Fall, 2022

ME 323 – Mechanics of Materials

Lecture 14 – Strain and stress in beams

Reading assignment: Ch.10 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

Instructor: Prof. Marcial Gonzalez

Last modified: 9/24/22 12:38:31 PM

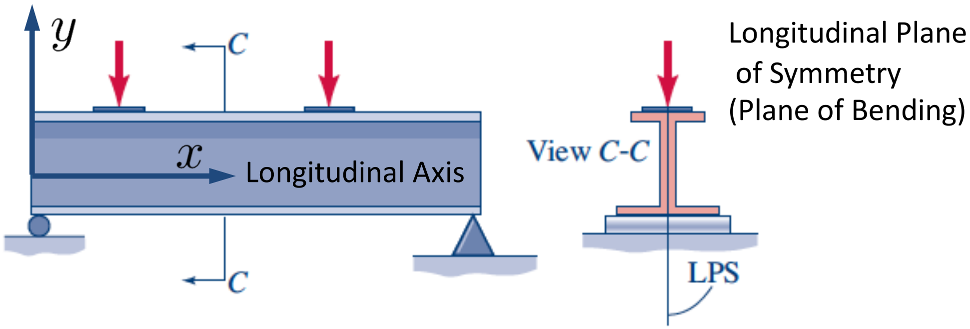
Equilibrium of beams

J. Bernoulli

L. Euler

Beam theory (@ ME 323)

- Geometry of the solid body: straight, slender member with constant cross section that is design to support transverse loads.



- Kinematic assumptions: Bernoulli-Euler Beam Theory

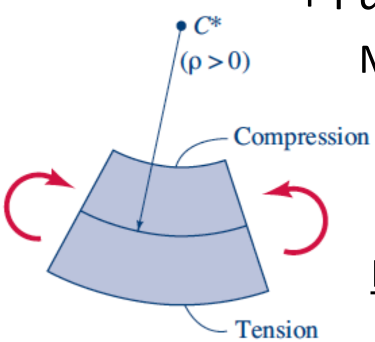
$$\epsilon_x(x,y) = -\frac{y}{\rho(x)}$$

- Material behavior: isotropic linear elastic material; small deformations.

$$\sigma_x(x,y) = E(x)\epsilon_x(x,y) = -\frac{E(x)y}{\rho(x)}$$

- Equilibrium:

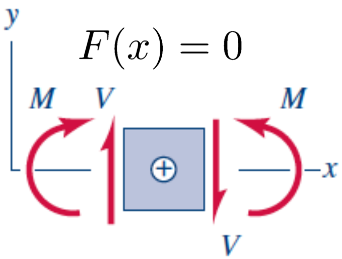
  + Pure bending ($V(x) = 0$)



Moment-curvature equation – Flexure formula

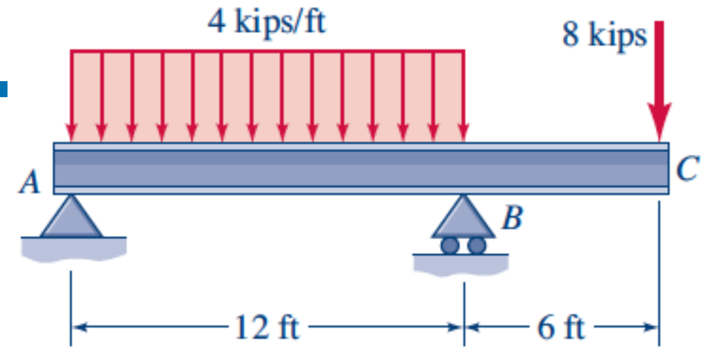
$$M(x) = \frac{E(x)I(x)}{\rho(x)} \qquad \sigma_x(x,y) = -\frac{M(x)y}{I(x)}$$

