Fall, 2022

ME 323 – Mechanics of Materials

Lecture 21 – Deflection of beams (cont.)

Reading assignment: Ch.9, Ch.11 lecturebook

PURDUE UNIVERSITY | Mechanical Engineering

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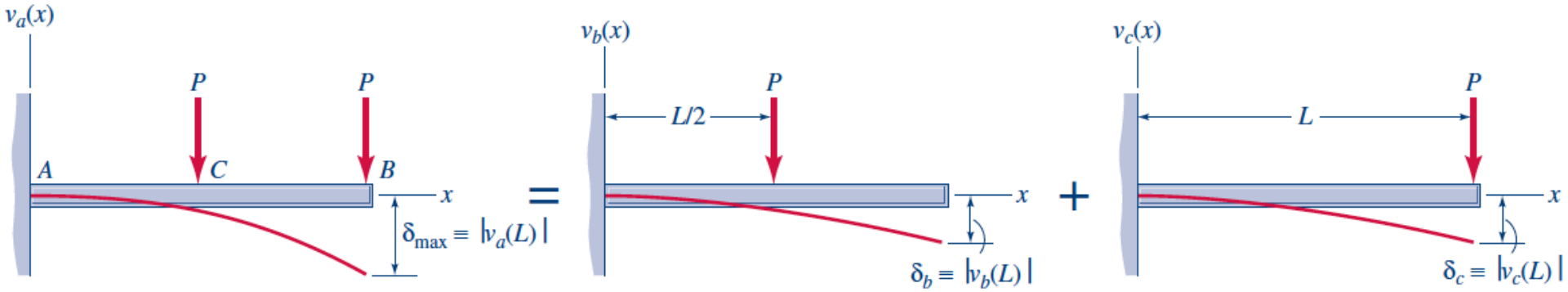
Deflection of beams

Method of superposition

The slope, deflection, reactions, internal shear and bending moment of a beam that simultaneously supports several different loads can be obtained by linear superposition, that is, by addition of the effect of the loads acting separately.

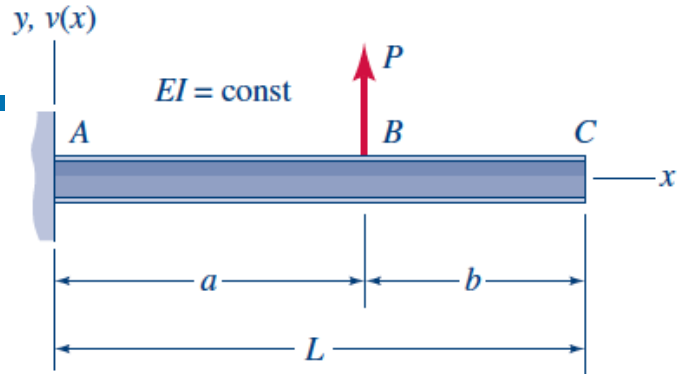
Recall: the principle of superposition can be used for linear elastic materials under small deformations.

Note: each individual sub-problem is in equilibrium, then their superposition is in equilibrium.



