Fall, 2022

ME 323 – Mechanics of Materials

Lecture 17 – Deflection of beams (cont.)

Reading assignment: Ch.9, Ch.11 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

Instructor: Prof. Marcial Gonzalez

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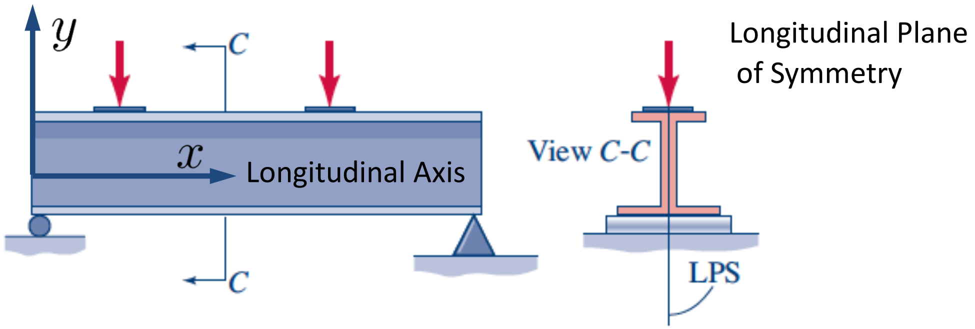
Deflection of beams

Beam theory (@ ME 323)

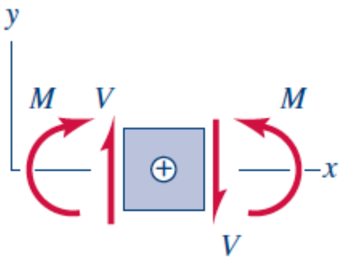
- Geometry of the solid body: straight, slender member with constant cross section that is design to support transverse loads.



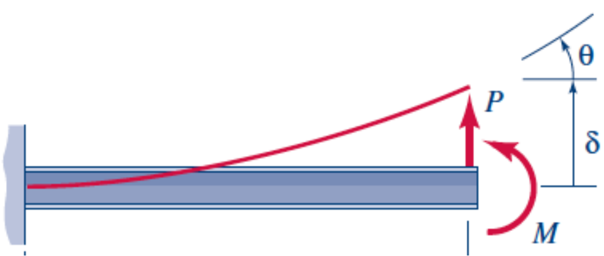
- Kinematic assumptions: Bernoulli-Euler Beam Theory

- Material behavior: isotropic linear elastic material; small deformations.

- Equilibrium:
  1) relate stress distribution (normal and shear stress) with internal resultants (only shear and bending moment)