$$F(x) = 0$$



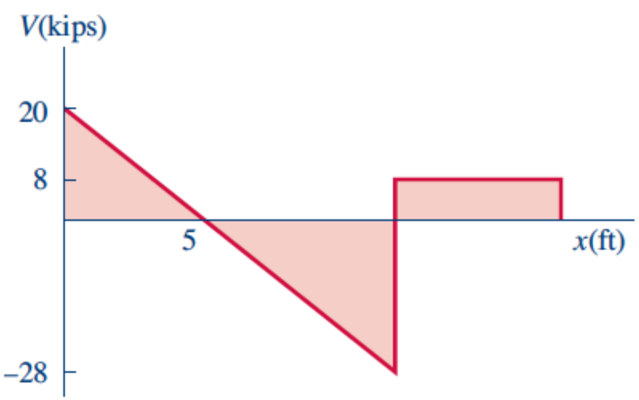
Note: the y-coordinate is measured from the centroid!!

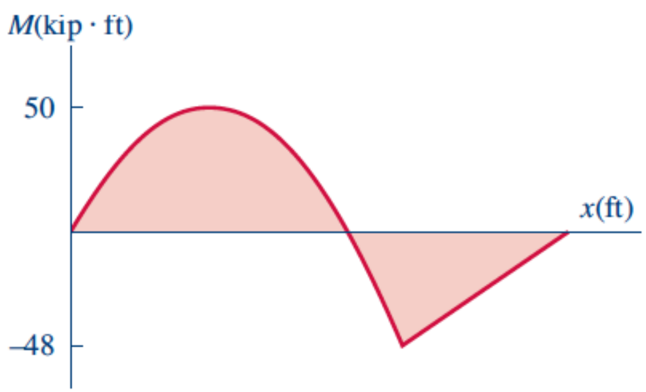
Equilibrium of beams

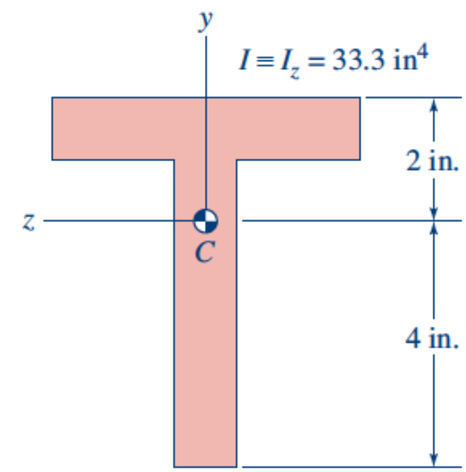


Example 21:

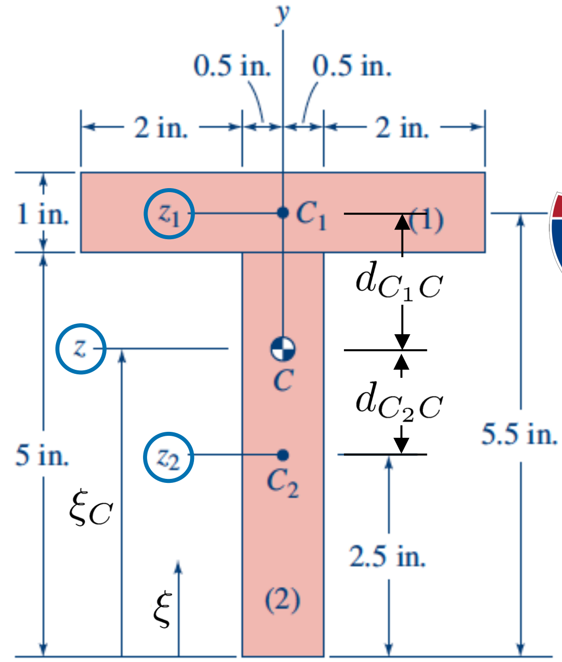
For a T-beam, determine the maximum tensile stress and the maximum compressive stress in the structure.