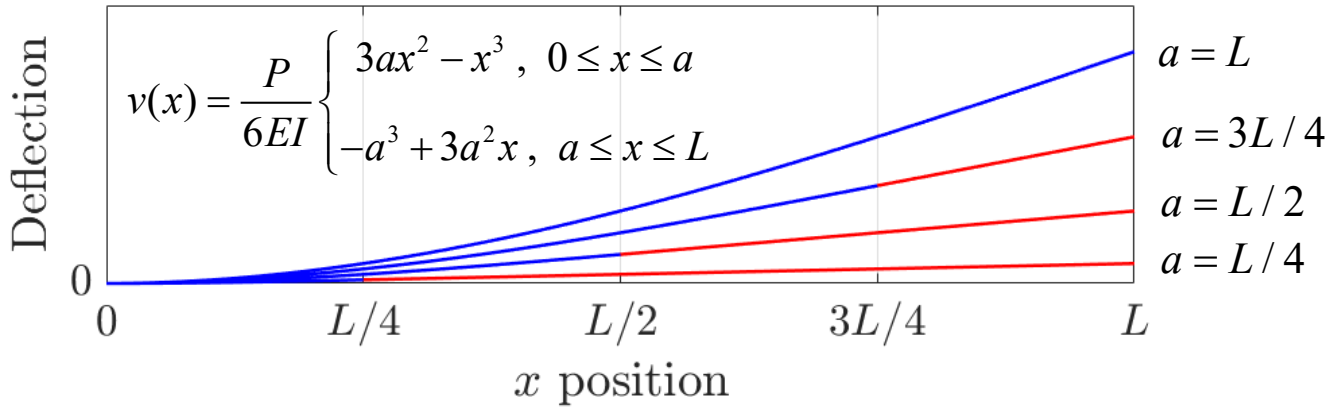
$$v_a(x) = v_b(x) + v_c(x)$$

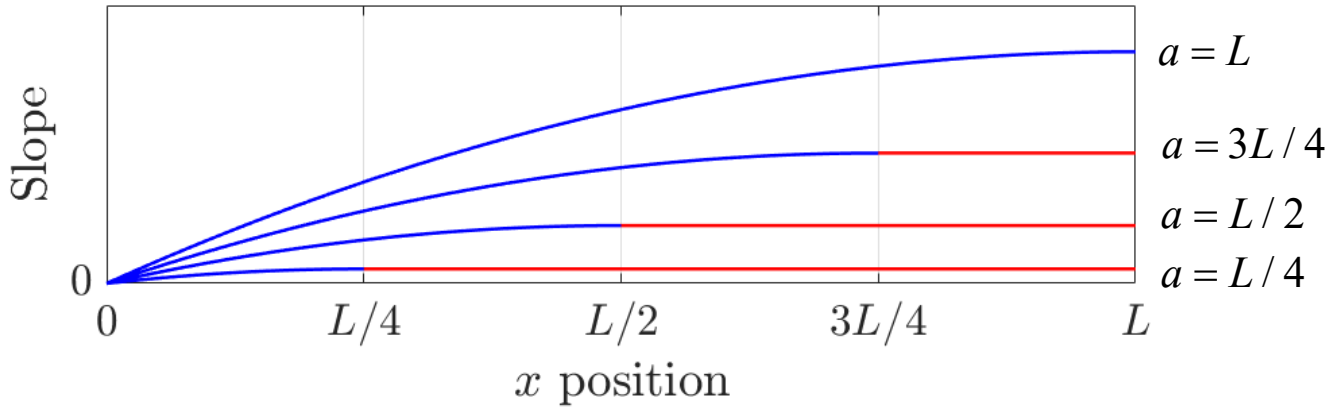
Deflection of beams



Example 31 (From Lecture 21)

The beam shown in the figure is completely fixed at end A. Determine an expression for the deflection curve $v(x)$.



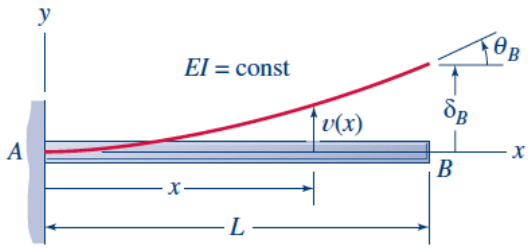


Deflection of beams

Method of superposition

DEFLECTIONS AND SLOPES OF BEAMS; FIXED-END ACTIONS

E



Notation

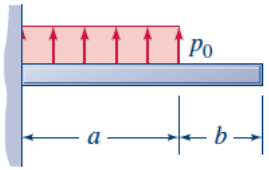
$v(x)$ = deflection in the y direction

$v'(x)$ = slope of the deflection curve

$\delta_B \equiv v(L)$ = deflection at end B

$\theta_B \equiv v'(L)$ = slope at end B

6



$$v = \frac{p_0 x^2}{24EI}(6a^2 - 4ax + x^2) \qquad 0 \le x \le a$$

$$v' = \frac{p_0 x}{6EI}(3a^2 - 3ax + x^2) \qquad 0 \le x \le a$$

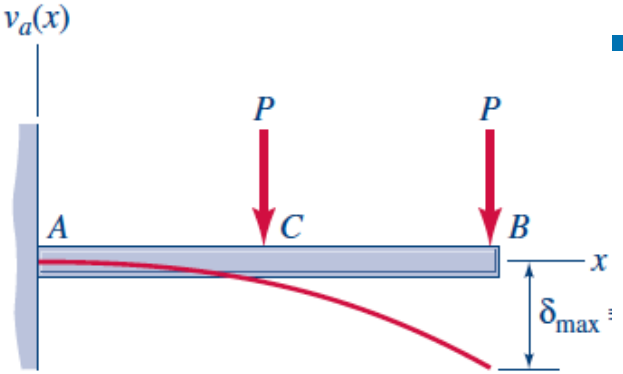
$$v = \frac{p_0 a^3}{24EI}(4x - a) \qquad v' = \frac{p_0 a^3}{6EI} \qquad a \le x \le L$$

