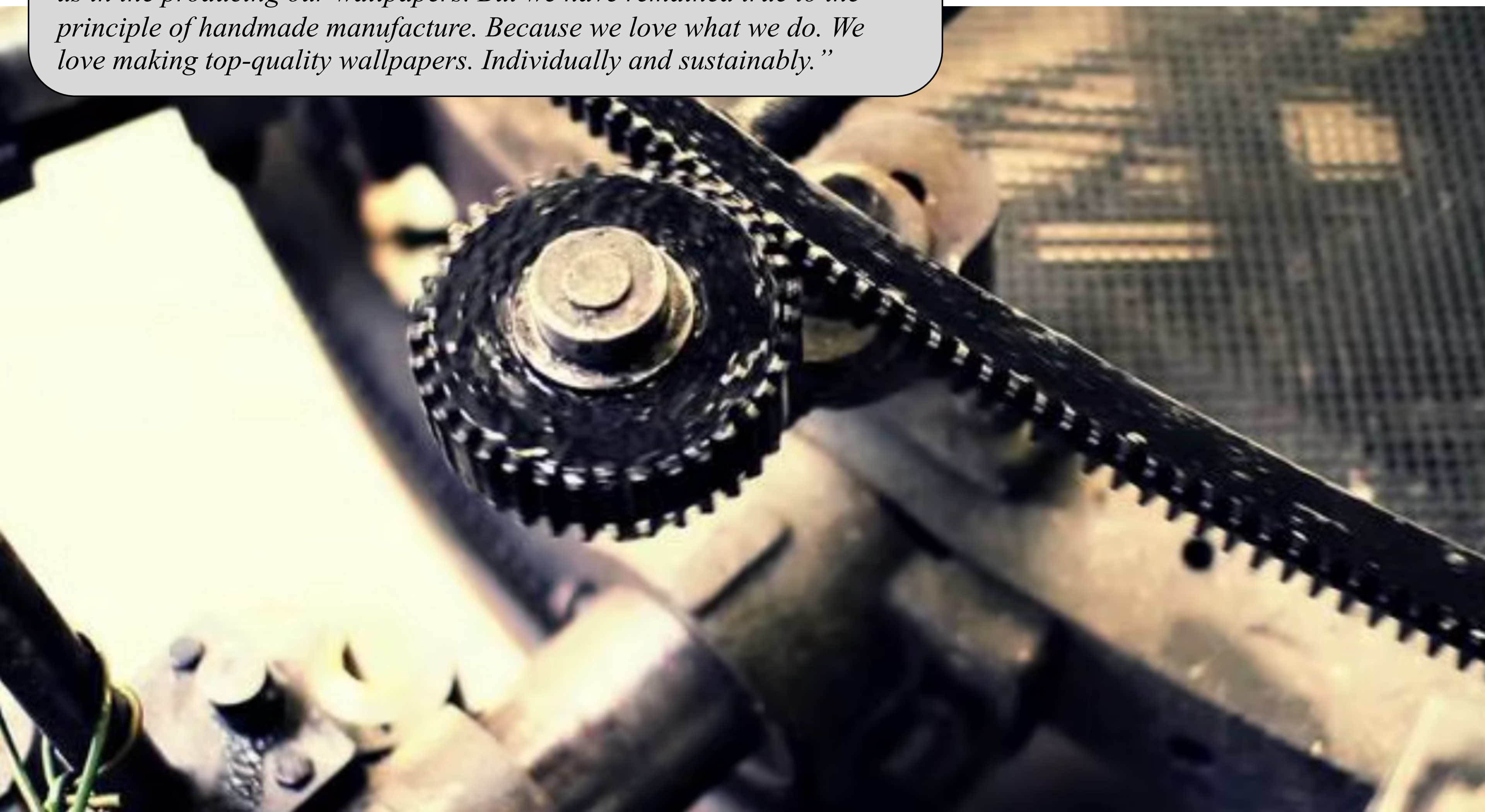
The machine-meets-handmade nature of many industries

About Hohenberger manufacture of wallcoverings:
"When Paul Taubert started making fine wall coverings in Hohenberg almost 90 years ago, most things were done by hand. Today, the small business has grown somewhat, and now state-of-the-art machines assist us in the producing our wallpapers. But we have remained true to the principle of handmade manufacture. Because we love what we do. We love making top-quality wallpapers. Individually and sustainably."