Equilibrium of beams

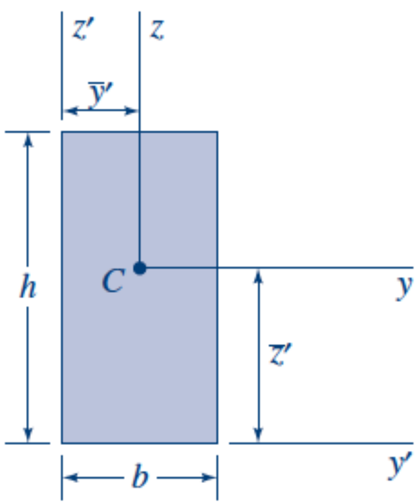


Example 22 (review Statics notes):

Determine the location of the centroid.

Determine the moment of inertia.

Rectangle



$$A = bh \qquad \bar{y}' = \frac{b}{2}$$

$$\bar{z}' = \frac{h}{2} \qquad I_y = \frac{bh^3}{12}$$

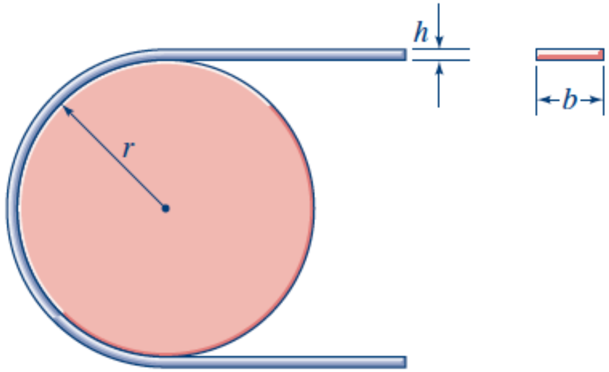
$$I_z = \frac{hb^3}{12} \qquad I_{yz} = 0$$

$$I_{y'} = \frac{bh^3}{3} \qquad I_p = \frac{bh}{12}(b^2 + h^2)$$

Equilibrium of beams

Example 23 (practice problem):

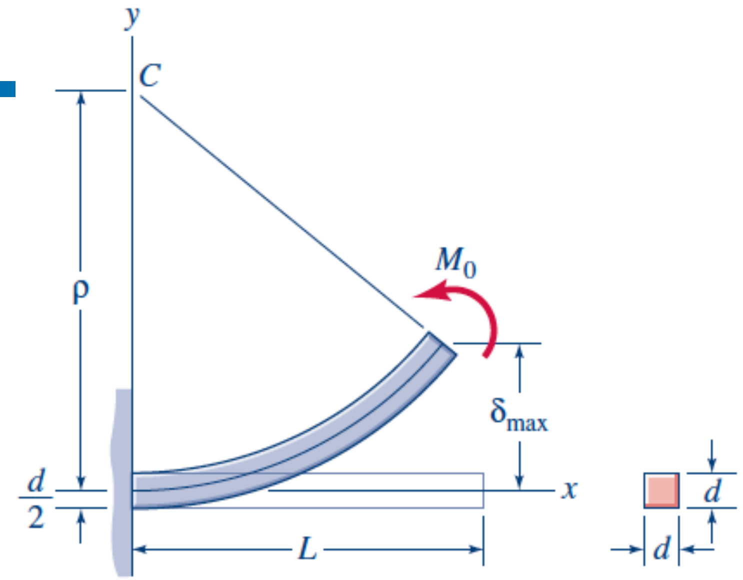
Determine the maximum tensile stress.



