

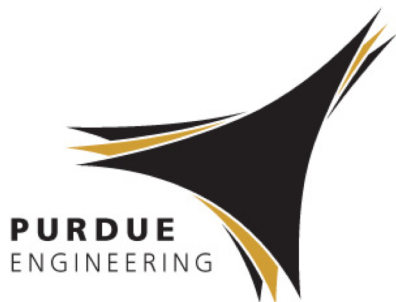
Fall, 2019

ME 323 –Mechanics of Materials

Lecture 14 – Equilibrium of beams (cont.)

Reading assignment: 5.1—5.4

News: ____



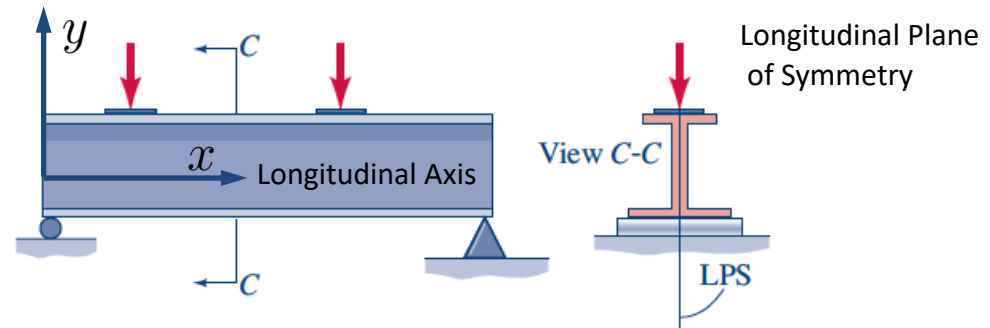
Instructor: Prof. Marcial Gonzalez

Last modified: 9/20/19 10:45:40 AM

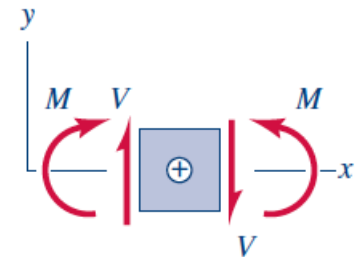
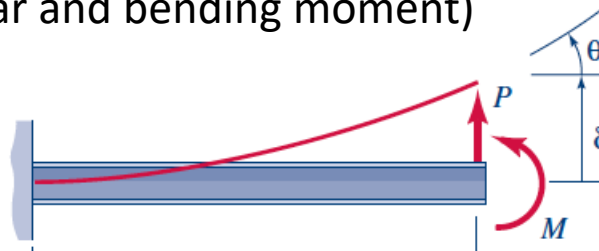
Equilibrium of beams

Beam theory (@ ME 323)

- Geometry of the solid body: straight, slender member with constant cross section that is design to support transverse loads.
- Kinematic assumptions: Bernoulli-Euler Beam Theory (Lecture 14)
Timoshenko Beam Theory, etc.
- Material behavior: isotropic linear elastic material; small deformations.