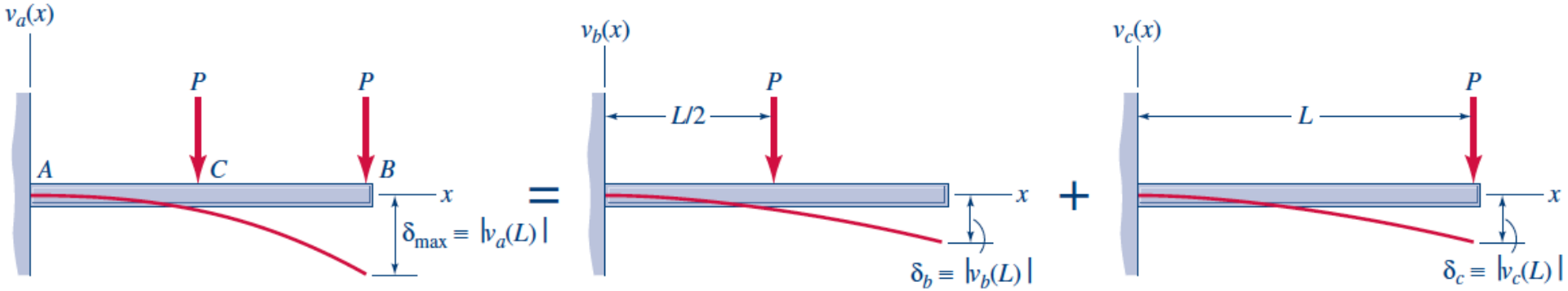
$$\delta_B = \frac{p_0 a^3}{24EI}(4L - a) \qquad \theta_B = \frac{p_0 a^3}{6EI}$$

Deflection of beams

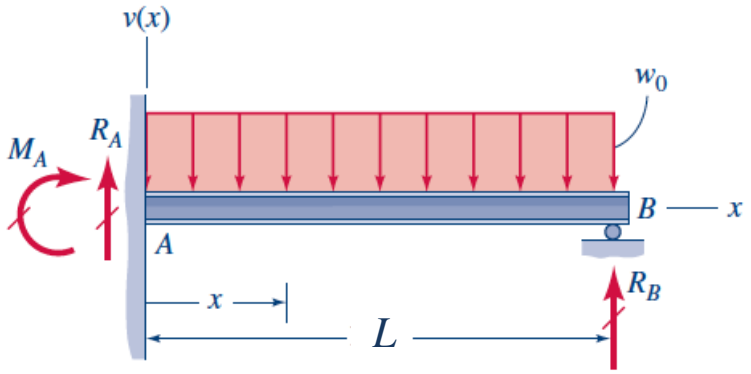
Example 35:

Determine an expression for the deflection curve $v(x)$ using the superposition method.



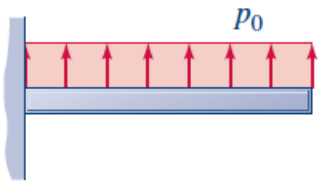


Deflection of beams



Example 36:

Determine an expression for the deflection curve $v(x)$ using the superposition method.



$$v = \frac{p_0 x^2}{24EI}(6L^2 - 4Lx + x^2)$$

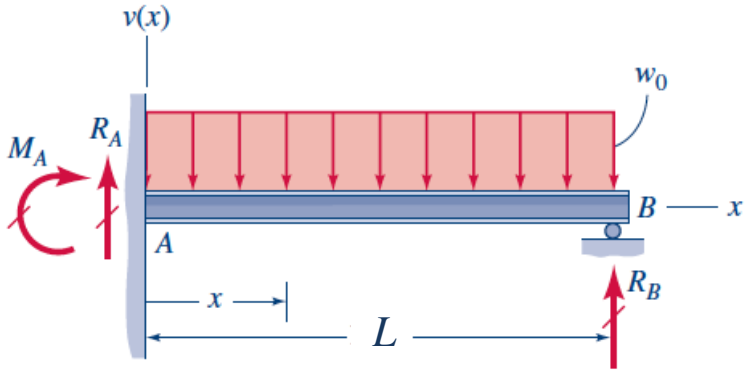
$$v' = \frac{p_0 x}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{p_0 L^4}{8EI} \qquad \theta_B = \frac{p_0 L^3}{6EI}$$