Video



Torsion

Torsional deformation

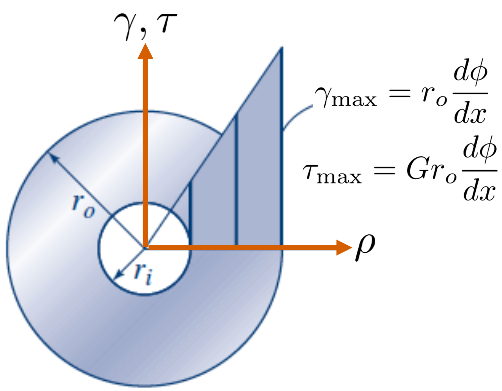
- <u>Geometry of the solid body</u>: straight, slender member with <u>circular</u> cross section that changes slowly along the length of the member.
- <u>Kinematic assumptions</u>: the axis remains straight and inextensible. Cross sections, which are plane and are perpendicular to the axis before deformation, remain plane and perpendicular after deformation. Radial lines remain straight and radial as the cross section rotates about the axis

Shear strain $$\gamma(x,\rho) = \rho \frac{d\phi(x)}{dx}$$

Total angle of rotation $$\phi = \int_0^L \frac{d\phi(x)}{dx} dx = \phi(L) - \phi(0)$$

- <u>Material behavior</u>: isotropic linear elastic material; small deformations.

Homogeneous: $$\tau(x,\rho) = G\gamma(x,\rho)$$



- <u>Equilibrium</u>: (*torque-twist equation*)

Homogeneous: $$\frac{T(x)}{G\ I_p(x)} = \frac{d\phi(x)}{dx} \quad \Rightarrow \tau(x,\rho) = \frac{T(x)\rho}{I_p(x)}$$

Homogeneous, constant cross section: $$\phi = \frac{TL}{GI_p} = f_t T$$

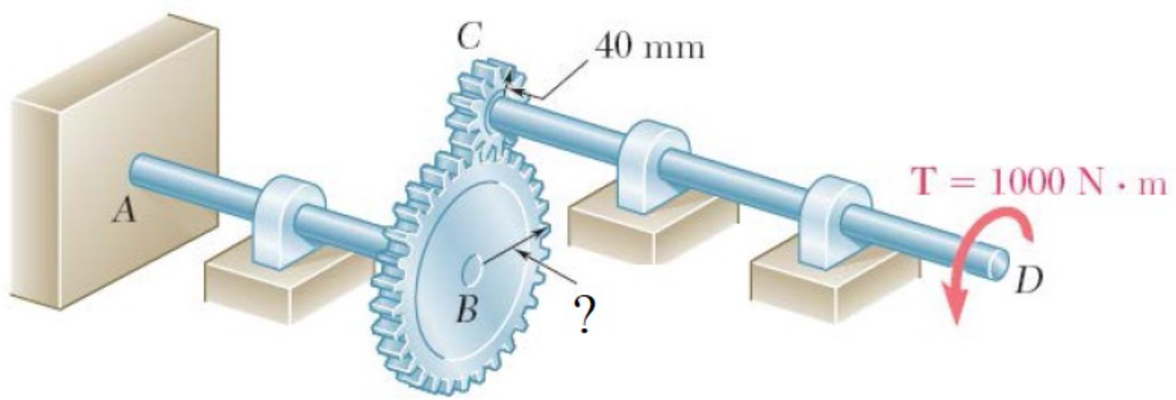
Example 17

A torque of magnitude T=1000 Nm is applied at D. The diameter of shaft AB is 67 mm and the diameter of shaft CD is 53 mm. The radius r_B of the gear at B it to be varied.

(a) Determine the value of radius r_B which would cause the magnitudes of the maximum shear stress in the two shafts to be equal.

(b) Determine the total rotation at D.

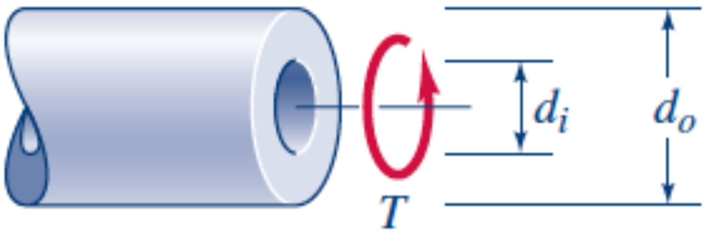
(c) Indicate the state of stress at 4 points in the cross section of shaft CD.



Torsion

Example 18 (Practice problem):

A tubular shaft of length L is made of a steel alloy with τ_{allow} and G. The total angle of rotation can not exceed ϕ_{allow}, determine the diameters for given T.



Torsion

Any questions?