Equilibrium of beams

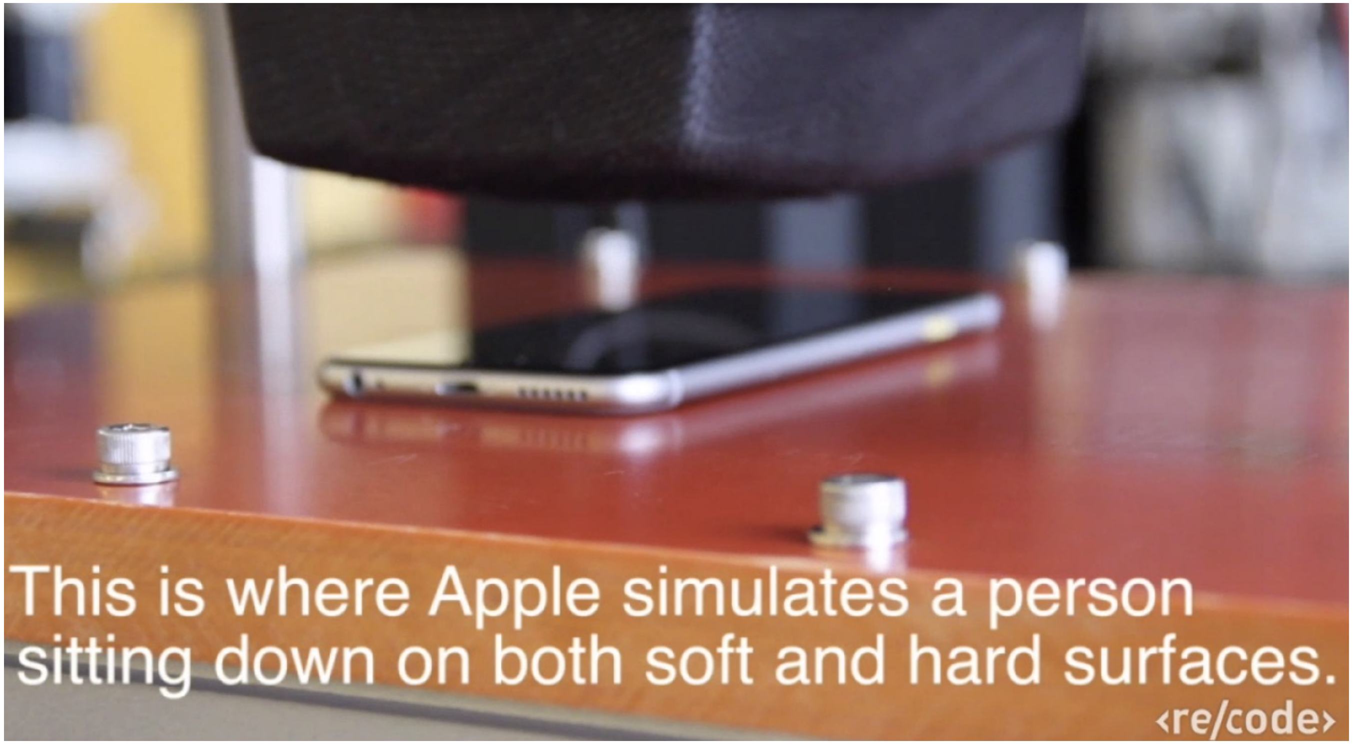
Example 24:

a) Determine the maximum tensile stress.
b) Determine the radius of curvature.
c) Determine the maximum deflection



Equilibrium of beams

Beam theory (@ Apple's test facility in Cupertino, CA)



https://youtu.be/mMJ4yZf8MoA

Equilibrium of beams

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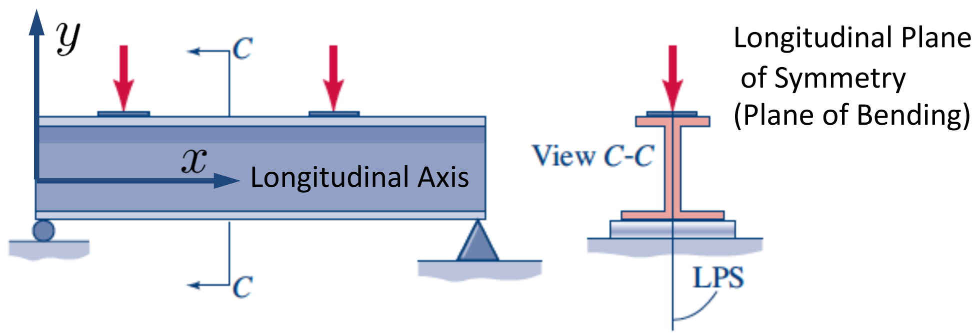