$$v = \frac{Px^2}{6EI}(3L - x) \qquad v' = \frac{Px}{2EI}(2L - x)$$

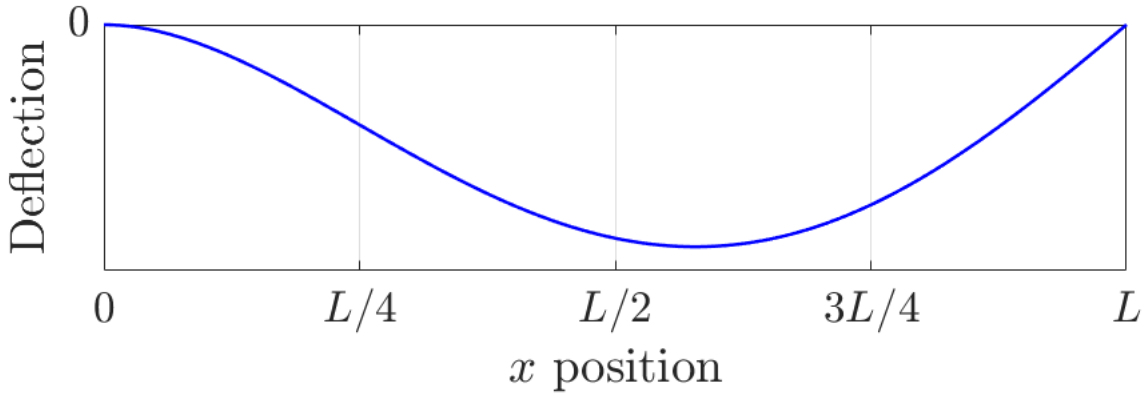
$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

Deflection of beams

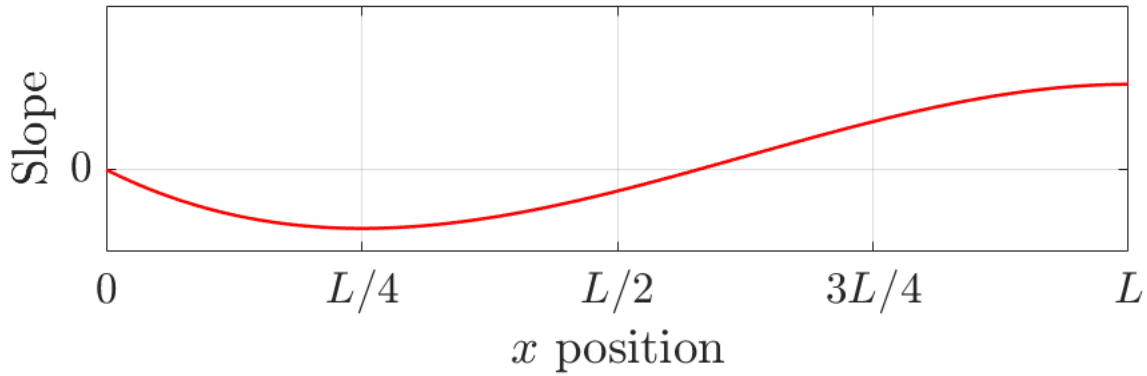


Example 36 (cont.):

Determine an expression for the deflection curve $v(x)$ using the superposition method.

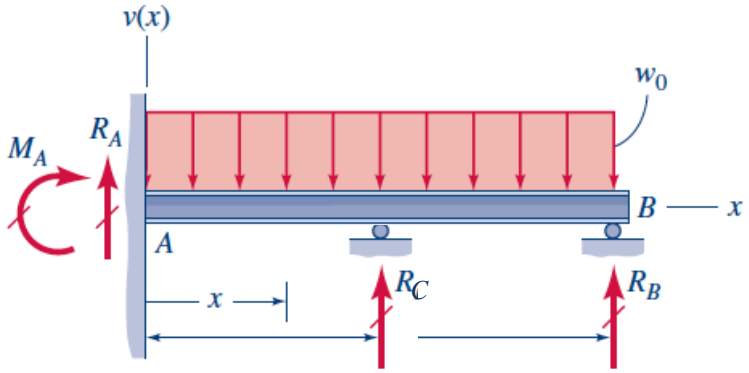


$$v(x) = \frac{w_0}{48EI}\left(-3L^2x^2 + 5Lx^3 - 2x^4\right)$$



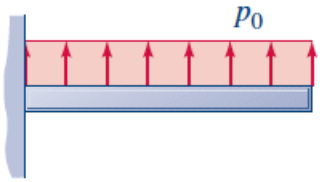
$$v'(x) = \frac{w_0}{48EI}\left(-6L^2x + 15Lx^2 - 8x^3\right)$$