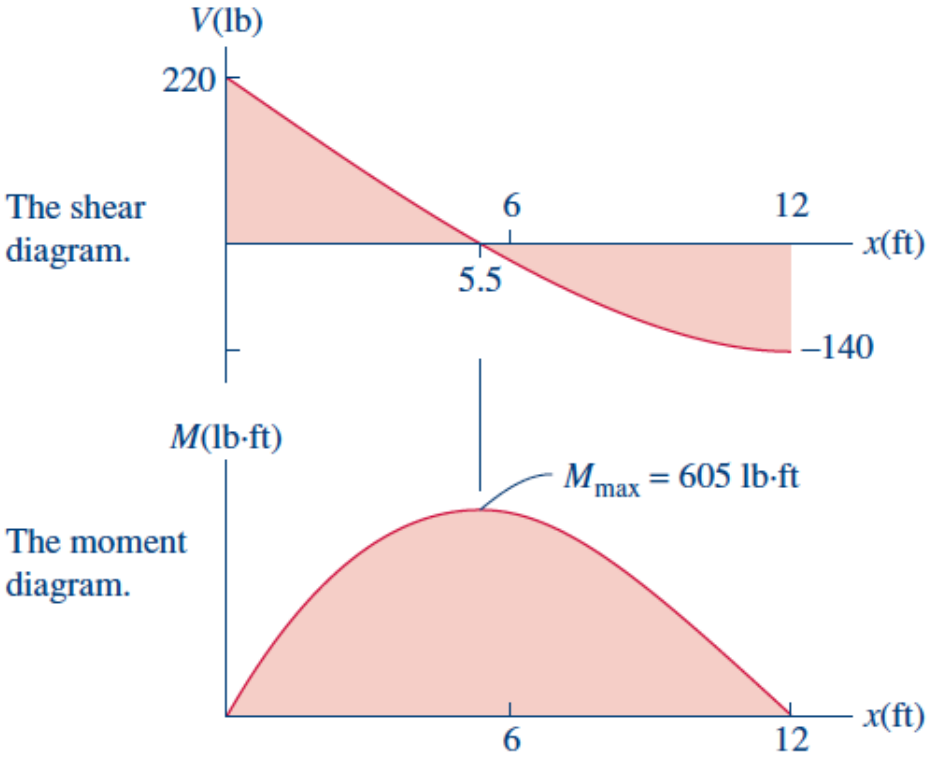
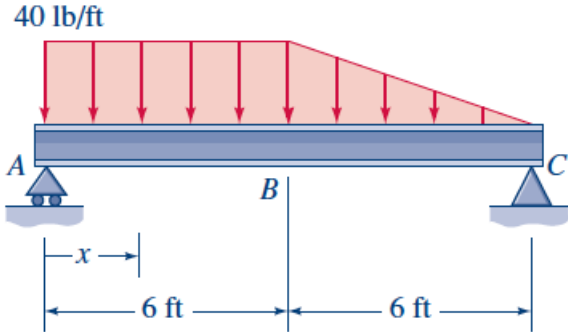
- Equilibrium:
 - 1) relate stress distribution (normal and shear stress) with internal resultants (only shear and bending moment)
 - 2) find deformed configuration



Shear and bending diagrams

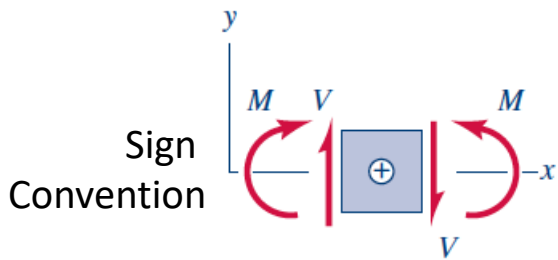
Example 19 (last lecture):

Draw shear force $V(x)$ and bending moment $M(x)$ diagrams



$$\frac{dV(x)}{dx} = p(x)$$

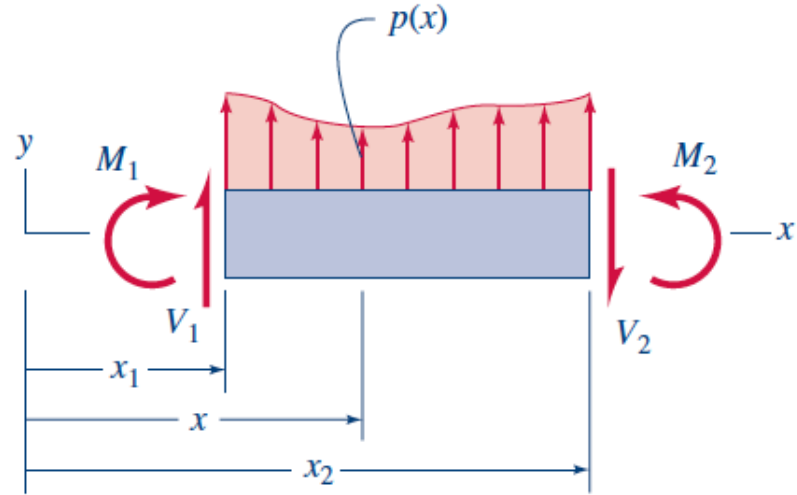
$$\frac{dM(x)}{dx} = V(x)$$



Equilibrium of beams

Equilibrium relationships

- Shear force and bending moments diagrams



Change in shear force from x_1 to x_2 is equal to the area under the load curve from 1 to 2

$$V_2 = V_{1+} + \int_{x_1}^{x_2} p(x) dx \longrightarrow V(x) = V_{1+} + \int_{x_1}^x p(\xi) d\xi \longrightarrow \frac{dV(x)}{dx} = p(x)$$