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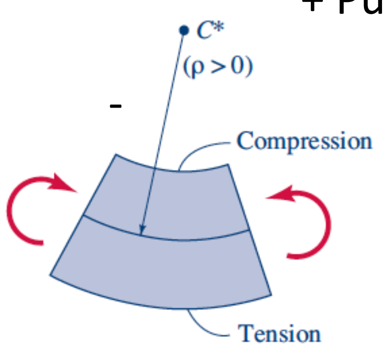

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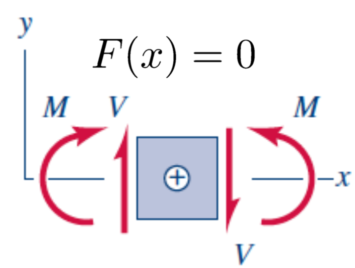
  + What if $V(x) \neq 0$?

  Most configurations will have a shear force.

  Q: What is the distribution of shear stresses?

$$F(x) = 0$$



Equilibrium of beams

Shear stress in beams

What about τ_{yx} ? Transverse and longitudinal shear stress!

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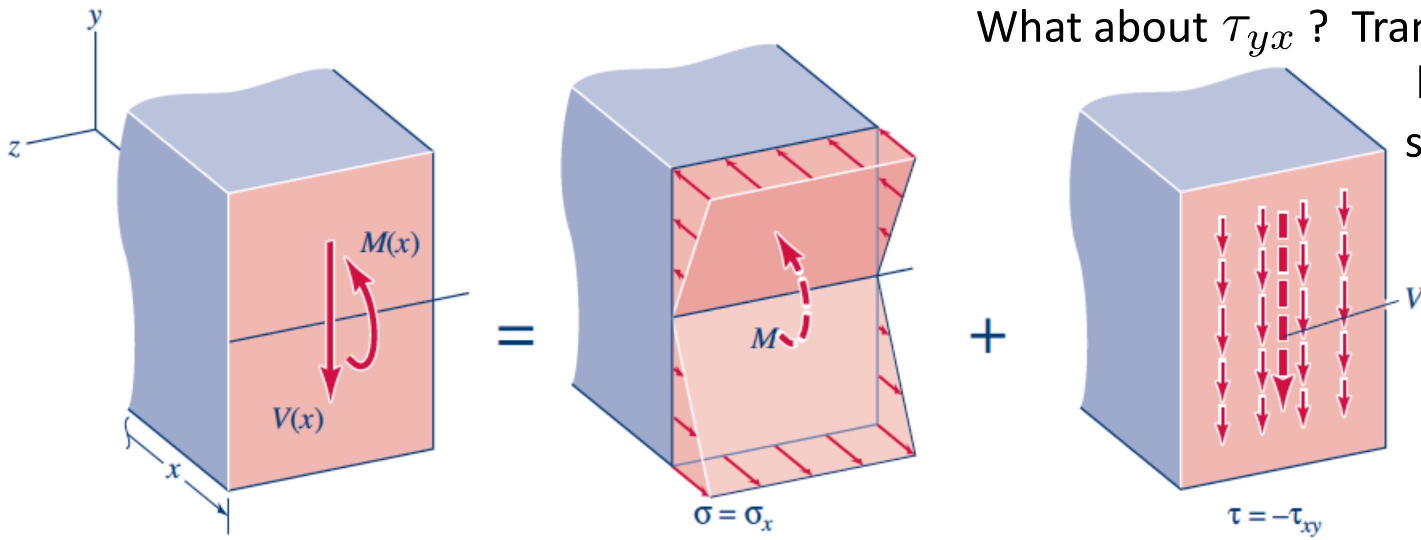

$$F = \int_A \sigma_x dA = 0 \qquad M(x) = -\int_A y\ \sigma_x dA \qquad V(x) = -\int_A \tau_{xy} dA$$

- <u>Kinematic assumptions</u>: Bernoulli-Euler Beam Theory
  - (from Lecture 13) cross sections remain plane and perpendicular to the deflection curve of the deformed beam; (**how is this possible if there are shear strains?**)
  - (now, in addition) the distribution of flexural stresses on a given cross section is *not affected* by the deformation due to shear.