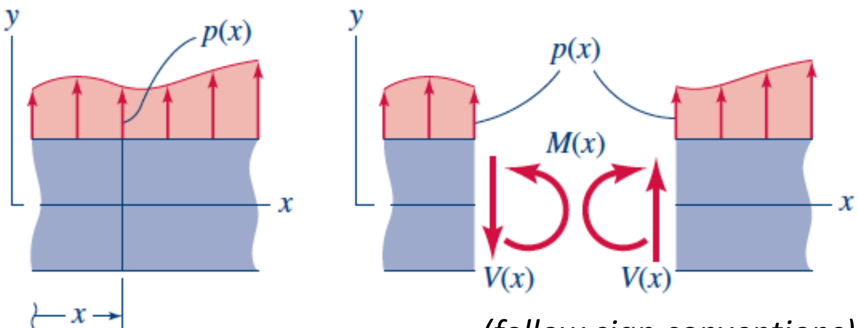
  2) find deformed configuration



Deflection of beams

Load-deflection equations

$$V'(x) = p(x) \qquad M'(x) = V(x)$$

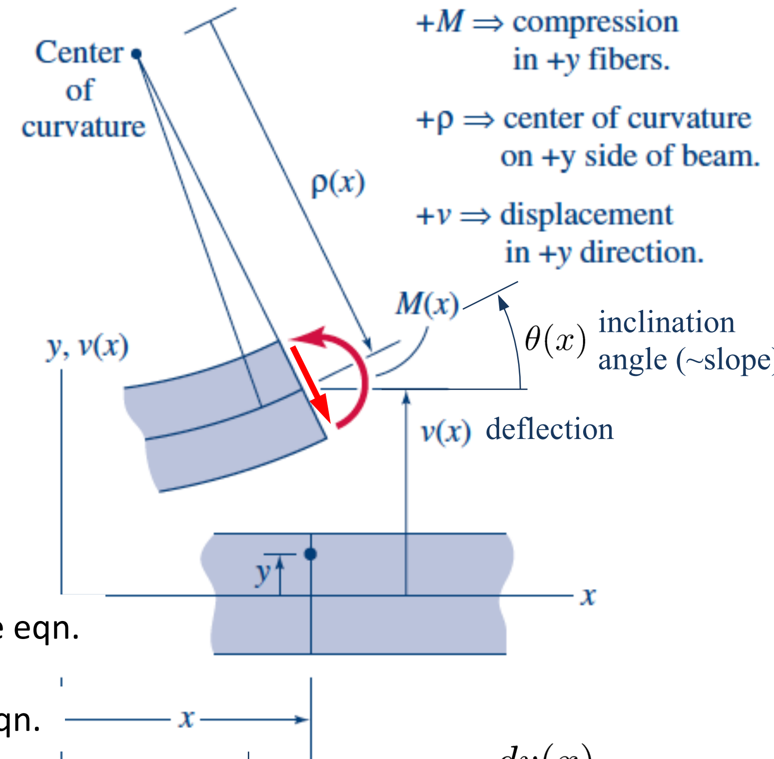


(follow sign conventions)

$$M(x) = E(x)I(x)v''(x)$$ Moment-curvature eqn.

$$V(x) = (E(x)I(x)v''(x))'$$ Shear-deflection eqn.

$$p(x) = (E(x)I(x)v''(x))''$$ Load-deflection eqn.



$$\theta(x) \approx \frac{dv(x)}{dx}$$

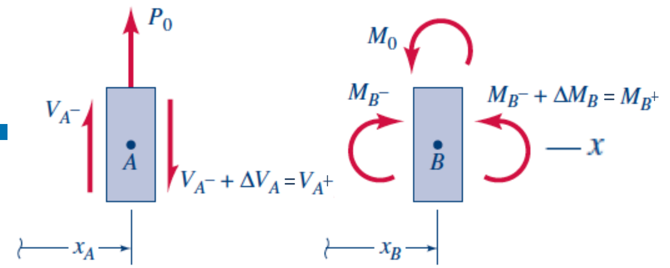
$$\frac{1}{\rho(x)} \approx \frac{d^2v(x)}{dx^2}$$

(constant cross-section and material properties)

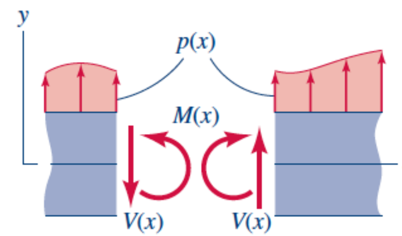
$$M = EIv'' \qquad V = EIv''' \qquad p = EIv''''$$

(2nd order) (4th order)

Deflection of beams

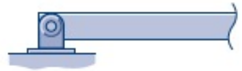
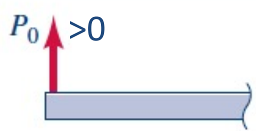
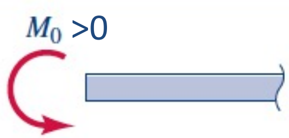


$$M(x_0^+) = M(x_0^-) - M_0$$

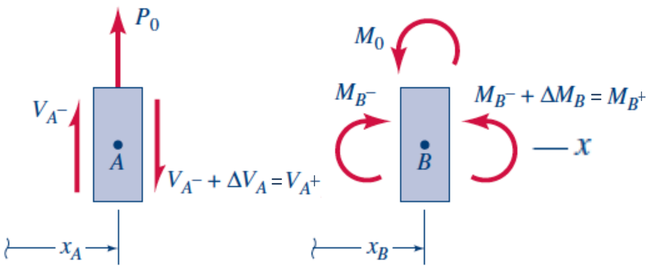


(follow sign conventions)

Boundary conditions

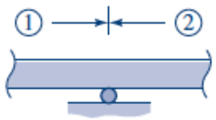
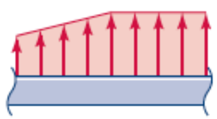
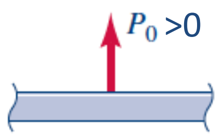
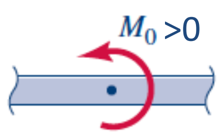
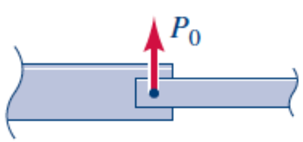
	Type	Symbol*	2nd Order	4th Order
BC	Fixed end		$v = 0$ $v' = 0$	$v = 0$ $v' = 0$
	Simple support		$v = 0$	$v = 0$ $M = 0$
	Free end		No BC	$V = 0$ $M = 0$
	Concentrated force		No BC	$V = P_0$ $M = 0$
	Concentrated couple		No BC	$V = 0$ $M = -M_0$
	Constrained rotation end		$v' = 0$	$V = 0$ $v' = 0$