Deflection of beams



Example 37:

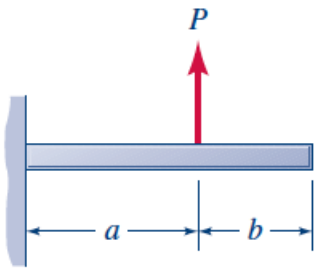
Determine an expression for the deflection curve $v(x)$ using the superposition method.



Previous slide …

$$v = \frac{Px^2}{6EI}(3L - x) \qquad v' = \frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$



$$v = \frac{Px^2}{6EI}(3a - x) \qquad v' = \frac{Px}{2EI}(2a - x) \qquad 0 \le x \le a$$

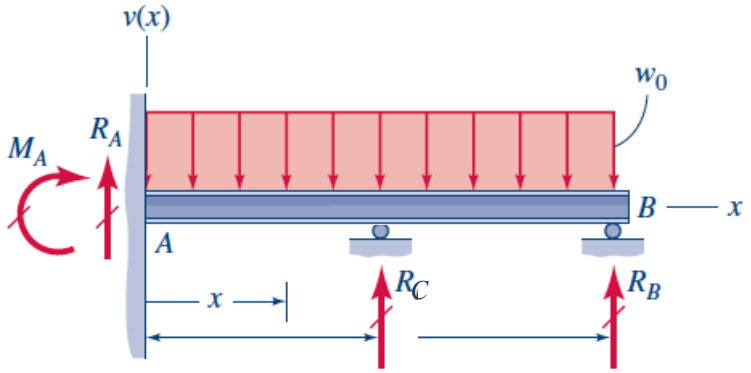
$$v = \frac{Pa^2}{6EI}(3x - a) \qquad v' = \frac{Pa^2}{2EI} \qquad a \le x \le L$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

Deflection of beams

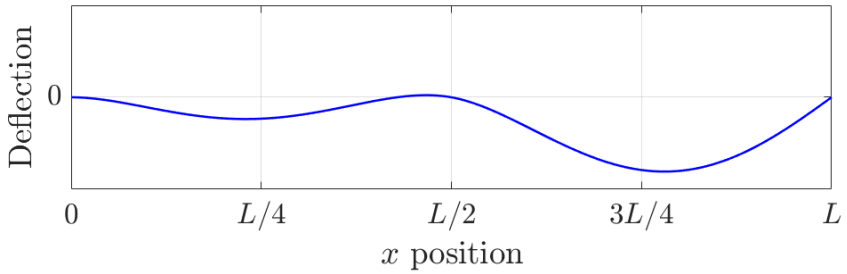
Example 37 (cont.):

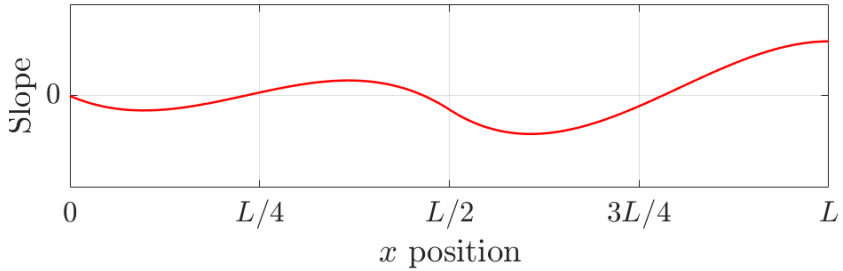
Determine an expression for the deflection curve $v(x)$ using the superposition method.



$$v(x) = \frac{w_0}{336EI}\begin{cases} -3L^2x^2 + 13Lx^3 - 14x^4 & , \quad 0 \le x \le L/2 \\ -4L^4 + 24L^3x - 51L^2x^2 + 45Lx^3 - 14x^4 & , \quad L/2 \le x \le L \end{cases}$$

$$v'(x) = \frac{w_0}{336EI}\begin{cases} -6L^2x + 39Lx^2 - 56x^3 & , \quad 0 \le x \le L/2 \\ 24L^3 - 102L^2x + 135Lx^2 - 56x^3 & , \quad L/2 \le x \le L \end{cases}$$





Deflection of beams

Any questions?