Equilibrium of beams

Shear stress in beams

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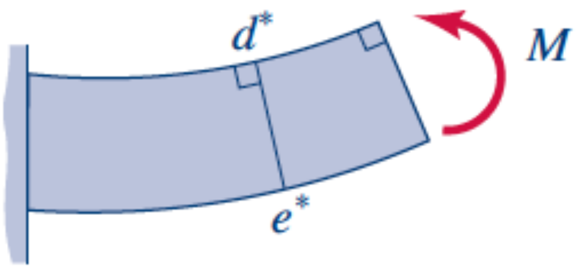
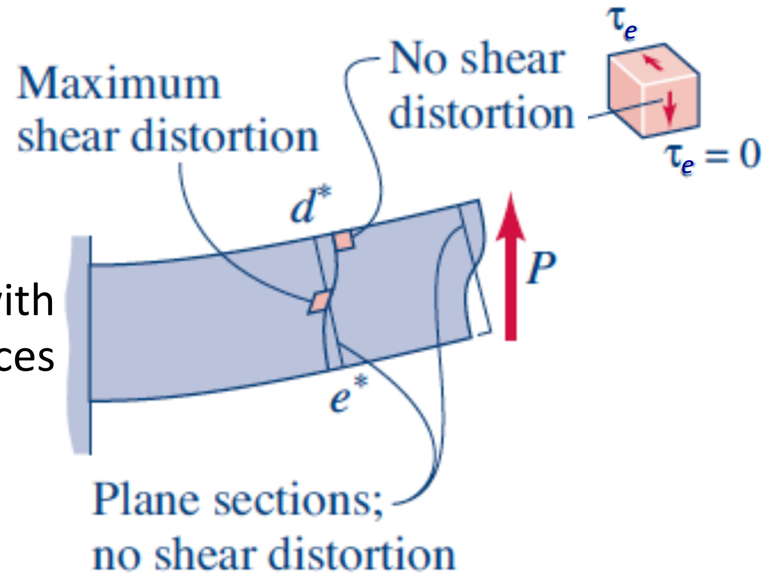
Pure bending