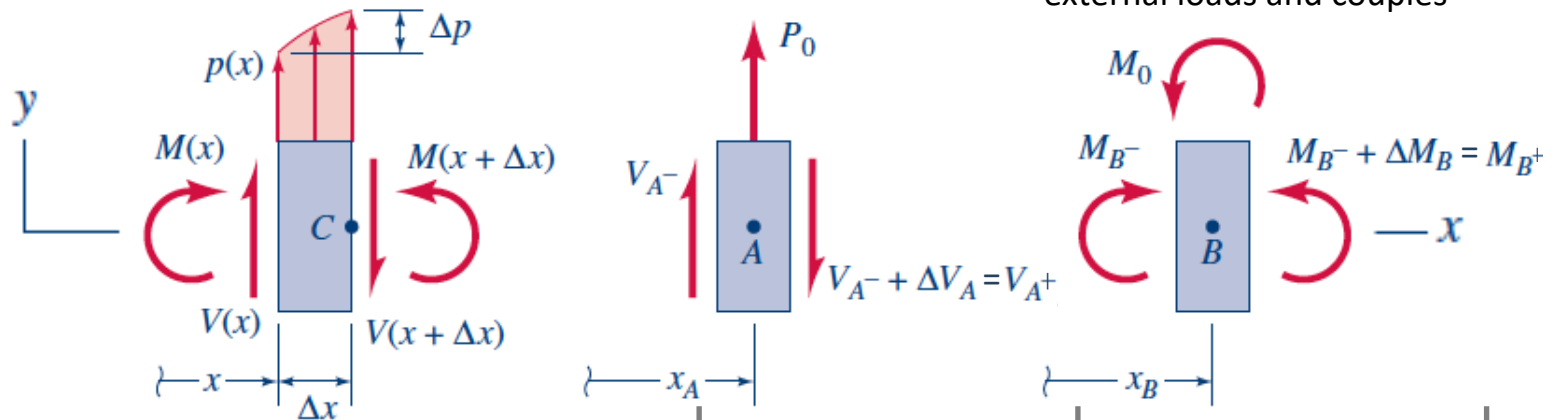
Change in bending moment from x_1 to x_2 is equal to the area under shear force curve from 1 to 2

$$M_2 = M_{1+} + \int_{x_1}^{x_2} V(x) dx \longrightarrow M(x) = M_{1+} + \int_{x_1}^x V(\xi) d\xi \longrightarrow \frac{dM(x)}{dx} = V(x)$$

Equilibrium of beams

Equilibrium relationships

- Sign convention!



$V(x) = V_{1+} + \int_{x_1}^x p(\xi) d\xi$	$\frac{dV(x)}{dx} = p(x)$	$\Delta V_A = P_0$	$\Delta V_B = 0$
$M(x) = M_{1+} + \int_{x_1}^x V(\xi) d\xi$	$\frac{dM(x)}{dx} = V(x)$	$\Delta M_A = 0$	$\Delta M_B = -M_0$

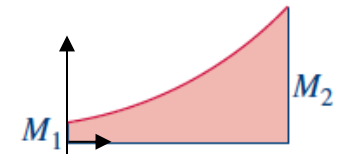
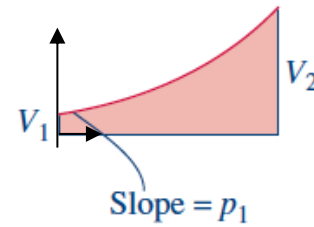
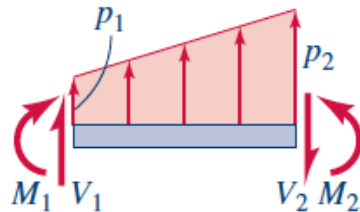
Note: become familiar with sign convention for external loads and for internal reactions.