Deflection of beams

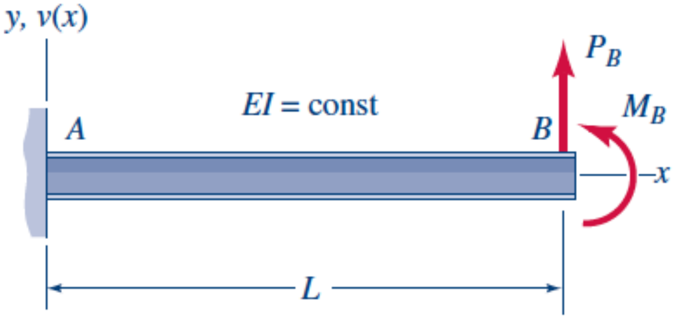


$$V(x_0^+) = V(x_0^-) + P_0 \qquad M(x_0^+) = M(x_0^-) - M_0$$

Continuity conditions

	Type	Symbol	2nd Order	4th Order
CC	Roller		$v_1 = v_2 = 0$ $v_1' = v_2'$	$v_1 = v_2 = 0$ $v_1' = v_2'$ $M_1 = M_2$
	Discontinuity in load function		$v_1 = v_2$ $v_1' = v_2'$	$v_1 = v_2$ $v_1' = v_2'$ $V_1 = V_2$ $M_1 = M_2$
	Concentrated force		$v_1 = v_2$ $v_1' = v_2'$	$v_1 = v_2, \quad v_1' = v_2'$ $V_2 - V_1 = P_0$ $M_1 = M_2$
	Concentrated couple		$v_1 = v_2$ $v_1' = v_2'$	$v_1 = v_2, \quad v_1' - v_2'$ $V_1 = V_2$ $M_2 - M_1 = -M_0$
	Pin, with force		$v_1 = v_2$	$v_1 = v_2$ $V_2 - V_1 = P_0$ $M_1 = M_2 = 0$

Deflection of beams



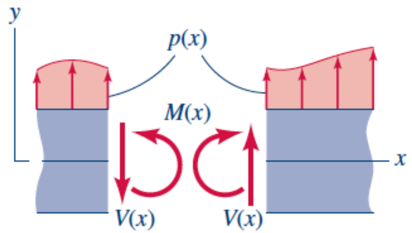
Example 30:

The beam shown in the figure is completely fixed at end A. Determine an expression for the deflection curve $v(x)$, and draw the shear force and bending moment diagrams.

Use the second-order method.

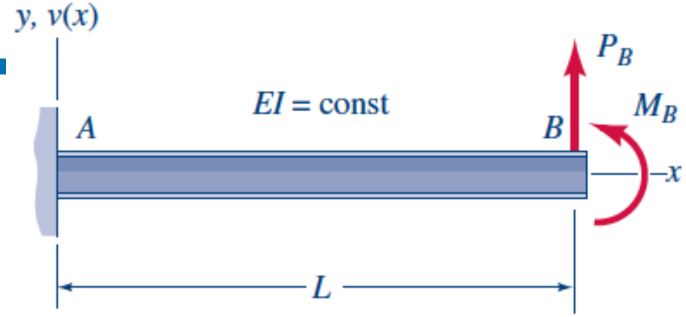
$$V(x_0^+) = V(x_0^-) + P_0$$

$$M(x_0^+) = M(x_0^-) - M_0$$



(follow sign conventions)

Deflection of beams



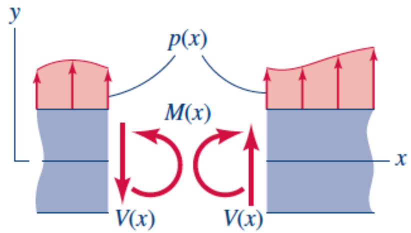
Example 31 (Practice problem):

The beam shown in the figure is completely fixed at end A. Determine an expression for the deflection curve $v(x)$.

Use the fourth-order method (4 unknowns, 4 B.C.).