Bending with shear forces



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Equilibrium of beams

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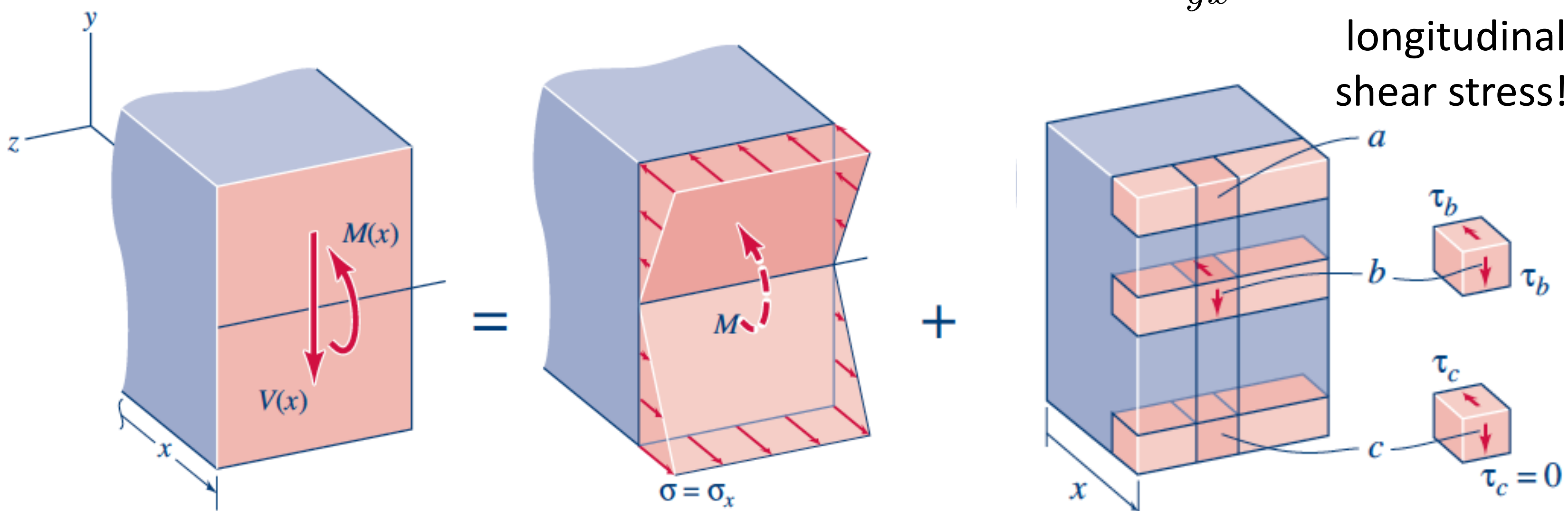

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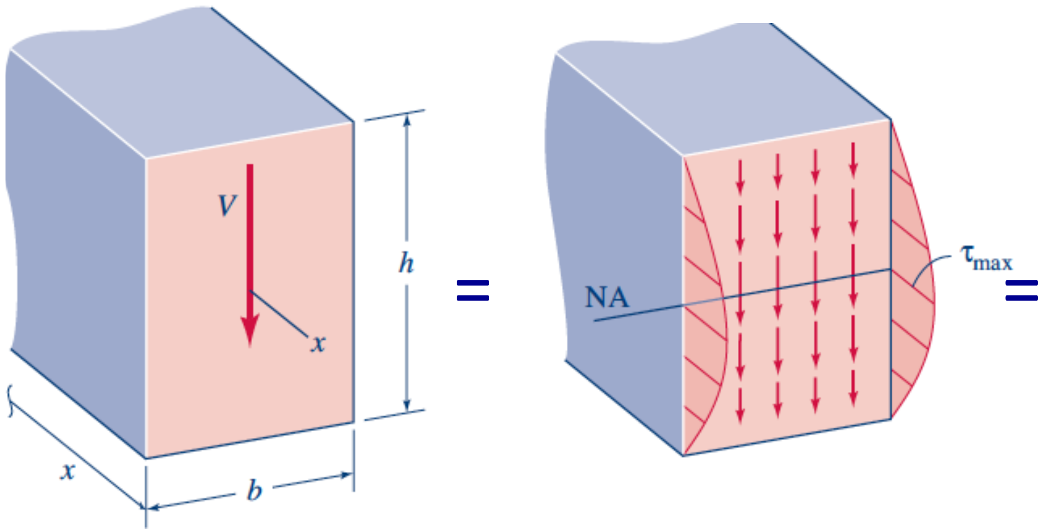
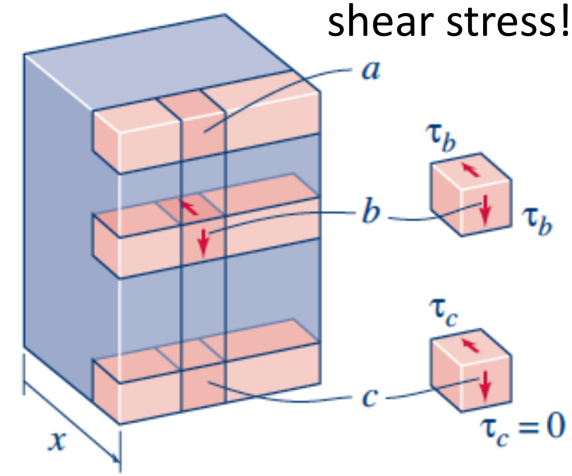
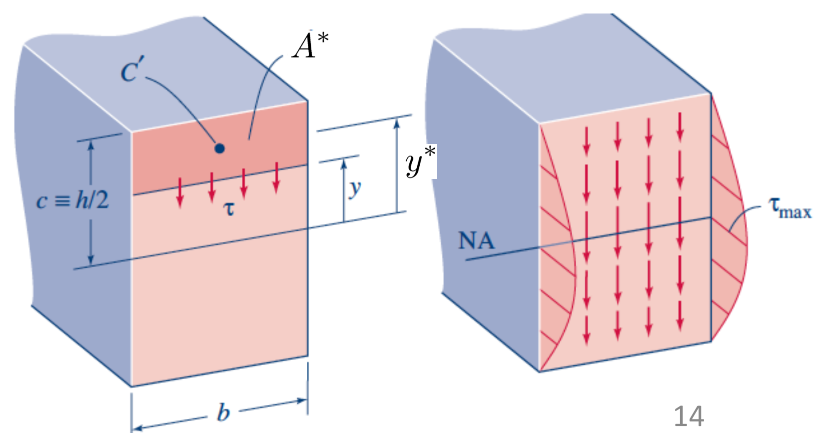