Equilibrium of beams

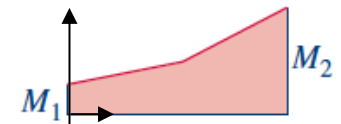
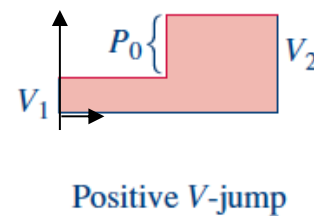
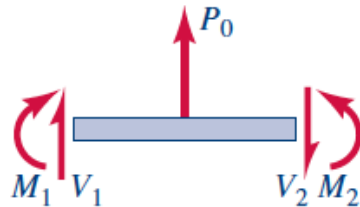
Shear and bending diagrams

- Graphical features

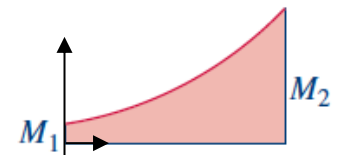
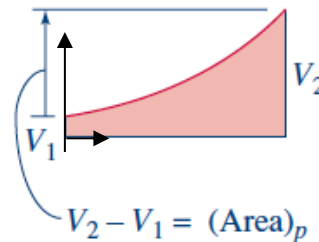
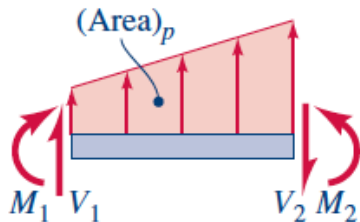
$$\frac{dV(x)}{dx} = p(x)$$



$$\Delta V = P_0$$



$$V_2 = V_1 + \int_{x_1}^{x_2} p(x) dx$$

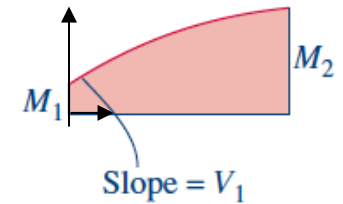
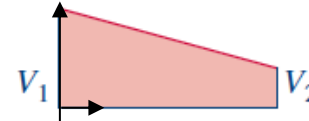
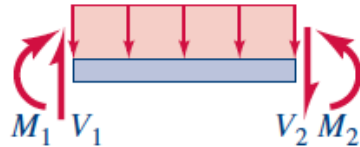


Equilibrium of beams

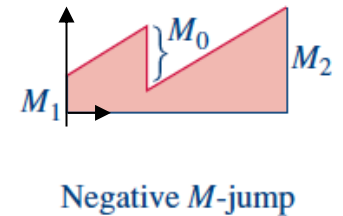
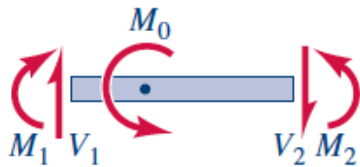
Shear and bending diagrams

- Graphical features

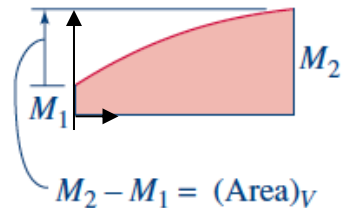
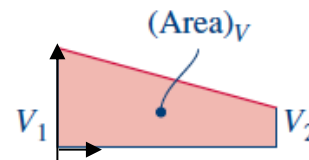
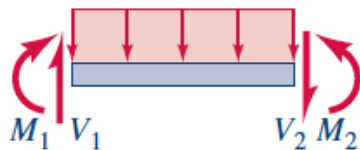
$$\frac{dM(x)}{dx} = V(x)$$



$$\Delta M = -M_0$$



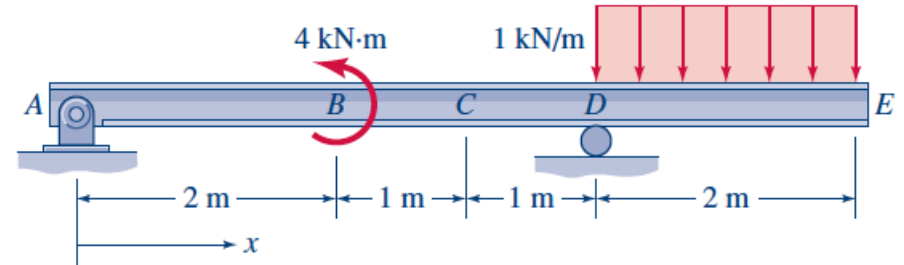
$$M_2 = M_1 + \int_{x_1}^{x_2} V(x) dx$$



Shear and bending diagrams

Example 20:

Determine $V(x)$ and $M(x)$



Equilibrium of beams

Any questions?