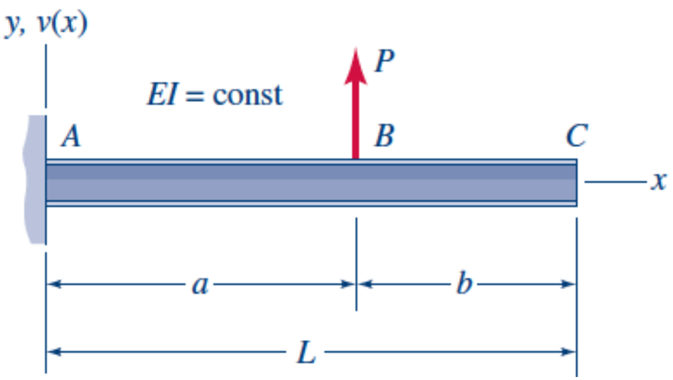
$$V(x_0^+) = V(x_0^-) + P_0$$

$$M(x_0^+) = M(x_0^-) - M_0$$



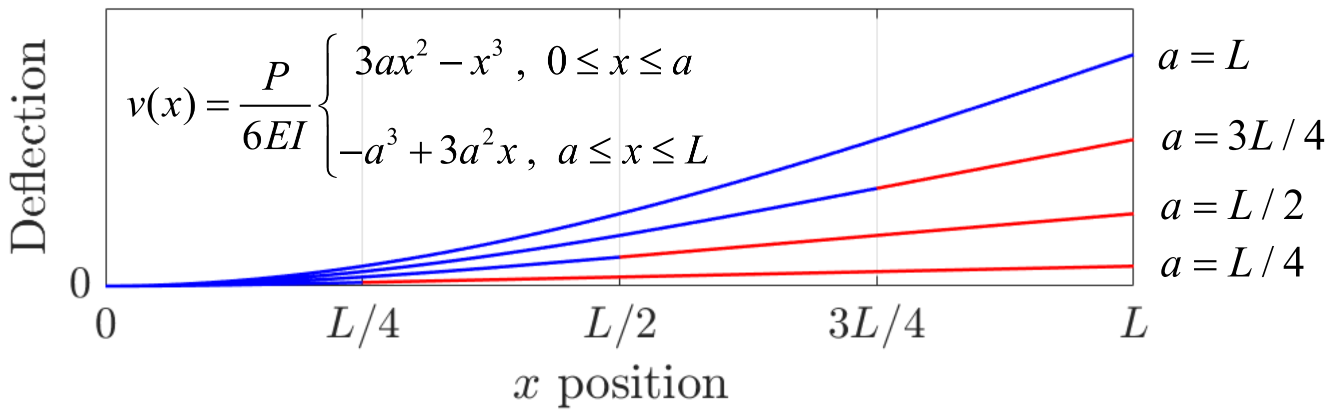
(follow sign conventions)

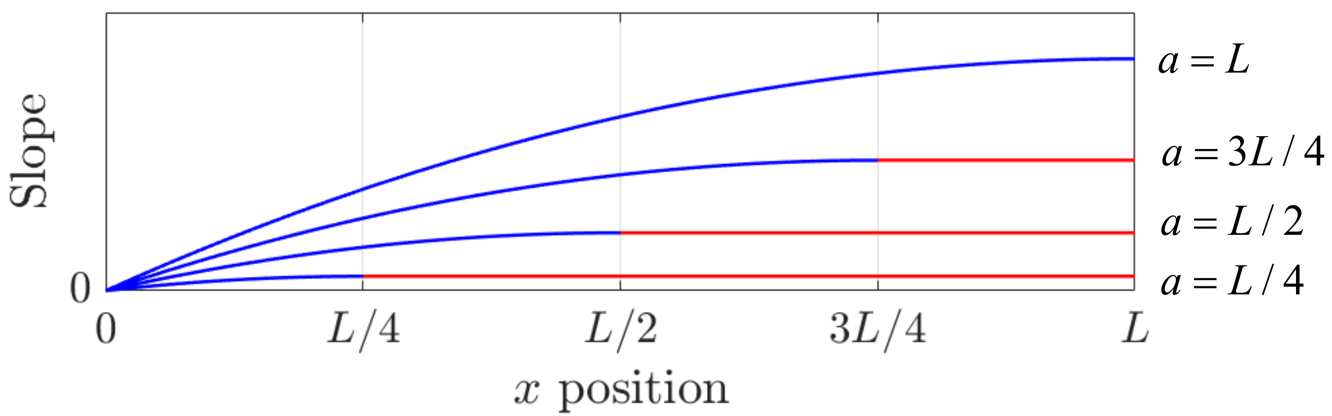
Deflection of beams



Example 32:

The beam shown in the figure is completely fixed at end A. Determine an expression for the deflection curve $v(x)$.





Deflection of beams – Indeterminate problems

Outline for 2nd order method (determinate or indeterminate):

- FBD
- Equilibrium for external forces and couples
- Find internal moment $M(x)$ for each section
- Integrate moment-curvature equation $EIv''(x) = M(x)$
- Apply boundary and continuity conditions
- Solve for unknowns
- Check units!
- Draw (i) deflection curve, (ii) shear force diagram, and (iii) bending moment diagram

Deflection of beams

Any questions?