- <u>Jourawski Theory (or Collignon Theory)</u>

$$\tau_{xy}(x, y) = \frac{V(x)Q(y)}{I\ t(y)}$$ Average transverse shear stress

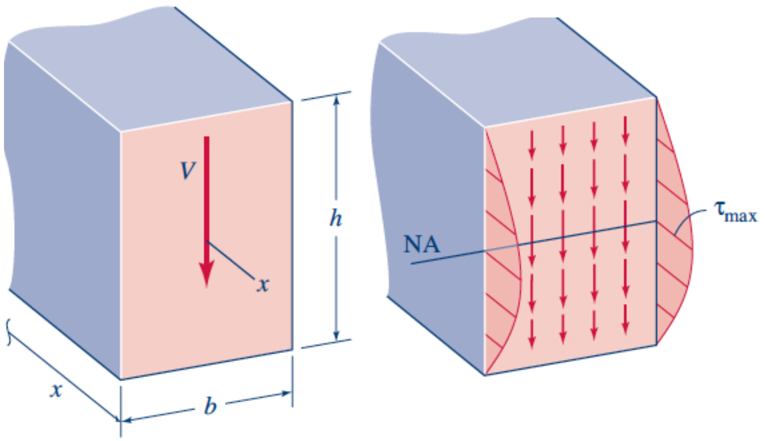
where $Q(y)$ is the first moment of area $A'(y)$ with respect to the neutral axis. $$Q(y) = \int_{A'(y)} \eta dA = A^* y^*$$



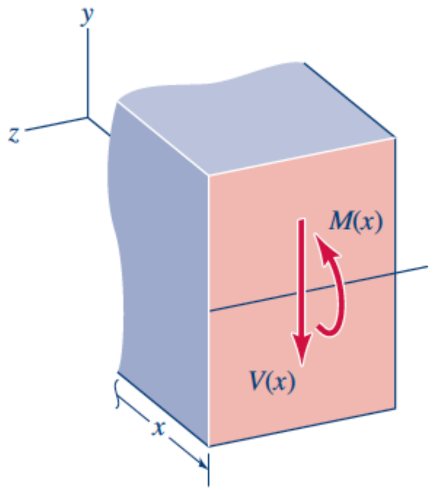
Equilibrium of beams

Example 25 (next class):

(a) Determine the maximum shear stress in the rectangular cross section.



(b) If the cross section has both bending moment and shear force as internal resultants, determine the state of stress at five points in the cross section.



Equilibrium of beams

Any questions?